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A Feasible SQP Method with Superlinear Convergence for General Constrained Optimization

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Abstract: In this study, optimization problems with general equality and inequality constraints are discussed. Firstly, the original problems are changed into parametric programming problems with only inequality constraints and these two problems are equivalent with each other if the parameter is suitable. Then, we give a new idea called first-order feasible condition, which is used to solve the changed problems. Under some reasonable conditions, the global and superlinear convergence is shown.

Key words: General constrained optimization, SQP method, feasible direction, global convergence, superlinear convergence

INTRODUCTION

It is well known that the Sequential Quadratic Programming (SQP) method is one of the most effective method to solve the inequality constrained optimization. Because of its superlinear convergence, there have existed a plenty of literatures about nonlinear programming. In order to promote the rate of convergence, it is risen some improved algorithms (Facchinei and Lucidi, 1995). But the almost above algorithms is applied to inequality constrained optimization. So it is needed to expanded to the general constrained optimization (Jian *et al.*, 2005; Jian and Zhu, 2003). However, as we know that the direction obtained by solving the QP subproblem is not a feasible direction. We need to revise the direction, the first-order feasible descent condition is proposed in Panier and Tits (1987) which will make it necessary to solve two or three QP problems per single iteration. Zhu and Jian (2005) gave an easier first-order feasible condition and obtained a revised feasible descent direction by using a line search which always can be implemented to establish a convex combination.

To solve optimization problems with general equality and inequality constraints, we use a penalty function as the merit function which to change the original problem into parametric programming problems with only inequality constraints. Then, based on the method in Zhu and Jian (2005), a feasible SQP method for general constrained optimization is established.

DESCRIPTION OF ALGORITHM

In this study, we consider the following nonlinear programming problem

$$\min \{f_0(x) | x \in R\} \tag{1}$$

where

$$R = \{x \in E^n | f_j(x) \le 0,\ j \in L_1;\ f_j(x) = 0,\ j \in L_2\}$$
$$L_1 = \{1, ..., m_1\},\ L_2 = \{m_1 + 1, ..., m\},\ L = L_1 \cup L_2$$

For the parameter $c>0$, the definition of the penalty function $F_c(x)$: $E^n \to E^1$ is

$$F_c(x) = f_0(x) + cF(x),$$

where

$$F(x) = \sum_{j \in L_2} f_j(x).$$

If $L_2 = \varnothing$, let $F(x) = 0$ and the problem (1) is the inequality constrained optimization.

Now, we consider the auxiliary programming of the problem (1)

$$\min \{F_c(x) | x \in R_+ = \{x \in E^n | f_j(x) \le 0,\ j \in L\}\} \tag{2}$$

Denote

$$I(x) = \{j \in L | f_j(x) = 0\}$$

Throughout this study, the following assumptions are assumed.

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H 1: $R_+ \neq \emptyset$, f_j (j = 0, ..., m) are continuously differentiable;
H 2: The vectors $(\nabla f_j(x), j\in\{jL_1 | f_j(x) = 0\} \cup L_2\}$ are linearly independent.
For the sake of simplicity, we denote

$$g(x;c) = \nabla F_c(x),\ g_j(x) = \nabla f_j(x),\ H(x;c) = \nabla^2 F_c(x),$$
$$H_j(x) = \nabla^2 f_j(x),\ N(x), = (g_j(x),\ j\in L)$$

$$D(x) = \text{diag}(D_j(x), j \in L) \qquad D_j(x) = \begin{cases} f_j^2(x), j \in L_1 \\ 0, j \in L_2 \end{cases}$$

$$Q(x) = (N(x)^T N(x) + D(x))^{-1} N(x)^T$$

$$\pi(x) = -Q(x)g_0(x) = (\pi_j(x),\ j\in L),$$
$$\pi(x;c) = -Q(x)g(x;c) = (\pi_j(x;c),\ j\in L)$$

According to Lemma 3 in Jian and Jhu (2003), we can obtain the following result.

Lemma 1: For any $x\in R_+$, $(N(x)^T N(x)+D(x))$ is a positive definite matrix, moreover, $\pi(x)$ and $\pi(x;c)$ satisfy the following conditions:

$$\pi_j(x) = \pi_j(x;c),\ \forall j\in L_1,\ \pi_j(x) = \pi_j(x;c)-c,\ \forall j\in L_2.$$

Lemma 2: If the parameter $c > \max\{|\pi_j(x)| : j\in L_2\}$, then x is a KKT point of the problem (1) if and only if x is a KKT point of the problem (2).

Now, we state the basic algorithm as follows.

Algorithm A:
Step 0 Initialization: Choose $x^1\in R_+$, Σ, a compact set of symmetric and positive definite matrices, $B_1 = B(x^1)\in\Sigma$, parameters ε, v, $\theta\in (0,1)$, $\delta>2$, $\alpha\in(0,1/2)$, $\tau\in (2,3)$, $\{\varepsilon', c_0)\subset(0,\infty)$.
Set k = 1;

Step 1: Adjust parameters c: Compute

$$t_k = \max\left\{\left|\pi_j(x^k)\right| : j \in L_2\right\} + c_0, c_k = \begin{cases} \max\left\{t_{k,} c_{k-1} + \varepsilon'\right\}, c_{k-1} < t_k, \\ c_{k-1}, c_{k-1} \geq t_k. \end{cases}$$

Step 2: Obtain $(d_0^k, \tilde{u}^k)$ by solving the following QP subproblem at x^k:

$$\begin{aligned} &\min g(x^k;c_k)^T d + \frac{1}{2} d^T B_k d \\ &\text{s.t.} f_j(x^k) + g_j(x^k)^T d \leq 0, j \in L. \end{aligned} \tag{3}$$

Denote $E_k = \{j\,|\,f_j(x^k)+g_j(x^k)^T d = 0\}$. If d_0^k, STOP.

Step 3: Compute $N_k, D_k, Q_k, d^k, \tilde{f}_j^k$ as follows:

$$N_k = N(x^k) = (g_j(x^k), j \in L), D_k = D(x^k) = \text{diag}(D_j^k, j \in L)$$

$$Q_k = Q(x^k) = (N_k^T N_k + D_k)^{-1} N_k^T, d^k = d_0^k - Q_k^T(\left\|d_0^k\right\|^\tau e + \tilde{f}^k)$$

$$\tilde{f}_j^k = \begin{cases} f_j(x^k + d_0^k), j \in E_k \\ 0, j \in L \setminus E_k \end{cases} \tag{4}$$

where $e = (1,...,1)^T$. If

$$g(x^k;c_k)^T d_0^k > \min\left\{-\left\|d_0^k\right\|^\delta, -\left\|d^k\right\|^\delta\right\} \tag{5}$$

go to step 5.

Step 4: Let $\lambda = 1$:

Lemma 5: If

$$F_{c_k}(x^k + \lambda d^k) \leq F_{c_k}(x^k) + \alpha\lambda g(x^k;c_k)^T d_0^k \tag{6}$$

$$f_j(x^k+\lambda d^k)\leq 0,\ j\in L \tag{7}$$

Let $\lambda_k = \lambda$ and go to step 7; Otherwise go to step Lemma 6.

Let $\lambda = 1/2\lambda$, if $\lambda<\varepsilon$, go to step 5, Otherwise repeat Lemma 5.

Step 5: Compute a direction s^k satisfying the first-order feasible condition by using d^k_0. Let

$$\begin{aligned} \rho_k &= -g(x^k;c_k)^T d_0^k \\ q^k &= \rho_k((1-\beta_k)d_0^k + \beta_k s^k) \end{aligned} \tag{8}$$

where β_k is the first number in the sequence $\left\{1, \frac{1}{2}, \frac{1}{4}, \dots\right\}$ satisfying

$$(1-\beta)g(x^k;c_k)^T d_0^k + \beta g(x^k;c_k)^T s^k \leq \theta g(x^k;c_k)^T d_0^k \tag{9}$$

Step 6: Compute the first number in the sequence $\left\{1, \frac{1}{2}, \frac{1}{4}, \dots\right\}$ satisfying:

$$F_{c_k}(x^k + \lambda q^k) \leq F_{c_k}(x^k) + \nu\lambda g(x^k;c_k)^T q^k \tag{10}$$

$$f_j(x^k+\lambda q^k)\leq 0,\ j\in L \tag{11}$$

Let $d^k = q^k$.
Step 7 Update:
Choose $B_{k+1}\in\Sigma$, $x^{k+1} = x^k+\lambda_k d^k$, $k = k + 1$. Go back to step 1.

Remarks: In step 5 the first-order feasible condition is defined as R: $g_j(x^k)^T s^k<0$, $j\in I(x^k)$. For example, according to the third step in the above algorithm, the following direction satisfying condition R:

$$s^k = -\|d_0^k\|Q^T e$$

where $e = \{1,1,...,1\}\in E^n$, as a matter of fact, because

$$N_k^T s^k = -\|d_0^k\|N_k^T N_k(N_k^T N_k + D_k)^{-1}e = -\|d_0^k\|e + \|d_0^k\|D_k(N_k^T N_k + D_k)^{-1}e,$$

it can be seen that when $j \in I(x^k), D_j^k = f_j^2(x^k) = 0$. So

$$g_j(x^k)^T s^k = -\|d_0^k\| < 0, j \in I(x^k).$$

GLOBAL CONVERGENCE OF ALGORITHM

Lemma 3: If $\{x^k\}$ is bounded, then $c_k \equiv c_{k_0} \triangleq c$, for $k\geq k_0$. Now we make another assumption and let it hold in the remainder of this study.

H 3: $\{x^k\}$ is bounded, B_k consistent positive definite, that is to say, there exists $b\geq a\geq 0$ such that $a\|y\|^2\leq y^T B_k y\leq b\|^2$, $\forall k$, $y\in E^n$ and $k = 1,2,...$

Lemma 4: (1) If $d_0^k = 0$, then x^k is a KKT point of the problem (2).
(2) If $d_0^k \neq 0$, then

$$g(x^k;c_k)^T d_0^k = -\rho_k < 0; g(x^k;c_k)^T q^k < 0$$
$$g_j(x^k)^T q^k < 0, j \in I(x^k)$$

i.e., q^k is a feasible direction of descent of the problem (2).
3) There exists a constant $a_0>0$, such that $g(x^k;c_k)^T q^k\leq -a_0\|q^k\|^2$.

Proof: (1) It is evident according to the definition of the KKT point.
(2) First of all, it is known by (3) that

$$g(x^k;c_k)^T d_0^k \leq -\frac{1}{2}(d_0^k)^T B_k d_0^k < 0, g_j(x^k)^T d_0^k \leq 0,$$
$$g_j(x^k)^T d_0^k \leq 0, j \in I(x^k),$$

then, by (9), there exists $\beta_k\in[0,1]$ such that

$$g_j(x^k)^T q^k = \rho_k g_j(x^k)^T((1-\beta_k)d_0^k + \beta_k s^k) \leq \rho_k\beta_k g_j(x^k)^T s^k < 0, j \in I(x^k),$$

and

$$g(x^k;c_k)^T q^k = \rho_k g(x^k;c_k)^T((1-\beta_k)d_0^k + \beta_k s^k) = \rho_k\left((1-\beta_k)g(x^k;c_k)^T d_0^k + \beta_k g(x^k;c_k)^T s^k\right) \leq \rho_k \cdot \theta g(x^k;c_k)^T d_0^k = -\rho_k^2 \cdot \theta < 0$$

3) Because $\{x^k\}$ is bounded and B_k is positive definite. It is clear that d^k_0, q^k, ρ_k are bounded. So, from (8), there exists a constant $a>0$, such that $\rho_k\geq\alpha\|q^k\|^2$, moreover, there exists a constants $a>0$ such that $g(x^k;c_k)^T q^k \leq -\theta$. $\rho^2_k\leq -a_0\|q^k\|^2$.

Now we turn to prove the global convergence of algorithm A. Suppose $\{x^k\}$ generated by the algorithm is an infinite sequence. Since there are only finitely many choices for sets $E_k\subset L$ and $d_0^k, \tilde{u}^k, q^k$ are bounded. Without loss of generality, assume that there exists an infinite subsequence K such that

$$x^k \to x^*, B_k \to B_*, d_0^k \to d_0^*, \tilde{u}^k \to \tilde{u}^*, \quad q^k \to q^*, E_k \equiv E, k \in K \tag{12}$$

where E is a constant sets.

To desire the global convergence of Algorithm A, the same as cite Zhu and Jiam (2005), similarly, strength the first-order feasible condition R: For $\{d_0^k\}_{k\in K}$, if $d^* \neq 0$, there exists $\delta>0$ such that

$$g_j(x^k)^T s^k \leq -\delta, \forall j \in I(x^*)$$

when $k\in K$ and k is large enough.

It is clear that s^k generated by the above method also satisfies this condition.

Theorem 1: The algorithm either stops at the KKT point of the problem (1) in finite iteration, or generates an infinite sequence $\{x^k\}$ any accumulation point x^* of which is a KKT point of the problem (1).

Proof: According to lemma 2, it is known that the KKT point of the problem (1) is the KKT point of the auxiliary problem (2) if the parameter c is large enough. So we only need to prove that the algorithm either stops at the KKT point of the problem (2) in finite iteration, or generates an infinite sequence $\{x^k\}$ any accumulation point x* of which is a KKT point of the auxiliary problem (2) if the parameter c satisfied some condition.

It is clear that the first part holds. Now we assume that the algorithm generates an infinite sequence $\{x^k\}$ of which x* is a given accumulation. Now, we prove $d^k_0 \to 0$, k∈K. First of all, in view of (5) (6) (10) and (2) of lemma 14, it is evident that $\{F_c(x^k)\}$ is monotonous decreasing. Hence, considering to $x^k \to x^*$, k∈K and assumption (2.1), it is holds that

$$F_c(x^k) \to F_c(x^*),\ k \to \infty \tag{13}$$

By the construction of Algorithm A, there are two cycles between step 1 and step 7, one of which generates $\{x^k\}$ with the form $x^{k+1} = x^k + \lambda_k q^k$ and the other generates it with the form $x^{k+1} = x^k + \lambda_k d^k$. we prove that the claim according the two cycles.

A: there exists $K_1 \in K(|K_1| = \infty)$, such that for all $k \in K_1$, $x^{k+1} = x^k + \lambda_k d^k$ is generated by step 4 and step 7, then from (5), (6), we know

$$0 = \lim_{k\in K_1}\left(F_c\left(x^{k+1}\right) - F_c\left(x^k\right)\right) \le \lim_{k\in K_1} \alpha\lambda_k g\left(x^k;c\right)^T$$
$$d_0^k \le \lim_{k\in K_1}\left(-\partial\varepsilon\left\|d_0^k\right\|^\delta\right) \le 0$$

so, $d_0^k \to 0, k \in K_1$. Since $d_0^k \to d_0^*, k \in K$, it is clear that $d_0^* = 0$. i.e., $d_0^k \to 0,\ k \in K$.

B: Without loss of generality, we assume that for all k∈K, $x^{k+1} = x^k + \lambda_k d^k$ is generated by step 6 and step 7. Suppose that desired conclusion is false, i.e., $d_0^k \neq 0$.

The corresponding QP problem at x* is

$$\begin{aligned} &\min g(x^*;c)^T d + \frac{1}{2} d^T B_* d \\ &\text{s.t.} f_j(x^*) + g_j(x^*)^T d \le 0, j \in L. \end{aligned} \tag{14}$$

It is clear that $d_0^* \neq 0$ is a unique solution of (14). So, $g\left(x^*;c\right)^T d_0^* < 0$.

According to the first-order feasible condition R' and the proof of lemma 13, it can be seen

$$g\left(x^*;c\right)^T q^* < 0, g_j\left(x^*\right)^T q^* < 0, j \in I_* \overset{\Delta}{=} I\left(x^*\right)$$

For k∈K, k large enough, we have

$$g\left(x^k;c\right)^T q^k \le \frac{1}{2} g\left(x^*;c\right)^T q^* < 0$$
$$g_j\left(x^k\right)^T q^k \le \frac{1}{2} g_j\left(x^*\right)^T q^* < 0, j \in I_*$$

Thereby it is easy to prove that the step size λ_k generated by step 6 satisfies $\lambda_k \ge \lambda_* = \{\lambda_k, k\in K\} > 0$, k∈K. So, from (10) (13), we get

$$0 = \lim_{k\in K}\left(F_c\left(x^{k+1}\right) - F_c\left(x^k\right)\right) \le \lim_{k\in K}$$
$$\left(\upsilon\lambda_k g\left(x^k;c\right)^T q^k\right) \le \frac{1}{2}\upsilon\lambda_* g\left(x^k;c\right)^T q^* < 0$$

It is a contradiction, which shows that $d_0^k \to 0, k \in K$, $k \to \infty$. Since $d_0^* = 0$ is the solution of (14), then x^* is a KKT point of the problem (1).

THE RATE OF CONVERGENCE

Here, we continue to discuss the convergence rate of the algorithm. For this purpose, we add an additional assumption on B_k.

H 4: $\{x^k\}$ is bounded with an accumulation point x* and $B_k \to B_*$, $k \to \infty$. The second-order sufficiency conditions with strict complementary are satisfied at the KKT point x^* and the corresponding multipliers μ^*.

According to H 4 and Proposition 1 in research Panier and Tits (1987), we can obtain the following conclusion:

Lemma 5: $\lim_{k\to\infty}\left\|x^{k+1} - x^k\right\| = 0$. Thereby, the entire sequence $\{x^k\}$ converges to x^*, i.e., $x^k \to x^*$, $k \to \infty$.

Lemma 6: For k large enough, then

(1) $\lim_{k\to\infty} d_0^k = 0$

(2) Assume $\tilde{u}_j^k$ and μ_j^* are the corresponding multipliers of d_0^k and x^*, then $\tilde{u}_j^k \to \mu_j^*, j \in I_* \setminus L_2, \tilde{u}_j^k \to \mu_j^* + c, j \in L_2$. $E_k \equiv I(x^*) \overset{\Delta}{=} I_*$

Proof: (1) Since $x^k \to x^*$, $B_k \to B_*$, $k \to \infty$, considering Theorem 1, it is clear to see that $\lim_{k\to\infty} d_0^k = 0$.

2) Since $(d_0^k, \tilde{u}^k)$ is a KKT point pair of (3), then

$$\begin{aligned} &g(x^k;c) + N_k\tilde{u}^k + B_k d_0^k = 0 \\ &(f_j(x^k) + g_j(x^k)^T d_0^k)\tilde{u}_j^k = 0, j \in L \\ &(D_k + \text{diag}(f_j(x^k) g_j(x^k)^T d_0^k, j \in L))\tilde{u}^k = 0 \end{aligned} \tag{15}$$

Denote

$$R_k = N_k^T N_k + D_k + \text{diag}(f_j(x^k) g_j(x^k)^T d_0^k, j \in L)$$

Then, from (15), we get

$$R_k \tilde{u}^k = -N_k^T (g(x^k;c) + B_k d_0^k) \tag{16}$$

Denote

$$N_* = N(x^*),\ D_* = D(x^*),\ Q_* = Q(x^*)$$

because $d_0^k \to 0$, we get $R_k \to N_*^T N_* + D_*$. It is obvious that $(N_*^T N_* + D_*)^{-1}$ exists. It follows that R_k^{-1} exists and $R_k^{-1} \to (N_*^T N_* + D_*)^{-1}$ for k large enough. So

$$\tilde{u}^k = -R_k^{-1} N_k^T (g(x^k;c) + B_k d_0^k) \to -Q_* g(x^*;c) = \pi(x^*;c)$$

In addition, since x^* is a KKT point of the problem (1), then

$$g_0(x^*) + N_* \mu^* = 0, f_j(x^*) \mu_j^* = f_j^2(x^*) \mu_j^* = 0, j \in L, D_* \mu^* = 0,$$

It follows that

$$N_*^T g_0(x^*) + N_*^T N_* \mu^* + D_* \mu^* = 0$$

i.e., $\mu^* = -Q_* g_0(x^*) = \pi(x^*)$. According to Lemma 1, we obtained that $\pi_j(x) = \pi_j(x;c), \forall j \in I_* \backslash L$, $\pi_j(x) = \pi_j(x;c) - c, \forall j \in L_2$. The first part of this lemma holds.

From $E_k = \{j \mid f_j(x^k) + g_j(x^k)^T d = 0\}$ and $d_0^k \to 0$ we obtained $E_k \subseteq I_*$. With the strict complementary condition, we know that if $j \in L_2$, then $\tilde{u}_j^k \to \mu_j^* > 0$;

If $j \in I_* \backslash L_2$, then $\tilde{u}_j^k \to \mu_j^* + c > 0$. So when k large enough, $\tilde{u}_j^k > 0$, $j \in I_*$, it follows that $I_* \subseteq E_k$. So $E_k \equiv L_*$.

Lemma 7: (A) For k large enough, there exists a constant $\eta > 0$ such that

$$\sum_{j \in I_*} \tilde{u}_j^k f_j(x^k) \le -\eta z_k, z_k = (\sum_{j \in I_*} f_j^2(x^k))^{\frac{1}{2}}, \nabla F_c(x^k)^T d_0^k \le -a \|d_0^k\|^2 \tag{17}$$

2) Denote

$$d_1^k = -Q_k^T (\|d_0^k\|^\tau e + \tilde{f}^k) \tag{18}$$

then d^k computed from (4) satisfies that

$$d^k = d_0^k + d_1^k, \|d^k\| \sim \|d_0^k\|, \|d_1^k\| = O(\|d_0^k\|)^2 \tag{19}$$

Proof: (A) From lemma 6, we know $\tilde{u}_j^k > 0$, so, there exists a constant $\eta > 0$ such that

$$\sum_{j \in I_*} \tilde{u}_j^k f_j(x^k) = -\sum_{j \in I_*} \tilde{u}_j^k |f_j(x^k)| \le -\eta z_k$$

Since d_0^k is a KKT point of (3), then

$$\begin{aligned} &g(x^k;c) + N_k \tilde{u}^k + B_k d_0^k = 0, \\ &(f_j(x^k) + g_j(x^k)^T d_0^k) \tilde{u}_j^k = 0, j \in L \end{aligned} \tag{20}$$

$$\begin{aligned} &g(x^k;c)^T d_0^k = -(d_0^k)^T B_k d_0^k + \sum_{j \in L} \tilde{u}_j^k f_j(x^k) \\ &\le -a\|d_0^k\|^2 - \eta z_k \le -a\|d_0^k\|^2 \end{aligned} \tag{21}$$

2) It is clear that $d^k = d_0^k + d_1^k$. If $j \in L \backslash E_k = L \backslash I_*$ then $\tilde{f}_j^k = 0$; If $j \in I_*$, then

$$\tilde{f}_j^k = f_j(x^k + d_0^k) = f_j(x^k) + g_j(x^k)^T d_0^k + O(\|d_0^k\|)^2 = O(\|d_0^k\|)^2,$$

Since $Q_k \to Q_*$, $\tau \in (2,3)$, so from (18), we know $\|d_1^k\| = O(\|d_0^k\|)^2$, thereby, $\|d^k\| \sim \|d_0^k\|$.

In order to obtain superlinear convergence, we must make further assumption.

H 5: Let $\{B_k\}$ satisfy

$$\begin{aligned} &\|P_k (B_k - \nabla_{xx}^2 \tilde{L}(x^k, \tilde{u}^k)) d_0^k\| = o(\|d_0^k\|) \Leftrightarrow \\ &\|P_k (B_k - \nabla_{xx}^2 \tilde{L}(x^*, \mu^*)) d_0^k\| = o(\|d_0^k\|) \end{aligned}$$

where

$$\begin{aligned} &P_k = E_n - A_k (A_k^T A_k)^{-1} A_k^T, A_k = (g_j(x^k), j \in I_*) \\ &\nabla_{xx}^2 \tilde{L}(x^k, \tilde{u}^k) = H(x^k; c_k) + \sum_{j \in I_*} \tilde{u}_j^k H_j(x^k), \\ &\nabla_{xx}^2 \tilde{L}(x^*, \mu^*) = H(x^*; c) + \sum_{j \in I} \mu_j^* H_j(x^*) \end{aligned}$$

Imitating the proof of Theorem 4.2 in Jian *et al.* (2005), we can obtain the following result.

For k large enough, step 5 and step 6 are no longer performed in the algorithm and $\lambda_k \equiv 1$, $x^{k+1} = x^k + d^k$ in step 4.

Moreover, in view of lemma 7 and the way of Theorem 5.2 in Facchinei and Lucidi (1995), we may obtain the following theorem.

Under all above-mentioned assumptions, the algorithm is superlinearly convergent, i.e., the sequence $\{x^k\}$ generated by the algorithm satisfies $\|x^{k+1} - x^*\| = o(\|x^k - x^*\|)$.

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