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ACI Code Provisions for Torsion Design of Reinforced concrete Beams, A Need for Revision

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Abstract: This study summarizes the critical provisions specified by the ACI 318-05 (2005) code, that are used for the design of reinforced concrete beam under pure torsion. Some parameters have been considered like the beam size, material properties and their effect on the required torsional reinforcement. The main emphasis is to highlight the drawback of this latest ACI code's provisions for the design of reinforced concrete beam under pure torsion. It was found that, before cracking, the required torsional reinforcement for beam with higher concrete strength is more than that of weaker concrete when the beam undergoes the same factored torsion. After cracking the required torsional reinforcement is not affected by the concrete strength, while the code specified the permitted maximum factored torsion as a function of the concrete strength.

Key words: ACI, beam, code, reinforced concrete, torsion

INTRODUCTION

Three dimensional modeling for the analysis of reinforced concrete structure always leads to produce a torsional moment in the main members of framed structure particularly when the infill slabs are taken into consideration in modeling the structure. The torsion is mainly developed in the edge beams and in some cases at intermediate beams due to unsymmetrical loads at the two side panels (slabs) or due to unequal spans of these panels. The value of this torsional moment may be so large that it may elevate the principal tensile stress in the beam and develops a crack that may case an unexpected failure of the member. According to the Sant-Venant's elastic theory for torsion (Timoshenko, 1952) that has been established in the mid of 19th century, the cracking torsion can be expressed as:

$$T_{cr} = aX^2Yf_t \tag{1}$$

Where a is shape factor, X and Y are the smaller and larger dimensions of the rectangular cross section, respectively and f_t is the tensile strength of the concrete.

Based on skew bending theory and the extensive study of Hsu (1968), it was proposed that $(0.4\sqrt{f'c})$ can be considered as the limiting torsion strength of plane concrete member, where f'c is the concrete cylinder compressive strength. A reduction factor (2.5) is proposed (Nawy, 2003) for the first cracking and the following equation can be used to predict the cracking torque:

$$Tc = \frac{1}{15}\sqrt{f'c}X^2Y \tag{2}$$

Based on the plastic theory, Nylander in 1955 utilize Nadui's plastic coefficient instead of Sant-Venant's elastic one (Nawy, 2003) to calculate the cracking torque in the form:

$$Tc = \left(\frac{1}{2} - \frac{1}{6}\frac{X}{Y}\right) X^2Y \tag{3}$$

The limitation of the above equations is that concrete is neither elastic nor perfectly plastic material. Space truss analogy was first developed by Rausch (1929) and later extended and refined by Hsu (1990 and 1993). It is assumed that the solid concrete beam section under torsion behave in a similar manner to the thin-wall section. Recently it has been concluded by Chiu *et al*. (2006) that the torsional cracking strengths of the specimens with hollow sections are smaller than those of the specimens with solid sections. After cracking of reinforced concrete section under torsion, the concrete separated by a series of helical cracks and the torsional strength is composed of that of the closed stirrups, longitudinal bars and the idealized inclined concrete compression struts which construct a space truss.

In the present study the flaws in the provisions of the ACI 318-05 (2005) code for the design of rectangular reinforced concrete beam under pure torsion is discussed, taken into consideration the effects of varying material properties, size of the cross section and variation of applied torsional moment.

ACI 318-05 CODE PROVISIONS

The provisions of this code for the design of reinforced concrete beam for torsion is identical to that of the older version ACI 318-02. According to this code the nominal torsional stress is calculated from the following equation:

$$v_{tn} = \frac{T_n}{2A_0 t} \quad (4)$$

Where (T_n) is the nominal torsional moment, (A_0) is the cross-sectional area bounded by the centerline of the shear flow and according to this code it is taken equal to ($0.85A_{0h}$), in which (A_{0h}) is the area enclosed by the outermost center line of the closed stirrups in the section and (t) is the wall thickness of the equivalent thin walled tube and is taken equal to:

$$t = A_{oh}/P_h \quad (5)$$

P_h is equal to the perimeter of the centerline of the closed stirrups.

The cross section of the member should be such that it satisfies the following:

$$\frac{T_u P_h}{1.7A_{0h}^2} \leq \frac{\phi 5\sqrt{f'c}}{6} \quad (6)$$

T_u is equal to the factored torque and $\phi = 0.75$. The code permit to neglect the torsional moment when the value of the nominal torque satisfies the following:

$$T_n \leq \frac{\sqrt{f'c}}{12}\left(\frac{A_{cp}^2}{P_{cp}}\right) \quad (7)$$

Where (A_{cp}) is the gross area bounded by the outer perimeter of the cross-section and (P_{cp}) is the outer perimeter of member cross-section. The limiting nominal torsional moment given in the right side of relation (7) is equal to one quarter of the nominal cracking torsional moment, thus the cracking torsion can be calculated by:

$$T_{cr} = \frac{\sqrt{f'c}}{3}\left(\frac{A_{cp}^2}{P_{cp}}\right) \quad (8)$$

The transverse reinforcement required for torsion (when the nominal torsional moment (T_n) exceeds the value given in Eq. 7) is calculated by using the relation:

$$A_t = \frac{T_n s}{2A_0 f_{yv} \cot\theta} \quad (9)$$

Where f_{yv} is the yield strength of the transverse reinforcement, s is spacing of the stirrups. The above equation is based on space truss analogy with compression diagonals at an angle (θ), assuming concrete carry no tension and steel yields. For non-prestressed members the code permits the use of θ equal to 45 degrees. A_t is equal to the area of one leg of the bar of closed stirrup.

The minimum area of two legs of transverse closed stirrups shall be computed by:

$$2A_t = \frac{1}{16}\sqrt{f'c}\,\frac{b_w s}{f_{yv}} \quad (10a)$$

But not less than $\frac{b_w s}{3f_{yv}}$ (10b)

The additional longitudinal reinforcement required for torsion shall not be less than:

$$A_l = \frac{T_n P_h}{2A_0 f_{yl}} \cot\theta \quad (11)$$

In which f_{yl} is the yield strength of the longitudinal reinforcement.

The minimum total area of longitudinal torsional reinforcement is computed by:

$$A_{lmin} = \frac{5\sqrt{f'c}A_{cp}}{12f_{yl}} - \left(\frac{A_t}{s}\right)P_h \frac{f_{yv}}{f_{yl}} \quad (12)$$

in the above equation $\left(\frac{A_t}{s}\right)$ shall not be substituted less than $\frac{b_w}{6f_{yv}}$.

APPLICATIONS AND DISCUSSION

The foregoing stated procedure for the design of rectangular reinforced concrete beam under pure torsion is programmed to calculate the required torsional transverse and longitudinal reinforcements. The designed five beams with different sizes are assigned a notation given in Table 1.

The required longitudinal reinforcement (A_l) and transverse reinforcements for the five rectangular beams are calculated under increasing incremental torsional

Table 1: Designations of the designed beams

Beam size (mm)	300×500	300×550	350×550	350×600	400×600
Beam No.	B1	B2	B3	B4	B5

Table 2: Cracking and maximum factored torsional moment of the beams

Beam No.	B1	B2	B3	B4	B5
ϕTcr (kN.m)	17.60	20.02	25.730	29.00	36.00
Tu_{max} (kN.m)	31.76	37.00	52.771	60.655	81.00
$Tu_{max}/\phi Tcr$	1.80	1.85	2.013	2.09	2.25
$v_{tu\ crack}$ (MPa)	1.730	1.689	1.524	1.495	1.388
$v_{tu\ max}$ (MPa)	3.125	3.125	3.125	3.125	3.125

moment. The concrete cylinder strength (f'c) is taken equal to (25 MPa) and the steel yield stress is taken equal to 400 MPa. Figure 1 shows the calculated longitudinal reinforcements for each beam under variable factored torsional moment. An increment of (1.0 kN m) of factored torsion is used starting from the limiting value specified in Eq. 7 up to the maximum permissible factored torsion specified in Eq. 6. It can be seen from Fig. 1 that before cracking, the required longitudinal reinforcement for all the beams is decreased with the increase of the applied torsional moment and this reduction is associated with the increase in the transverse reinforcement according to Eq. 9 and that is shown in Fig. 2. When the value of (Tu) exceeds 85% of (ϕT_{cr}), the required longitudinal reinforcement begins to increase linearly with increasing value of (Tu). For all the beams, when (Tu) is less than ($0.85\phi T_{cr}$), the require A_l are controlled by the minimum longitudinal reinforcement (A_{lmin}) computed by Eq. 12. This equation contains two terms, the first one is constant and depends on the material properties f'c and fy as will as on the cross section area of the member (A_{cp}), while the second term (with negative sign) is depends on the value of (A_t/s) which is calculated from Eq. 9 which in turn increases linearly with the increase in the value of (Tu) as shown in Fig. 2, this explains the reduction of (A_l) with the increase of (Tu) up to the value of ($0.85\phi T_{cr}$). The constant values of (A_l) at the first few steps of loading for beams B3, B4 and B5 are due to the fact that (A_t/s) substituted in Eq. (12) is controlled by the minimum value of (A_t/s) specified by Eq. (10b), which is a constant value. For the five beams, the factored cracking torque (ϕTcr), maximum permissible factored torque (Tu_{max}), ratio of (Tu_{max}/ϕTcr), cracking stress ($v_{tucrack}$) and stress at maximum permissible torque ($v_{tu\ max}$) are given in Table 2. Table 2 shows that stresses at maximum torque are equal to (3.125 MPa) for all the beam sizes, but this is not the case for the cracking stresses which are reduced with the increase in the size of the beam although the concrete strength for all the beams are equal to (25 MPa) and for this the author put a big question mark.

Figure 3 shows the variation of the total volume ratio of torsional reinforcement (VRS) with the factored torsion for the five beams using f'c = 25 MPa. This volume ratio of torsional reinforcement is calculated by:

$$VRS = \frac{A_l}{X.Y} + \frac{\frac{2A_t}{s}(X1 + Y1)}{X.Y} \qquad (13)$$

Figure 3 shows that when the torsional moment is less than cracking moment, the values of (VRS) are same for all the beams and they are equal to 0.52%.

When the value of (Tu) exceeds ($0.85\phi T_{cr}$), the values of (VRS) start to increase linearly with the increase in the (Tu) values but with different rate of increase. The rate of increase of (VRS) for beam with larger size is less than that of smaller one.

The variation of (VRS) with factored torsion (Tu) for beam (B5) using different concrete cylinder strength (f'c) is shown in Fig. 4. It can be seen that before cracking the required (VRS) remains constant with the increase in the value of (Tu) and its value depend on the concrete strength and for this particular beam size it range from 0.0466 for f'c = 20 MPa to 0.066 for f'c = 40 MPa. The figure also shows that at the initial steps of loading for beams with concrete strength (30 and 40 MPa), as marked by the circle, they have slightly larger value of (VRS) and this is because for concrete strength higher than 28.4 MPa, Eq. 10a will control the minimum (A_t/s) value, so for small torsional moment it gave a larger ratio of steel volume. It can also be noticed that the length of the range of constant (VRS) is not constant for different concrete strength and this is because the cracking torsional moment is a function of the concrete strength as stated before. When (Tu) exceed ($0.85\phi T_{cr}$), the required (VRS) increased linearly and it is not affected by the concrete strength, while the maximum applied torsional moment is limited by the concrete strength as specified by Eq. 6. Here the following question is emerge, why for a particular beam size the volume ratio of total torsional reinforcement after cracking is not affected by the concrete strength, while the maximum permissible torsional moment is controlled by the concrete strength?

For the same beam (B5) the variation of (VRS) with concrete strength (f'c) under factored torsional moment equal to 20 kN.m, which is less than (ϕT_{cr}), is plotted in Fig. 5. The figure shows that the required total torsional reinforcement before cracking is proportional to the concrete strength, i.e., the designed torsional reinforcement for beam with higher concrete strength is more than that of weaker concrete strength under the same torsional moment and this is illogical.

Figure 6 shows the variations of (VRS) with the ratio ($Tu/\phi T_{cr}$) for beam (B5) using different concrete strength.

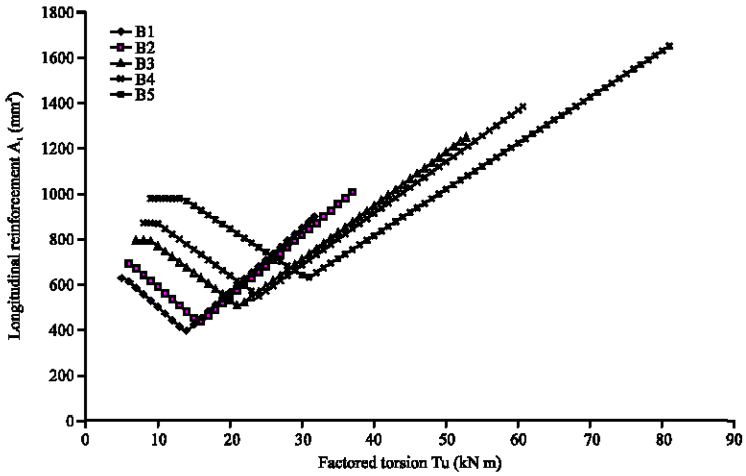

Fig. 1: Variation of A_l with Tu for different beam sizes

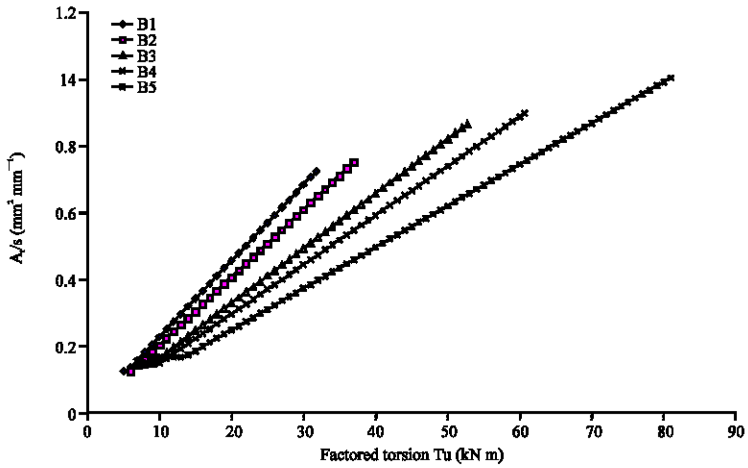

Fig. 2: Variation of A_t/s with Tu for different beam sizes

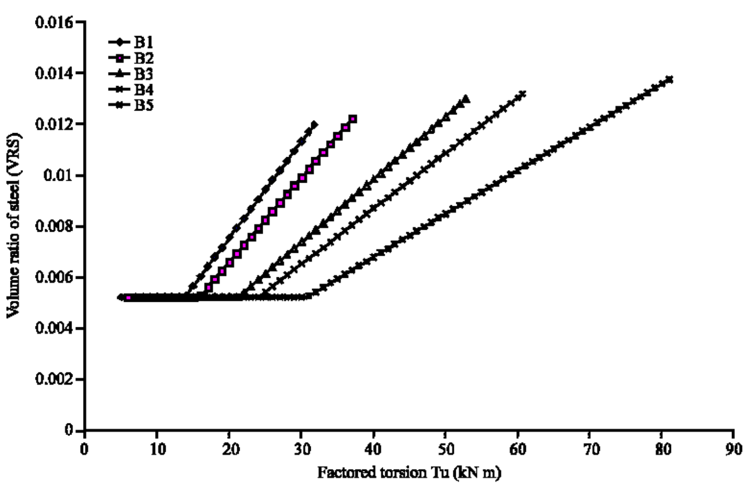

Fig. 3: Variation of VRS with Tu for different Beam sizes

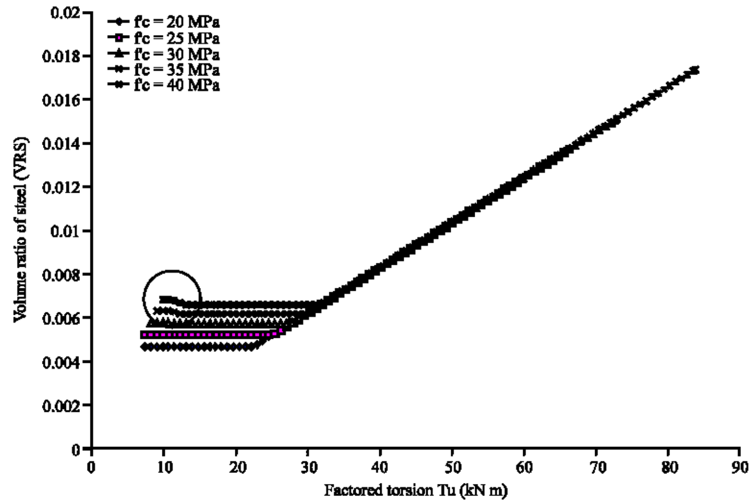

Fig. 4: Variation of VRS with Tu for beam B5 using different concrete strength

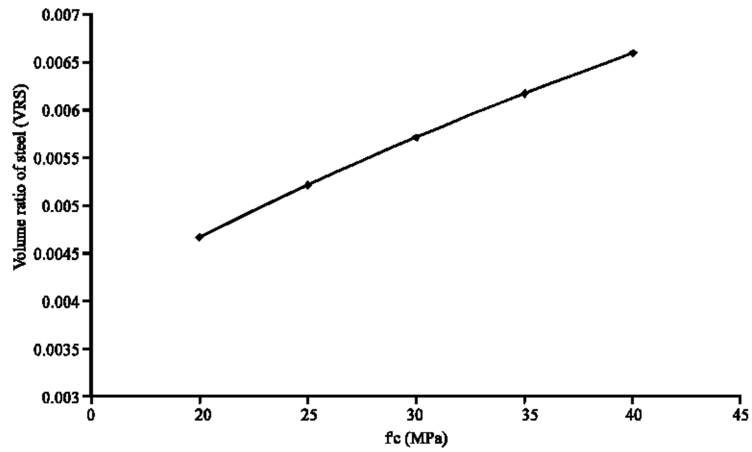

Fig. 5: Variation of VRS with concrete strength for beam B5 under factored torsion of 20 kN.m

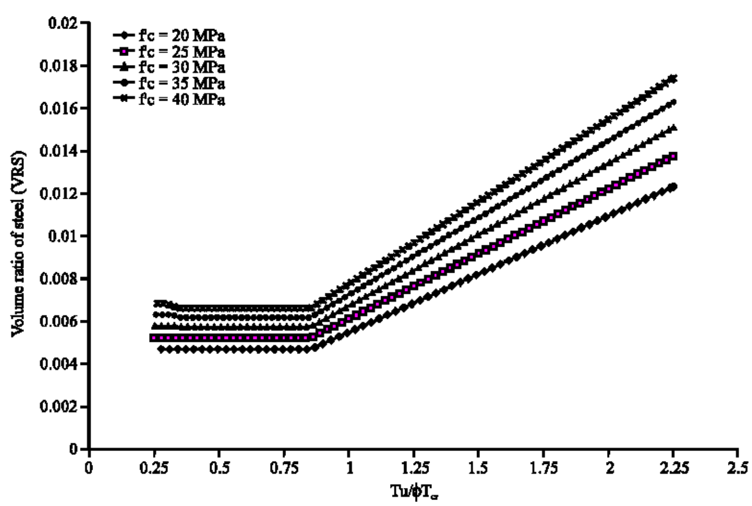

Fig. 6: Variation of VRS with ($Tu/\phi T_{cr}$) for Beam B5

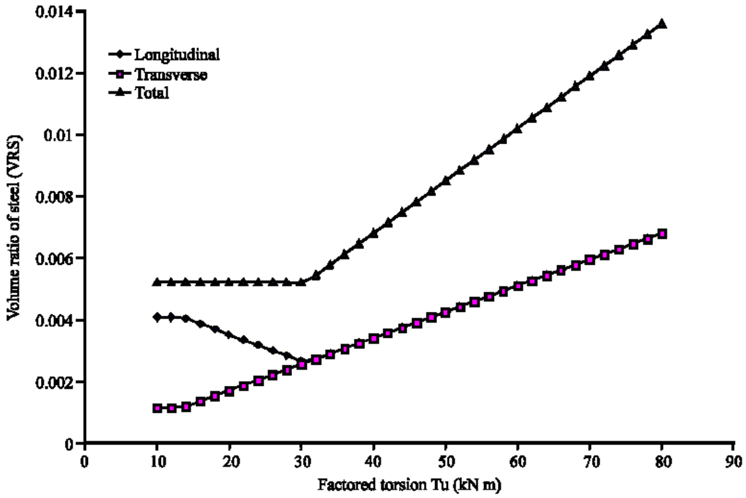

Fig. 7: Variation of volume ratios of steel with Tu for Beam B5

It can be seen that for a certain ratio of ($Tu/\phi T_{cr}$) and for all the permissible range of loading for this beam ($Tu/\phi T_{cr}$ range from 0.25 to 2.25), the (VRS) value for beam with higher concrete strength is more than that of smaller concrete strength.

Figure 7 shows the variations of volume ratio of longitudinal, transverse and total torsional reinforcement with (Tu) for Beam B5, using f'c = 25 Mpa and fy = 400 MPa. It can be seen that when (Tu) exceed ($0.85\phi T_{cr}$), at any particular torque the volume fraction of the transverse reinforcement is equal to that of the longitudinal reinforcement while; before cracking the two fractions are completely different.

CONCLUSIONS

From the study of some aspects of the design provisions of the ACI 318-05 code for the design of rectangular reinforced concrete beams under pure torsion, the followings can be stated: Firstly, for a particular rectangular beam, when the applied torsion is less than 85% of the factored cracking torsion, the longitudinal reinforcement is decreasing by the increase of the applied torsional moment, while the transverse reinforcement increases with the applied torsion such that the total volume of torsional steel remains constant for a considerable range of loading up to ($0.85\phi T_{cr}$). No justification has been given by the code for the variation of longitudinal and transverse reinforcement with the applied torsion before cracking. Secondly, it was found that before cracking the volume ratio of total torsional reinforcement for a higher concrete strength is more than that of low one and this is a questionable condition. Thirdly, the cracking stress varies inversely with the beam size using constant concrete strength. And finally it was found that after cracking the required torsional reinforcement is not affected by the concrete strength, while, the maximum permissible torsion is controlled by the concrete strength. This study has shown that some provisions of the design of rectangular reinforced concrete beam under pure torsion as specified by the (ACI 318-05) code is not logical and for sure requires some revisions. More study to understand the behavior of reinforced concrete beams under torsional moment is also required.

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