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Convex Characterization of Simulated Droughts and Floods of Water Bodies

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Abstract: Convexity is considered as one of the basic descriptors of shapes. In this study, the characterization of the convexity of simulated droughts and floods of water bodies is performed. First, concepts of mathematical morphology employed to generate simulated droughts and floods of water bodies. The average convexity measures of the generated simulated droughts and floods are computed. It is observed that droughting reduces the convexity of water bodies, while flooding increases the convexity of water bodies. A power law relationship is observed between the average convexity measures of the simulated droughts/floods and the level of droughting/flooding and areas of water bodies. The scaling exponent of this power law, which is named as a fractal dimension, indicates the rate of change of convexity of simulated droughts/floods of water bodies over varying levels of droughting/flooding.

Key words: Water bodies, mathematical morphology, simulated drought and floods, convexity, fractal dimension

INTRODUCTION

Convexity is considered as one of the basic descriptors of shapes. Convexity in image processing has been studied for quite some time (Valentine, 1964; Stern, 1989; Boxer, 1993; Held and Abe, 1994; Popov, 1996; Zunic and Rosin, 2004; Rosin and Mumford, 2004; Rahtu *et al*., 2004; Kolesnikov and Fränti, 2005; Rahtu *et al*., 2006; Varošanec, 2007) and has numerous applications, including shape decomposition (Latecki and Lakämper, 1999; Rosin, 2000), camouflage breaking (Tankus and Yeshurun, 2000), object indexing (Latecki and Lakämper, 2000), measurement of border irregularities measurement in medical image analysis (Lee *et al*., 2003), handwritten word recognition (Kapp *et al*., 2007) and estimation of derivatives of holomorphic functions (Li, 2007).

An object P is said to be convex if it has the following property: If points A and B belong to P, then all points from the line segment [AB] belong to P as well. The smallest convex set which includes P is called the convex hull of P and is denoted as CH(P). The convexity measure C(P) is defined to be:

$$C(P) = \text{Area}\ (P)/\text{Area}\ (CH(P)) \tag{1}$$

Convexity measures have the following properties (Zunic and Rosin, 2004):

- Convexity measures have the range of (0,1)
- The convexity measure of a given object equals 1 if and only if this object is convex
- There are objects whose convexity measure is arbitrarily close to 0
- The convexity measure of an object is invariant under similarity transformations of the object.

In this study, the characterization of the convexity of simulated droughts and floods of water bodies is performed. It is shown that a power law relationship exists between the average convexity measures of simulated droughts/floods of water bodies and the level of droughting/flooding.

MATHEMATICAL MORPHOLOGY

Mathematical morphology is a branch of image processing that deals with the extraction of image components that are useful for representational and descriptional purposes. The fundamental morphological operators are discussed in Matheron (1975), Serra (1982) and Soille (2003). Morphological operators generally require two inputs; the input image A, which can be in binary or gray scale form and the kernel B, which is used to determine the precise effect of the operator.

Dilation sets the pixel values within the kernel to the maximum value of the pixel neighborhood. Binary dilation fills the small holes inside particles and gulfs on the boundary of objects, enlarges the size of the particles and may connect neighboring particles (Duchene and Lewis, 1996). The dilation operation is expressed as:

$$A \oplus B = \{a+b: a \in A, b \in B\} \tag{2}$$

Erosion sets the pixels values within the kernel to the minimum value of the kernel. Binary erosion removes isolated points and small particles, shrinks other particles, discards peaks on the boundaries of objects and disconnects some particles (Duchene and Lewis, 1996). Erosion is the dual operator of dilation:

$$A \ominus B = (A^c \oplus B)^c \qquad (3)$$

where A^c denotes the complement of A and B is symmetric with respect to reflection about the origin.

Drought and flood simulation is implemented by performing erosion and dilation, respectively, on water bodies using square kernels. Erosion reduces the area of water bodies, mimicking droughting, while dilation increases the area of water bodies, mimicking flooding. The level of droughting/flooding is indicated by the kernel size.

Figure 1 shows a number of water bodies situated in the flood plain region of Gothavary River, India. The water bodies were traced from IRS 1D remotely sensed data. Due to the impracticalities of dealing with incomplete water bodies, only the complete water bodies are considered (Fig. 2). Simulated droughts (Fig. 3) and floods (Fig. 4) of the water bodies for levels of 1 to 15 are computed. The areas of the generated simulated droughts and floods are shown in Table 1.

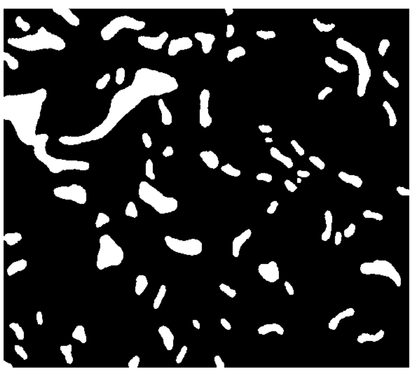


Fig. 1: Water bodies traced from IRS 1D remotely sensed data

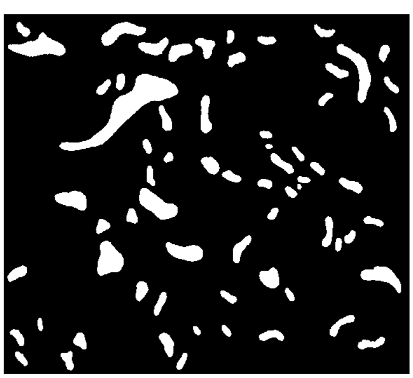
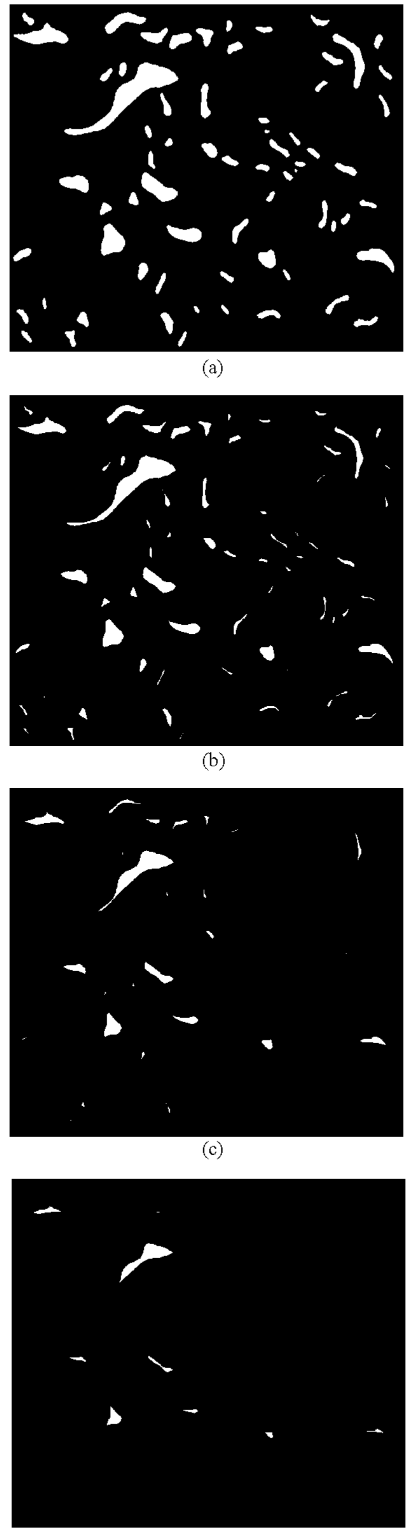


Fig. 2: The water bodies after removal of incomplete water bodies



(a)

(b)

(c)

(d)

Fig. 3: The generated simulated droughts of the water bodies at droughting levels of: (a) 3 (b) 7 (c) 11 (d) 15

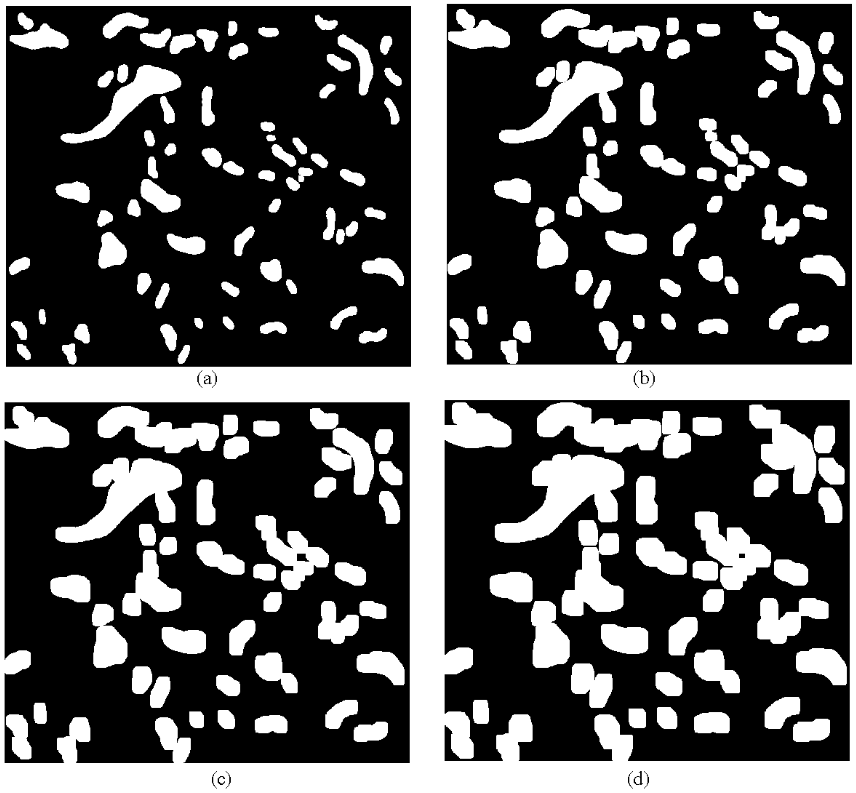


Fig. 4: The generated simulated floods of the water bodies at flooding levels of: (a) 3 (b) 7 (c) 11 (d) 15

Table 1: Areas of the generated simulated droughts and floods of the water bodies

	Area (pixels)	
Levels	Drought	Flood
1	137837	137837
2	123336	152789
3	109565	167979
4	96328	183647
5	83764	199532
6	71844	215862
7	60839	232283
8	51141	249127
9	42863	265919
10	35980	283048
11	30230	300047
12	25380	317358
13	21220	334503
14	17737	351937
15	14794	369093

CHARACTERIZATION OF CONVEXITY OF WATER BODIES

The convex hulls of the generated simulated droughts (Fig. 5) and floods (Fig. 6) of the water bodies are computed using the convex hull computing neural network (CHCNN) algorithm proposed in Leung *et al.* (1997). The algorithm is based on a two-layered neural network, topologically similar to ART, with is an adaptive training strategy called excited learning. CHCNN provides a parallel on-line and real-time processing of data which after training, yields two closely related approximations, one from within and one from the outside, of the desired convex hulls. The accuracy of the approximated convex hull is approximately $O[K^{-1/(n-1)}]$, where K is the number of neurons in the output layer of the CHCNN. When K is taken to be sufficiently large, CHCNN can generate any accurate approximate convex hull.

The average convexity measures of the simulated droughts and floods are computed (Table 2). It is observed that droughting reduces the convexity of water bodies, while flooding increases the convexity of water bodies.

Log-log plots of the average convexity measures of the simulate droughts/floods C against the level of droughting/flooding r is drawn (Fig. 7 and 8). Power law relationships are observed in both plots. These power laws take the following form:

$$C = c*r^{D} \quad (4)$$

These power law relationships arise as a consequence of the fractal properties of the convexity of simulated droughts and floods of water bodies. In

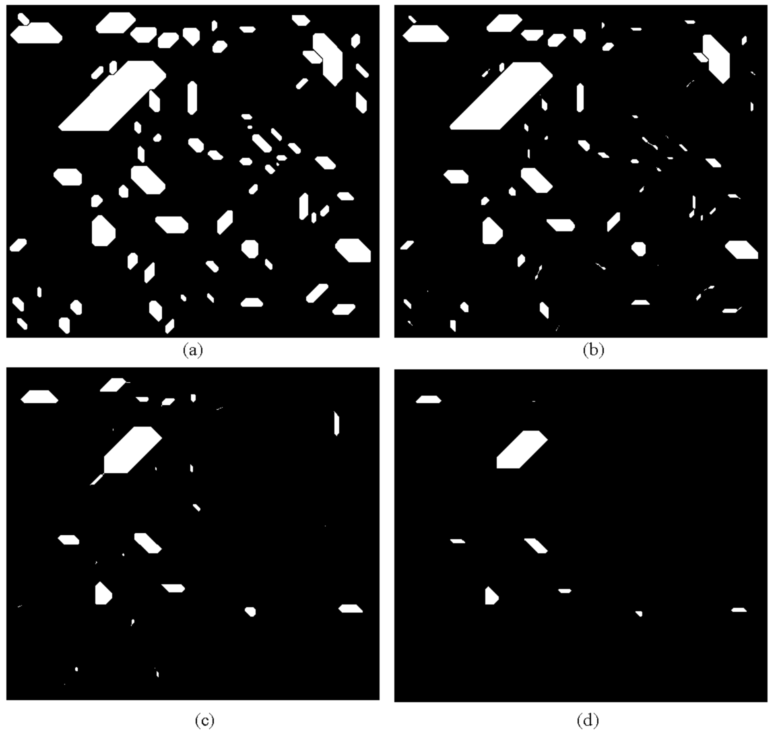


Fig. 5: Convex hulls of the corresponding simulated droughts in Fig. 3

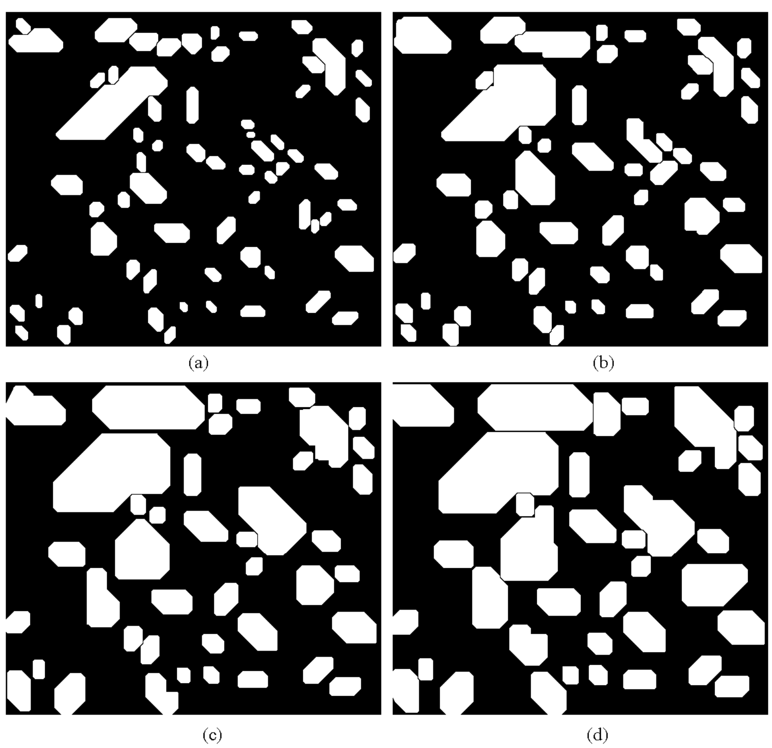


Fig. 6: Convex hulls of the corresponding simulated floods in Fig. 4

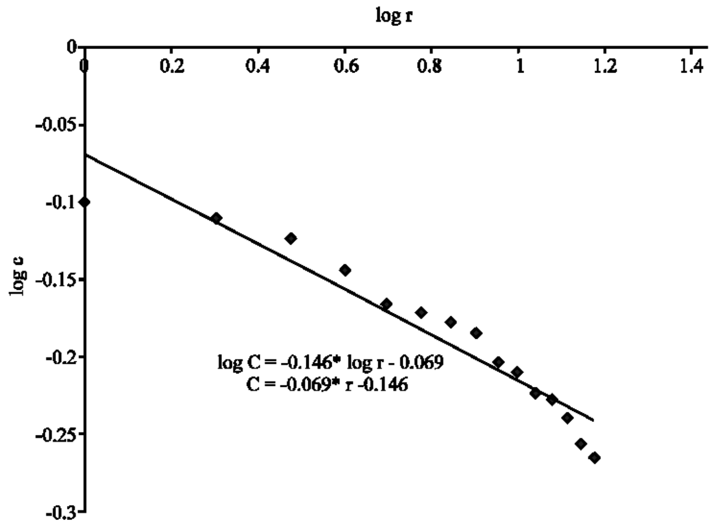


Fig. 7: Log-log plot of the average convexity measures of the simulated droughts against the droughting level

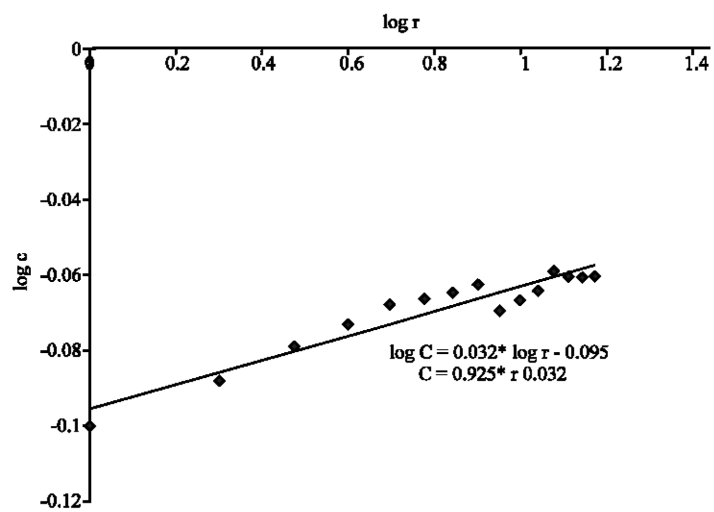


Fig. 8: Log-log plot of the average convexity measures of the simulated floods against the flooding level

Table 2: Average convexity measures of the generated simulated droughts and floods of the water bodies

	Convexity measures	
Levels	Drought	Flood
1	0.794670	0.794670
2	0.774333	0.817806
3	0.751657	0.832742
4	0.717467	0.844875
5	0.681786	0.855180
6	0.672416	0.858126
7	0.707331	0.860983
8	0.760615	0.864736
9	0.822571	0.852731
10	0.761111	0.858281
11	0.755434	0.863198
12	0.795720	0.871762
13	0.766094	0.871172
14	0.753631	0.870377
15	0.600934	0.870888

Eq. 4, c is a constant of proportionality, while D is the fractal dimension of the convexity of simulated droughts/floods of water bodies, which indicates the rate of change of convexity of simulated droughts/floods of water bodies over varying levels of droughting/flooding. D has a positive value for flooding and a negative value for droughting.

CONCLUSION

In this study, the characterization of the convexity of simulated droughts and floods of water bodies is performed. It was observed that droughting reduces the convexity of water bodies, while flooding increases the convexity of water bodies. A power law relationship was observed between the average convexity measures of the simulated droughts/floods and the level of droughting/flooding. The scaling exponent of this power law, which was named as a fractal dimension, indicates the rate of change of convexity of water bodies over varying levels of droughting/flooding.

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