Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 7 (7): 1071-1077, 2007
ISSN 1812-5654

Blood Flow Through an Artery with Mild Stenosis: A Two-layered Model, Different Shapes of Stenoses and Slip Velocity at the Wall

R. Ponalagusamy
Department of Mathematics, National Institute of Technology,
Tiruchirappalli-620 015, Tamilnadu, India

Abstract: A mathematical model for blood flow through stenosed arteries with axially variable peripheral layer thickness and variable slip at the wall has been considered. The model consists of a core surrounded by a peripheral layer. It is assumed that the fluids of both the regions (core and peripheral) are Newtonian having different viscosities. For such models, in literature, the peripheral layer thickness and slip are assumed a priori based on experimental observations. In the present analysis, analytic expressions for the thickness of the peripheral layer, slip and core viscosity have been obtained in terms of measurable quantities (flow rate (Q), centerline velocity (U), pressure gradient (-dp/dz) and plasma viscosity (μ_p)). Using the experimental values of Q, U, (-dp/dz) and μ_p the values of the peripheral layer thickness, red cell concentration in the core, core viscosity and slip velocity at the wall have been determined. The theoretically obtained peripheral layer thickness has been compared with its experimental value. It is found that the agreement between the two is very good (error <1.0%). It is important to mention that in the present analysis, core viscosity has been obtained by two methods. First by calculating from the formula obtained in the present analysis and the second by calculating the red cell concentration in the core and then using concentration versus relative viscosity curve. A comparison of these two values of the core viscosity shows a reasonably good agreement between them (difference up to 14%). The analysis developed here could be used to determine the more accurate values of the apparent viscosity of blood, agreeability, rigidity and deformability of red cells. This information of blood could be useful in the development of new diagnosis tools for many diseases.

Key words: Blood flow, slip velocity, stenosed arteries, a two-fluid model, different shapes of stenoses

INTRODUCTION

Many investigators (Chaturani and Kaloni, 1976; Chaturani and Upadhya, 1979; Shukla *et al*., 1980b; Majhi and Usha, 1984; Chaturani and Biswas, 1983; Philip and Chandran, 1996) have theoretically studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity or peripheral plasma layer thickness on the flow variables such as velocity, wall shear stress and flow resistance. In these models, the peripheral layer thickness and slip velocity are assumed a priori based upon the experimental observations. Wang and Bassingthwaighte (2003) have considered a two fluid model for blood flow through a slightly curved tube by assuming a constant peripheral plasma layer. Using experimental data, they have calculated curve-fitted values of peripheral layer thickness and the ratio of core viscosity to the peripheral plasma viscosity. It would be of interest to obtain the analytic expression for them (slip and peripheral layer thickness) in terms of the measurable flow variables (flow rates, pressure gradient, etc.).

To understand the flow patterns in stenosed arteries, Young (1968), Macdonald (1979) and Deshpande *et al.* (1979) etc., have analyzed the flow of blood through an arterial stenosis. Lee and Fung (1970) have obtained the numerical results for the streamlines and distribution of velocity, pressure, vorticity and the shear stress for different Reynolds number in blood flow through locally constricted tubes. In these models, the flow of blood is represented by one-layered model. Bugliarello and sevilla (1970) and Bugliarello and Hayden (1963) have experimentally observed that when blood flows through narrow tubes there exists a cell free plasma layer near the wall. In view of their experiments, it is preferable to represent the flow of blood through narrow tubes by a two-layered model instead of one-layered model.

Blood Flow experiments (Bugliarello and Hayden, 1963; Bennet, 1967) indicate the existence of slip at the tube wall. Nubar (1971), Brunn (1975) and Hyman (1973) have also reported the existence of slip at the blood vessels wall. Oldroyd (1956) has reviewed the several treatments of slip at the walls of the capillary tubes. In

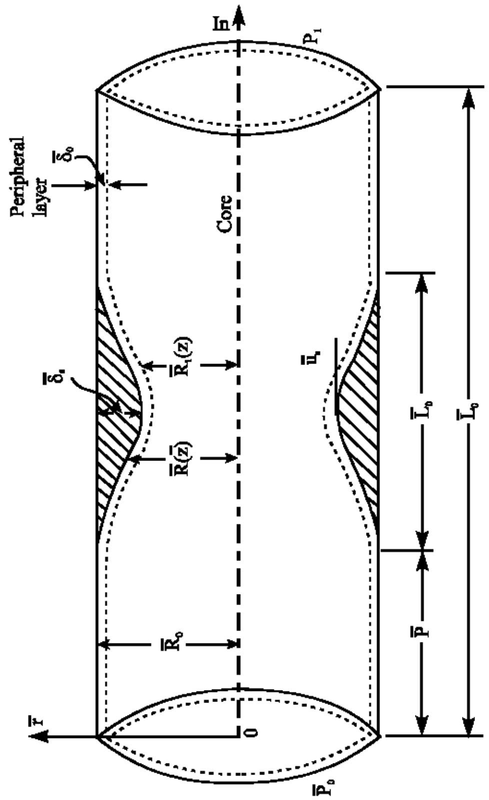

Fig. 1: Geometry of arterial stenosis with plasma layer and axially variable slip velocity

view of theoretical and experimental observations implying the existence of slip at the wall, it is improper to ignore the slip in blood flow. It is also noted that in the literature, there is no direct formula to calculate the slip velocity. It is, therefore, worthwhile to find a formula for determining the slip velocity at the wall.

Shukla *et al.* (1980a, b) have taken two-layered models and analysed the influence of peripheral plasma viscosity on flow characteristics. Chaturani and Kaloni (1976), Chaturani and Upadhya (1979) and Ponalagusamy (1986) have considered the flow of blood represented by a two-layered model and have obtained the apparent viscosity. In all these models, the peripheral layer thickness is assumed a priori. It is, therefore, of interest to obtain an analytic expression for the calculation of peripheral layer thickness.

The focus of this investigation is to obtain analytical expressions for slip velocity, peripheral layer thickness and core viscosity in terms of measurable flow variables. (pressure gradient tube radius, flow rate etc.).

Formulation of the problem: Consider an axially symmetric, steady, laminar and fully developed flow of blood through an artery with mild stenosis as shown in Fig. 1. Here the flow of blood (incompressible fluid) is represented by a two-layered model (a core of red blood cell suspension surrounded by a peripheral layer of plasma (Fig. 1)). It is assumed that the fluids of the peripheral layer and core are Newtonian.

We shall take the cylindrical coordinate system ($\bar z, \bar r, \bar\theta$) whose origin is located on the vessel (stenosed artery) axis. The consistency function $\bar\mu(\bar r)$ may be written as

$$\bar\mu(\bar r) = \bar\mu_c \text{ for } 0\ \bar r \le \bar R_1(\bar z) \tag{1}$$

$$= \bar\mu_P \text{ for } \bar R_1(\bar z) \le \bar r \le \bar R_1(\bar z) \tag{2}$$

where $\bar\mu_c$ and μ_p are the viscosities of the central core fluid and the plasma respectively and $\bar R_1(\bar z) \le \bar r \le \bar R_1(\bar z)$ are the radii of the central core region and the artery in the stenotic region. The non-dimensional variables are

$$r = \frac{\bar r}{\bar R_0} \qquad z = \frac{\bar z}{\bar z_0} \qquad R = \frac{\bar R}{\bar R_0}$$

$$R_1 = \frac{\bar R_1}{\bar R_0} \qquad p = \frac{\bar p}{\bar\rho_p \bar U_0^{\,2}} \qquad u_c = \frac{\bar\mu_c}{U_0}$$

$$u_p = \frac{\bar u_p}{\bar U_0} \qquad v_c = \frac{\bar v_c \bar z_0}{\bar U_0 \bar\delta_s} \qquad v_p = \frac{\bar v_p \bar z_0}{\bar U_0 \bar\delta_s}$$

$$\delta_s = \frac{\bar\delta_s}{R_0}$$

where $\bar u$ and $\bar v$ are velocity components in the axial $\bar z$ and radial $\bar r$ directions, $\bar p$ the pressure, $\bar\rho$ is the density, $\bar R_0$ is the radius of the normal artery, $\bar z_0$ the one-fourth length of the stenoisis $\bar L_0 \bar U_0$ the average velocity in the normal artery region and $\bar\delta_s$ is the maximum height of the stenosis (Fig.1). The quantities in the peripheral layer and in the central core are denoted by subscripts p and c, respectively. '-over a letter denotes the corresponding dimensional quantity. As per discussion made by Young (1968), the appropriate equations describing the flow in the case of a mild stenosis ($\delta_s/R<<1$),subject to the additional conditions (a) $Re_p(\delta_s/L_0)<<1$, (b) $2R_0/L_0 \sim o(1)$, are

for region $0 \le r \le R_1(Z)$,

$$O = -\frac{\partial p}{\partial z} + \frac{\beta}{Re_P \mu}\left[\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u_c}{\partial r}\right] \tag{3}$$

$$O = -\frac{\partial p}{\partial r} \tag{4}$$

for region $R_1(z) \le r \le R(z)$

$$O = -\frac{\partial p}{\partial z} + \frac{\beta}{Re_p}\left[\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u_p}{\partial r}\right] \tag{5}$$

$$O = -\frac{\partial p}{\partial r} \tag{6}$$

where $\mu = \frac{\bar{\mu}_p}{\bar{\mu}_c}$, $\beta = \frac{\bar{z}_0}{\bar{R}_0}$ and $Re_p = \frac{\bar{U}_0 \bar{R}_0 \bar{\rho}_p}{\bar{\mu}_p}$.

The boundary conditions are

$$\left.\begin{array}{llll} \text{(i)} & u_p = u_s & \text{at} & r = R(z) \\ \text{(ii)} & u_p = u_c & \text{at} & r = R_1(z) \\ \text{(iii} & \tau_p = \tau_c & \text{at} & r = R_1(z) \\ \text{(iv)} & \frac{\partial u_c}{\partial r} = 0 & \text{at} & r = 0 \end{array}\right\} \tag{7}$$

where u_s (= $\frac{\bar{u}_s}{\bar{U}_0}$) is the non-dimensional slip velocity (axial) and τ is the shear stress. It may be remarked that u_s is a function of z. The geometry of the stenosis (non-dimensional form) is given by (Ponalagusamy, 1986),

$$R(z) = 1 - A[L_0^{n-1}(z-d) - (z-d)^n], \text{ for } d \le z \le d+L_0 \tag{8}$$
$$= 1, \text{ otherwise}$$

Where n (≥ 2) is a parameter determining the shape of the stenosis, R(z) is the radius of the artery in the stenotic region, L_0 is the length of the stenosis, d indicates its location and A is given by

$$A = \frac{\delta_s}{R_0 L_0^n}\frac{n^{n/(n-1)}}{(n-1)}$$

Here $\boldsymbol{\delta_s}$ denotes the maximum heights of the stenosis at

$$z = d + \frac{L_0}{n^{1/(n-1)}}$$

such that the ratio of the stenotic height to the radius of the normal artery is much less than unity. It is of interest to note that an increase in the value of n leads to the change of stenosis shape. When n = 2, the geometry of stenosis becomes symmetri at
$z = d + \frac{L_0}{2}$.

Solution: Using boundary conditions (7), the solutions of Eq. (3) and (5) can be obtained as

$$u_c = \frac{q(z)Re_p}{4\beta}\left[R^2 - R_1^2 - (1-\mu)\mu r^2\right] + u_s(z) \tag{9}$$

$$u_p = \frac{q(z)Re_p}{4\beta}\left[R^2 - r^2\right] + u_s(z) \tag{10}$$

where $q(z) = -\frac{\partial p}{\partial z}$

The flow rate through the peripheral layer Q_p is defined as

$$Q_p = 2\int_{R1(z)}^{R(z)} r u_p(z,r)\,dr \tag{11}$$

which, on using Eq.(10), gives

$$Q_p = \frac{q(z)Re_p}{8\beta}\left[R^4 - 2R^2R_1^2 + R_1^4\right] + u_s\left(R^2 - R_1^2\right) \tag{12}$$

Similarly, the flow rate through core region Q_c can be written as

$$Q_c = \frac{q(z)Re_p}{8\beta}\left[2R^2R_1^2 - 2R_1^4(1-\mu) - \mu R_1^4\right] + u_s R_1^2 \tag{13}$$

The total flow rate Q is,

$$Q = Q_c + Q_p = \frac{q(z)Re_p}{8\beta}\left[R^4 - R_1^4 \cdot (1-\mu)\right] + R^2 u_s(z) \tag{14}$$

From Eq. (14), we obtain

$$q(z) = \frac{8\beta\left[Q - R^2 u_s(z)\right]}{Re_p\left[R^4 - R_1^4(1-\mu)\right]} \tag{15}$$

Integrating Eq. (15) and using the conditions p = p_0 at z = 0 and p = p_1 at z = L (Fig. 1) and simplifying, we get

$$\lambda = \frac{p_0 - p_1}{Q} = \frac{8\beta}{Re_p}\int_0^L \frac{dz}{\left[R^4 - R_1^4(1-\mu)\right]} - \frac{8\beta}{Q\,Re_p}\int_0^L \frac{R^2 u_s}{\left[R^4 - R_1^4 \cdot (1-\mu)\right]}dz \tag{16}$$

Where $L = \frac{\bar{L}_0}{\bar{Z}_0}$ and ë is the resistance to flow. The wall shear stress τ_w can be defined as (in dimensionless form)

$$\tau_w = \frac{1}{Re_p}[\left|\frac{\partial u_p}{\partial r}\right| \text{ at } r = R \quad (17)$$

Which, on using Eq. (10) and (15), gives

$$\tau_w = \frac{4R\left[Q - R^2 u_s(Z)\right]}{Re_p\left[R^4 - R_1^4 - (1-\mu)\right]} \quad (18)$$

ANALYTIC EXPRESSIONS FOR SLIP VELOCITY, CORE VISCOSITY AND PERIPHERAL LAYER THICKNESS

Btrunn (1975) has indicated that the introduction of a thin solvent layer near the wall produces the same effect as that of the slip at the wall. In the case of one layered model ($R = R_1$) with slip at the wall, the flow rate $Q_{1L}$and wall shear stress τ_{w1L} (from Eq. (14) and (18) can be obtained as

$$Q_{1L} = \frac{\rho^* q(z) Re}{8\beta} R^4 + R^2 u_s \quad (19)$$

$$\tau_{w1L} = \frac{4\left[Q_{1L} - R^2 u_s\right]}{\rho^* Re R^3} \quad (20)$$

where $\rho^* = \frac{\bar{\rho}_p}{\bar{\rho}}$, $Re = \frac{\bar{\rho}\bar{U}_0\bar{R}_0}{\bar{\mu}^*}$ and $\bar{\rho}$ and $\bar{\mu}^*$ are the density and viscosity of the fluid when the flow is one-layered. For the two-layered model without slip at the wall ($u_s = 0$), the flow rate Q_{2L} and the wall shear stress τ_{w2L} (from Eq. (14) and (18) can be obtained as

$$Q_{2L} = \frac{q(z) Re_p R^4}{8\beta}\left[1 - (1 - \frac{\delta}{R})^4 (1-\mu)\right] \quad (21)$$

$$\tau_{w2L} = \frac{4Q_{2L}}{Re_p R^3\left[1 - (1 - \frac{\delta}{R})^4 (1-\mu)\right]} \quad (22)$$

Where $\delta = \frac{\bar{\delta}}{\bar{R}_0}$ is the non-dimensional peripheral layer thickness which is a function of axial distance z. Since the two models (one-layered with slip and two-layered without slip) represent the same phenomena, the flow rates and wall shear stresses can be equated as

$$Q_{1L} = Q_{2L} = Q^*, \; \tau_{w1L} = \tau_{w2L}$$

$$\frac{\rho^* q(z) Re}{8\beta} R^4 + R^2 u_s = \frac{q(z) Re_p R^4}{8\beta}\left[1 - (1 - \frac{\delta}{R})^4 (1-\mu)\right] \quad (23)$$

$$\frac{4\left[Q_{1L} - R^2 u_s\right]}{\rho^* Re R^3} = \frac{4Q_{2L}}{Re_p R\left[1 - (1 - \frac{\delta}{R})^4 (1-\mu)\right]} \quad (24)$$

From Eq. (23) or (24), one can obtain u_s as

$$u_s(Z) = \frac{q(z) R^2}{8\beta}\left[Re_p\left[1 - (1 - \frac{\delta}{R})^4 (1-\mu)\right] - \rho^* Re\right] \quad (25)$$

From Eq. (21), the expression for core viscosity $\bar{\mu}_c$ can be obtained as

$$\bar{\mu}_c = \frac{\bar{\mu}_p (1 - \frac{\delta}{R})^4}{\left[(1 - \frac{\delta}{R})^4 + \frac{8\beta Q^*}{q(z) Re_p R^4} - 1\right]} \quad (26)$$

In Eq. (25) and (26), the peripheral layer thickness is an unknown quantity which can be determined in the following manner. For a two-layered model without slip at the wall ($u_s = 0$), the expression for velocity in the core region is obtained from Eq. (9) as

$$u_c = \frac{q(z) Re_p R^2}{4\beta}[1 - (1 - \frac{\delta}{R})^2 (1-\mu) - \mu\left(\frac{r}{R}\right)^2 \quad (27)$$

The centreline velocity U (at r = 0.0) from Eq. (27) can be obtained as

$$U = \frac{q(z) Re_p R^2}{4\beta}\left[1 - (1 - \frac{\delta}{R})^2 (1-\mu)\right] \quad (28)$$

Elimination of μ from Eq. (21) and (28) gives

$$\frac{\delta}{R} = 1 \pm \left[\frac{q(z) Re_p R^4 - 8\beta Q^*}{q(z) Re_p R^4 - 4\beta U R^2}\right]^{1/2} \quad (29)$$

All the quantities on right hand side of Eq. (29) are measurable experimentally, hence is known. Once we know the value of δ, $\bar{\mu}_c$ and u_s can be calculated from Eq. (25) and (26).

RESULTS AND DISCUSSION

It has been noticed from Eq. (28) that the centerline velocity U is always less than $\frac{q(z)Re_p R^2}{4\beta}$. Since only those values of $\frac{\delta}{R}$ are of interest which are real and less than or equal to unity, the following condition can be established from Eq. (29).

$$q(z)Re_pR^4 \geq 8\beta Q^* \tag{30}$$

And the Eq. (29) reduces to

$$\frac{\delta}{R} = 1 - \left[\frac{q(z)Re_pR^4 - 8\beta Q^*}{q(z)Re_pR^4 - 4\beta UR^2}\right]^{\frac{1}{2}} \tag{31}$$

Since the experimental values of pressure gradient, flow rate and centerline velocity for flow through an arterial mild stenosis at different cross-sections for various values of stenotic height and shapes and red blood cells concentrations are not available, the variation of slip velocity, peripheral layer thickness, the core viscosity with the axial distance cannot be obtained. However, to show the procedure and to see the accuracy of the method, we have used the experimental data of flow through a uniform tube. First, we write Eq. (25), (26) and (31) in the dimensional form as

$$\bar{u}_s = \frac{\bar{q}_0(\bar{R}_0)^2}{8}\left[\frac{1}{\bar{\mu}_p}\left\{1-(1-\frac{\bar{\delta}_0}{\bar{R}_0})^4(1-\mu)\right\} - \frac{1}{\mu}\right] \tag{32}$$

$$\bar{\mu}_c = \frac{\bar{\mu}_p\left\{1-\frac{\bar{\delta}_0}{\bar{R}_0}\right\}^4}{\left[\left\{1-\frac{\bar{\delta}_0}{\bar{R}_0}\right\}^4 + \frac{8\bar{Q}^*\bar{\mu}_p}{\pi\bar{q}_0(\bar{R}_0)^4} - 1\right]} \tag{33}$$

$$\frac{\bar{\delta}_0}{\bar{R}_0} = 1 - \left[\frac{\frac{\pi\bar{q}_0(\bar{R}_0)^4}{8\bar{\mu}_p} - \bar{Q}^*}{\left\{\frac{\pi\bar{q}_0(\bar{R}_0)^4}{8\bar{\mu}_p} - \frac{\pi\bar{U}(\bar{R}_0)^4}{2}\right\}}\right] \tag{34}$$

Where $\bar{q}_0$ is the pressure gradient and $\bar{\delta}_0$ is the peripheral plasma layer thickness in the normal artery region. For blood with 6% and 40% red blood cell concentration, we have the following data from Bugliarello and sevilla (1970) and Bugliarello and Hayden (1963).

Table 1: Comparison of peripheral layer thickness $\bar{\delta}$

Tube diameter	$\bar{\delta}_0$ μm Present	Experimental Result (12)	Experimental Result (13)	Difference (%)
40 μm	3.2030	3.200	-	0.094
66.6 μm	12.8405	-	12.876	0.276

Table 2: Core fluid viscosity $\bar{\mu}_c$

Tube diameter	Whole blood concentration (%)	Concentration in core (%)	$\bar{\mu}_c$ poise: By the formula (Eq.45) $\bar{\mu}_c$ (I)	By concentration curve (20) $\bar{\mu}_c$ (II)	Difference (%)
40 μm	40	56.709	0.04766	0.05417	13.65
66.6 μm	6	15.894	0.02267	0.02160	4.95

Table 3: Slip velocity $\bar{u}_s$

Tube diameter	$\bar{u}_s$ cm sec^{-1}: With $\bar{u}_s$ (I)	With $\bar{u}_s$ (II)	Difference (%)
40μm	0.6862	0.6612	3.624
66.6μm	0.8157	0.8179	2.697

Table 4: Wall shear stress $\bar{\tau}_{w0}$

Tube diameter	$\bar{\tau}_{w0}$ * 10^3 dyne cm^{-2}: With $\bar{\mu}_c$ (I)	With $\bar{\mu}_c$ (II)	Difference (%)
40 μm	33.7496	34.7101	2.846
66.6 μm	7.1327	7.0992	0.472

Table 5: Γ_{w0} and flow resistance $\bar{\lambda}_0$ at $\bar{z} = \frac{\bar{L}}{2}$ ($\bar{L}$ = 5 cm)

Tube diameter	$\bar{\lambda}_0$ * 10^7 (dyne-sec) cm^{-2}: With $\bar{\mu}_c$ (I)	With $\bar{\mu}_c$ (II)	Difference (%)
40 μm	87.7332	90.230	2.846
66.6 μm	7.8116	7.75	0.5

For 40 μm Diameter

C = 40% and $\bar{Q}^* = 19.2342 * 10^{-6}\,cm^3/sec$

$\bar{\delta}_0$ = 3.2 μm and $\bar{q}_0 = 167.5 * 10^3\,dyne/cm^3$

$\bar{\mu}_0 = 0.0144P$(at 25.5°C) and $\bar{U} = 2.37\,cm/sec$.

For 66.6 μm Diameter

C = 6%, $\bar{Q}^* = 45.6546 * 10^{-6}\,cm^3/sec$,

$\bar{\delta}_0$ = 12.87 μm and $\bar{q}_0 = 14.2655 * 10^3\,dyne/cm^3$,

$\bar{\mu}_0 = 0.0143$(at 25.5°C) and $\bar{U} = 2.38\,cm/sec$.

Using these value, the peripheral layer thickness is computed (Table 1) for blood flow in 40 and 66.6 μm tube diameter from Eq. (46). One can easily see from this table that the peripheral layer thickness, obtained from the present analysis, has a good agreement with the experimental observation (Bugliarello and sevilla, 1970; Bugliarello and Hayden, 1963), the error is less than 1%.

With the help of the obtained values of the peripheral layer thickness, the core viscosity and red blood cell concentration in the core have been computed (Table 2). It is of interest to note that in the present analysis, the core viscosity has been obtained by two methods. The first by calculating from Eq. (33) and the second by determining red blood cell concentration in the core and then using concentration versus relative viscosity curve. A comparison of these two values of the core viscosities shows a reasonably good agreement (difference between them is 14 and 5% for 40 and 66.6 μm diameter tube respectively) between them. Substituting the obtained values of peripheral layer thickness and core viscosities in the Eq. (32), One can get two values of the slip velocity (Table 3). Agreement between these two values of slip velocity is reasonably good (difference up to 4%).

The wall shear stress $\bar{\tau}_{w0}$ and flow resistance $\bar{\lambda}_0$ have been computed by using the values of the peripheral layer thickness, obtained in the present analysis and core viscosity, obtained by two methods (mentioned above). Thus two values of $\bar{\tau}_{w0}$ and $\bar{\lambda}_0$ are quite close to each other (difference up to 3% Table 4 and 5).

CONCLUSIONS

A two-layered model of blood flow through a stenosed artery with axially variable peripheral layer thickness and variable slip velocity at the wall has been considered. The model consists of a core (red cell suspension) surrounded by a peripheral plasma layer. Both the fluids (core and peripheral layer) are assumed to be Newtonian having different viscosities. The analytic expressions for peripheral layer thickness, core viscosity, slip velocity, wall shear stress and resistance to flow have been obtained (Eq. 31, 26, 25, 18 and 16). It may be mentioned that we could not analyze their (peripheral layer thickness, core viscosity, etc.,) variation with the axial distance in the strenotic region because of the non-availability of the required experimental data (pressure gradient and centerline velocity at difference cross-section of the stenosed arteries for various values of stenotic heights and different shapes, flow rates and concentrations). It is, therefore, of interest to conduct such experiments to provide the required data which, in turn, will help in understanding the flow of blood through a stenosed artery.

It is of interest to mention that measuring the thickness of peripheral plasma layer experimentally is difficult because its thickness is not constant even for the steady flow through uniform tubes, due to the random motion of the suspended particle (red blood cell); whereas the reliable values of pressure gradient, plasma viscosity and centerline velocity can be measured for a given flow rate, tube size and concentration. Therefore, it is preferable to use these reliable measurements for the computation of the value of peripheral layer thickness (Eq. 34).

It is worth mentioning that there are two methods to determine the core viscosity. The first method is by using Eq. (33) which, in turn, uses the value of the obtained peripheral layer thickness (Eq. 34) and the experimental data for pressure grandient, plasma ciscocity and the flow rate. The second method is by computing the red blood cell concentration in the core and then using concentration vs viscosity curve. It is important to note that the first method is more convenient than the second. Further, the core viscosities obtained by tow methods differ by 5-14%. The values of the apparent viscosity of blood, agreeability, rigidity and deformability of red cells can be determined by the present analysis more accurately than the other existing analyses (Das and Seshadri, 1970; Thomas, 1965) because in the present analysis, the core viscosity is obtained by calculating the actual red cell concentration in the core which is different from the concentration of whole blood.

The present analysis could also serve as the check for the experimentally measured rheologic values of blood. It may be mentioned at this stage that the variation of peripheral layer thickness, core viscosity and slip velocity with the axial distance in the stenotic region has not been analyzed due to the non-availability of the experimental values of pressure gradient and the centerline velocity at different cross-sections of the stenosed arteries for various values of stenotic heights, flow rates and concentrations. It would be of interest to conduct such experiments to provide this vital data which, in turn, could be useful in the understanding of the rheology of blood. This rheologic information of blood in turn could be exploited for the development of new diagnostic tools for many diseases such as myocardial infarction, hypertension, renal, etc. (Dintenfass, 1977).

REFERENCES

Bennet, L., 1967. Red cell slip at wall *in vitro*. Science, 15: 1554-1556.

Brunn, P., 1975. The velocity slip of polar fluids. Rheol. Acta, 14: 1039-1054.

Bugliarello, G. and J.W. Hayden, 1963. Detailed characteristics of the flow of blood *in vitro*. Trans. Soc. Rheol., 7: 209-230.

Bugliarello, G. and J. Sevilla, 1970. Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes. Biorhelogy, 17: 85-107.

Chaturani, P. and P.N. Kaloni, 1976. Two-layered poiseuille flow model for blood flow through arteries of small diameter and arterioles. Biorheology, 13: 243-250.

Chaturani, P. and V.S. Upadhya, 1979. A two-layered model for blood flow through small diameter tubes. Biorheology, 16: 109-118.

Chaturani, P. and D. Biswas, 1983. A theoretical study of blood flow through stenosed arteries with velocity slip at the wall. Proc. First Internat. Symposium on Physiological Fluid Dynamics, IIT Madras, India, pp: 23-26.

Das, R.N. and V. Seshadri, 1970. A semi-empirical model for flow of blood and other Particulate suspensions. Bull. Math. Biol., 37: 459-490.

Deshpande, M.D, D.P. Giddens and R.F. Mabon, 1979. Steady laminar flow through modeled vascular stenosis. J. Biomech., 9: 65-174.

Dintenfass, L., 1977. Viscosity factors in hypertensive and cardio-vascular diseases. Cardiovascular Med., 2: 337-363.

Fry, D.L., 1968. Acute vascular endothelial changes associated with increased blood velocity gradients. Circulat. Res., 22: 165-197.

Goldsmith, H.L., 1972. Theoretical and Applied Mechanics. Proc. 13th IUTAM Congress. Becker, E. and G.K. Mikhailov (Eds.), Springer-Verlag, New York.

Hyman, W.A., 1973. The role of slip in the rhelogy of blood. Biorhelogy, 10: 57-60.

Lee, J.S. and Y.C. Fung, 1970. Flow in locally constricted tubes at low Reynolds Number. J. Mech. Trans. ASME, 37: 9-16.

Macdonald, D.A., 1979. On steady flow through modeled vascular stenosis. J. Biomech., 12: 13-20.

Majhi, S.N. and L. Usha, 1984. Fahraeus-Lindqvist effect and generalized poiseuille flow with or without wall slip. Proc. 13th Nat. Conf. Fluid Mech. Fluid Power, REC, Tiruchirappalli, pp: 125-128.

Nubar, Y., 1971. Blood flow, slip and viscometry. J. Biophys., 11: 252-264.

Oldroyd, J.G., 1956. Rheology. Eirich, F.R. (Ed.), Vol. 1, Academic Press, New York, pp: 653-682.

Philip, D. and Peeyush Chandra, 1996. Flow of eringen fluid (simple micro fluid) through an artery with mild stenosis. Int. J. Eng. Sci., 34: 87-99.

Ponalagusamy, R., 1986. Blood Flow Through Stenosed Tube. Ph.D Thesis, IIT, Bombay, India.

Shukla, J.B., R.S. Parihar and S.P. Gupta, 1980a. Biorhelogical aspects of blood flow through artery with mild stenosis:Effects of peripheral layer. Biorheology, 17: 403-410.

Shukla, J.B., R.S. Parihar and S.P. Gupta, 1980b. Effects of peripheral layer viscosity on blood flow through the artery with mild stenosis. Bull. Math. Biol., 42: 797-805.

Thomas, D.G., 1965. Transport characteristics of suspensions. VIII. J. Colloid. Sci., 20: 267-277.

Wang, C.Y. and J.B. Bassingthwaighte, 2003. Blood flow in small curved tubes. J. Biomech. Eng., 125: 910-913.

Young, D.F., 1968. Effects of a time-dependent stenosis on flow through a tube. J. Eng. Ind. Trans. AMSE., 90: 248-254.