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Portfolio Value-at-Risk with Time-Varying Copula: Evidence from Latin America

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Abstract: Model risk in the estimation of value-at-risk is a challenging threat for the success of any financial investments. The degree of the model risk increases when the estimation process is constructed with a portfolio in the emerging markets. The proper model should both provide flexible joint distributions by splitting the marginality from the dependencies among the financial assets within the portfolio and also capture the non-linear behaviours and extremes in the returns arising from the special features of the emerging markets. In this study, we use time-varying copula to estimate the value-at-risk of the portfolio comprised of the Bovespa and the IPC Mexico in equal and constant weights. The performance comparison of the copula model to the EWMA portfolio model made by the Christoffersen back-test shows that the copula model captures the extremes most successfully. The copula model, by estimating the portfolio value-at-risk with the least violation number in the back-tests, provides the investors to allocate the minimum regulatory capital requirement in accordance with the Basel II Accord.

Key words: Time-varying copula, portfolio value-at-risk, Latin American equity markets, portfolio GARCH

INTRODUCTION

In this research, we estimate the portfolio value-at-risk by using copula method. The paper presents an application of copula for the portfolio risk estimation for the portfolio constructed with two Latin American equity markets, namely Mexico and Brazil. As much as we know, there are not any research papers to estimate neither the value-at-risk for a single market index nor a portfolio using time series data from Latin American equity markets. We present an application for portfolio risk estimation with a recent methodology, namely copula. In addition, we present a guide to choose the proper methods among different copula methods by using akaike comparisons.

Parametric models like GARCH and its derivatives have certain assumptions, whether normal or asymmetric, on the distributional characteristics of the individual financial time series or portfolio returns. On the other hand, financial markets in the emerging economies might show abnormal behaviours due to their chaotic and dynamic climates. For that reason, modeling value-at-risk in the emerging markets requires flexible and non-parametric approaches to reach a successful estimation performance.

The copula method is based on the Sklar (1959) theorem which describes the copula as an indicator of the dependencies among the variables. According to Dowd (2004), the strength of the copula comes from its feature that it does not have any assumptions on the joint distributions among the financial assets in a portfolio. It creates N marginal distribution for the joint distribution with N dynamics. In fact, the normality is rarely an adequate assumption in finance. For example, Longin and Solnik (2001) and Ang and Chen (2002) empirically show that asset returns are more highly correlated during volatile markets and during market downturns. For that reason, the deviation from normality might lead to inadequate value-at-risk estimations.

The copula as an risk measurement technique has been started to use in the financial risk estimation in recent years. Frey and McNeil (2003), Hamerle and Rasch (2005) and Goorbergh *et al*. (2005) use the copula in the option valuation. The method is employed by Junker *et al*. (2006) in the analysis of term structures of the interest rates; Giesecke and Goldberg (2004) and Meneguzzo and Vecchiato (2004) in the credit risk analysis and Neslehova *et al*. (2006) for the calculation of the operational risk at the banks. The researches using copula for the estimation of value-at-risk go back from the last five years. The copula methodology used in the first researches does not include conditionality, in other words, it does not have time-varying feature. Empirical and methodological discussions for the constant copula can be found in Cherubini *et al*. (2004), Rockinger and Jondeau (2001), Fortin and Kuzmics (2002) Embrechts *et al*. (2002 and 2003), Chen and Fan (2002), Cherubini and Luciano (2001), Fermanian and Scaillet (2003) and Li (2000) and Rosenberg (2003).

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Patton (2001) constructs the conditional copula by allowing the first and second conditional moments to vary on time. After the methodological expansion of Patton (2001), the conditional copula has been started to use in the estimation of value-at-risk. Jondea and Rockinger (2006) use normal GARCH based copula for the value-at-risk estimation of the portfolio composed of international equity indices. Junker *et al.* (2006) employ copula to model the yield curve on the US interest rates between 1982-2001. Chen and Fan (2006) use copula method to construct a semi-parametric model based on the Markov chain GARCH.

Poon *et al.*. (2004) use Gaussian and Gumbel copulas for estimation of loss of the portfolio with linear assets. Diversification breakdown in a portfolio is examined by Loretan and English (2000), Campbell *et al.* (2002) and Ang and Chen (2002). Rockinger and Jondeau (2001) use a dependence measure to check if the linear dependence varies with the time by applying the Plackett's copula with the returns of European stock indices, the S&P500 and the Nikkei 225. They underline that the disadvantage of the Plackett's copula is that it cannot be easily used with portfolios composed of financial assets more than two. Cherubini and Luciano (2001) estimate the value-at-risk using the copula and the historical empirical distribution in the estimation of marginal distributions.

The copula can be seen as an alternative for the multivariate GARCH models. Lee and Long (2005), on the other hand, combine copula with multivariate GARCH model, which allows very flexible joint distributions. They propose copula-multivariate GARCH model with uncorrelated dependent errors to compare with three MGARCH models and empirically show that the mixed model outperforms multivarate GARCH in terms of in-sample model selection criteria and out-of-sample multivariate density forecast. Palaro and Hotta (2006) use a mixed model with the conditional copula and multivariate GARCH to estimate the value-at-risk of a portfolio composed of Nasdaq and S&P500 indices. The copula method is used with expected short fall to estimate the fat tails. Embrechts *et al.* (2005) apply the copula method to create value-at-risk scenarios for the worst cases. Juri and Wuthrichts (2002) combine the copula with extreme value theory, Mendes and Souza (2004) do with the stress scenarios to calculate the value-at-risk amount.

We write Matlab codes to estimate the models used in this research. The Matlab codes constructed by Patton (2001, 2006a and b) and Sheppard (2006) for the conditional copula are the references for us to create the software for the models employed. Quantile copula multivariate GARCH is calculated with two steps maximum probability in parallel to the method used by Bauwens *et al.* (2006).

MATERIALS AND METHODS

Conditional copula was constructed based on the Sklar's (1959) dependency theory. Under the assumption that for t = 1,........T, we assume that Eq. 6 represents the historical data till time t. Under this assumption, Sklar (1959) theorem can be expressed Eq. 1.

$$\xi_t = \sigma\ \{X_{1t-1}, X_{2t-1}, \ldots\ldots, X_{nt-1}, X_{t-2}, X_{2t-2}, \ldots\ldots, X_{nt-2}, \ldots..\}$$

$$F_1(X_{1t}, X_{2t} \ldots\ldots\ldots\ldots X_{nt} \mid \xi_t) = C_t(F_{1t}(X_{1t} \mid \xi_t), F_{2t}(X_{2t} \mid \xi_t), \ldots.., F_{nt}(X_{nt} \mid \xi_t \mid \xi_t) \tag{1}$$

In the equation, C_t should have copula function for each t. Patton (2001 and 2006a) assumes that conditional mean is created on autoregressive process, while conditional variance is done based on GARCH (1,1) process. Symmetric Joe-Clayton (SJC) can be expressed with Eq. 2 (Patton, 2006a). In the equation, τ^U and τ^L represent the tail distributions.

$$C_{SJC}(u,v \mid \tau^U, \tau^L) = 0.5.(C_{SJC}(u,v \mid \tau^U, \tau^L) + (C_{SJC}(1\text{-}u, 1\text{-}v \mid \tau^U, \tau^L) + u + v - 1) \tag{2}$$

The equation of $\tau^U = \tau^L$ makes the model symmetric. On the other hand, Patton (2001, 2006a, 2006b) uses copula to model the conditional dependency varying in time, as well. The minimum and maximum dependency values of conditional symmetric Joe-Clayton copula are expressed on Eq. 3 and 4, respectively (Patton, 2006a).

$$\tau_t^U = \Lambda\left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U . \frac{1}{10}\sum_{j=1}^{10}\left|u_{t-i} - v_{t-i}\right|\right) \tag{3}$$

$$\tau_t^L = \Lambda\left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L . \frac{1}{10}\sum_{j=1}^{10}\left|u_{t-i} - v_{t-i}\right|\right) \tag{4}$$

The equation of $\Lambda(x) \equiv (1 + e^{-x})$ is the logistic transformation that fixes τ^U and τ^L parameters to take (0,1) values. For reliable empirical evidence, we should create marginal distribution for each stock index and a conditional copula function for the whole portfolio. Marginal distribution is calculated with normal GARCH (1,1) expressed in Eq. 5.

$$\begin{gathered} X_t = \varepsilon t \\ h_t^x = \omega_x + \beta_x h^x_{t-1} + \alpha_x \varepsilon^2_{t-1} \\ \varepsilon_t / h_t^x \sim N(0,1)\,(x) \end{gathered} \tag{5}$$

X_t represents the logarithmic difference of the financial asset. After estimating marginal distributions, the joint distribution of two financial assets is reached. The correlation parameter for conditional symmetric Joe-Clayton copula, ρ, is expressed with Eq. 6.

$$P_t = \Lambda(\omega_\rho + \beta_\rho \rho_{t-1} + \alpha_\rho 1/\rho \sum_{j=1}^{p} \varnothing^{-1}(u_{t-j}) \varnothing^{-1}(v_{t-j})) \quad (6)$$

$\Lambda(x)$ is the hyperbolic tangent function fixing ρ_t between (-1,1). The Eq. 7 is, on the other hand, dependency parameter that enables to capture the changes in the dependency.

$$1/\rho \sum_{j=1}^{p} \varnothing^{-1}(u_{t-j}) \varnothing^{-1}(v_{t-j})) \quad (7)$$

Christoffersen (1998) test focuses on the probability of failure rate. The importance of testing conditional coverage arises with volatility clustering in financial time series. Christoffersen test might be more proper to detect fat-tail in the returns as compared to the alternative models like Kupiec test. To apply the test, we firstly define $p^\alpha = \Pr(y_t < VaR_t(\alpha))$ and test H_0: $p^\alpha = \alpha$ against H_1: $p^\alpha \neq \alpha$. We consider $\{1(y_t < VaR(\alpha)\}$ which has a binomial likelihood $L(p^\alpha) = (1-p^\alpha)^{n_0} (p^\alpha)^{n_1}$ (Saltoglu, 2003).

where

$$n_0 = \sum_{t=R}^{T} 1(y_t > VaR_t(\alpha)) \text{ and } n_1 = \sum_{t=R}^{T} 1(y_t < VaR_t(\alpha))$$

Under the null hypothesis, it becomes $L(\alpha) = (1-\alpha)^{n_0} \alpha^{n_1}$. Thus the likelihood ratio test statistics is in equation below.

$$LR = -2\ln(L(\alpha)/L(\hat{p})) \xrightarrow{d} \chi(1) \quad (8)$$

We estimate VaR with α = 0.01 confidence interval and backtest VaR models with Kupiec Christoffersen out-of-sample forecasting test. We chose 99% confidence level in accordance to Basel II requirement.

RESULTS

Data: In this research, we examine the performance of copula methodology for Latin American equity portfolio for the period from 02.01.2001 to 28.02.2007 with 1498 daily observations. We choose time-varying copula and EWMA to estimate the value-at-risk of the portfolio comprised of the Bovespa and the IPC Mexico in equal and constant weights. Data is received from Bloomberg. Figure 1 shows estimated equity indexes in log-differenced series and Fig. 2 shows scatter plot of two stock indexes. There is positive but non-constant correlation between returns of two indexes, but they have different tails. In that framework, data might present a valuable opportunity to observe using effects of copula which employs marginal distributions effects.

Empirical results: Table 1 shows results of unit root tests of Phillips-Peron (Phillips and Peron, 1988) and

Table 1: Unit root test results

Capula	Phillips-Peron test I(1)	Augmented D-F test I(1)
Bovespa	0.689875	0.698002
IPS Mexico	1.985460	1.857960
LLBovespa	-36.929700*	-36.947500*
LLIPS Mexico	-34.798100*	-34.868600*

*Stationary in 1% confidence interval

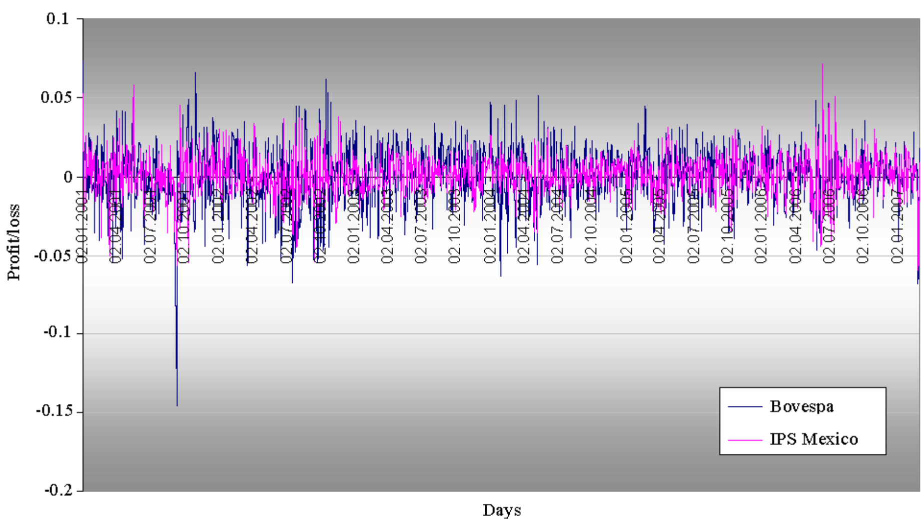

Fig. 1: Log-differenced series (Bovespa and IPS Mexico)

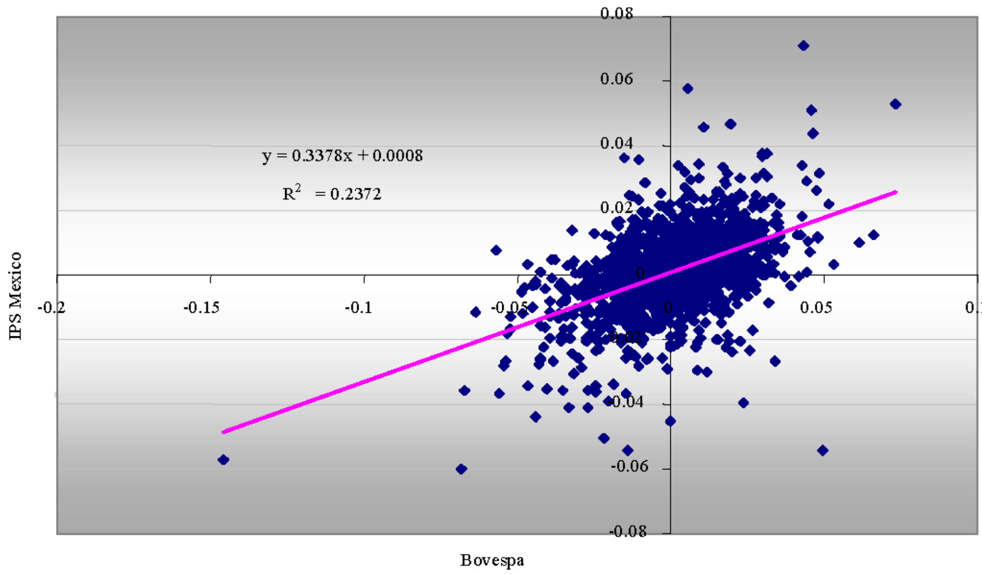

Fig. 2: Scatter plot of Bovespa and IPS Mexico

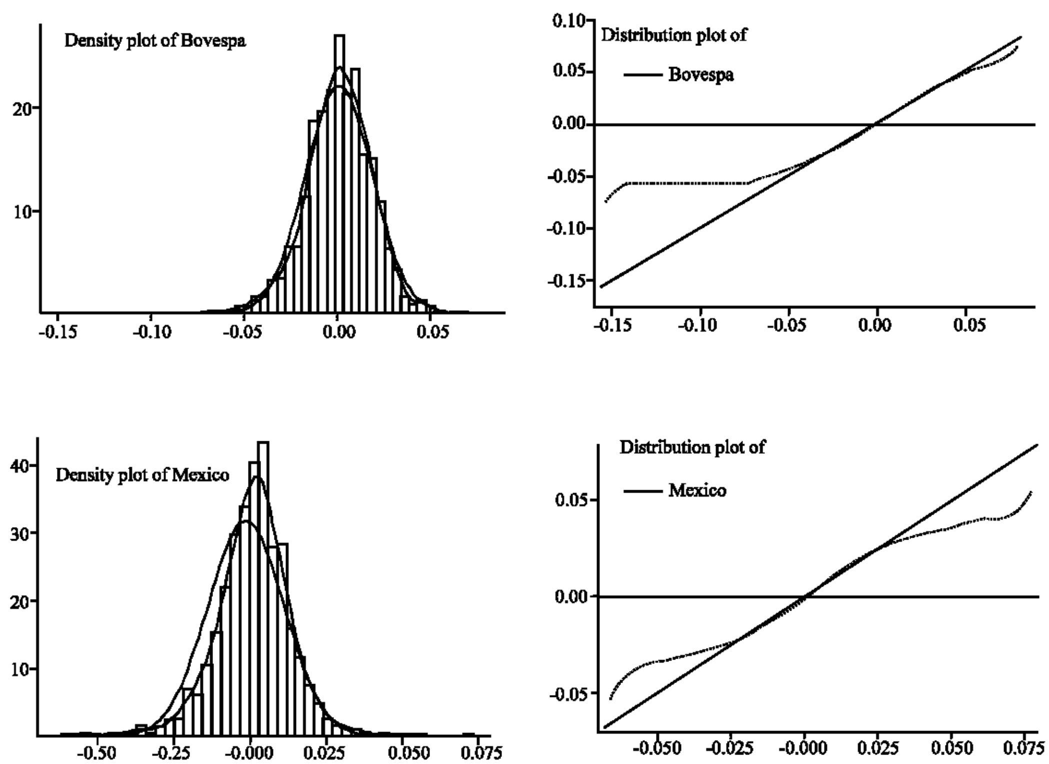

Fig. 3: Density and distribution plots of Bovespa and IPS Mexico

Augmented Dickey Fuller (Dickey and Fuller, 1981). Series are not stationary at I(0) where stationary at I~(1). I(1) represents log-difference series indicating that portfolio VAR should be estimated with log-differenced series.

Descriptive statistics of the series are presented in Table 2. Kurtosis and skewness for both series are not close to normal distribution based on Jargue-Bera statistics and series are left tailed as drawn in Fig. 3. Distributions of sensity plot of the series also clearly indicate that they are not normally distributed. Linear correlation between series is 48.7% which underlines the fact that copula can be applied to improve forecasting with marginal distribution effects.

In Table 3, akaike values of alternative copula models are showed for the portfolio. Student-t copula is the most proper copula model among non-conditional copula models. On the other hand, symmetrized joe-clayton copula is the best model as the conditional model and among all copula models. Therefore, we choose symmetrized joe-clayton copula to estimate the alue-at-risk of the portfolio.

In Table 4, parameters for the symmetrized joe-clayton copula and conditional symmetrized Joe-Clayton copula are reported. For standard symmetrised joe-clayton copula, τ^U and τ^L parameters are 0.45 and 0.20 (Patton, 2001) respectively. In Table 4, symmetrized Joe-Clayton copula τ^U and τ^L paremeters are 0.34191 and 0.23436. Copula likelihood also indicates that conditional symmetrized joe-clayton copula is better than unconditional symmetrized Joe-Clayton copula.

Figure 4 displays contour plot of symmetrized Joe-Clayton Copula for the portfolio. Copula simulates marginal distributions and the simulated distribution of the portfolio captures empirical distribution shown in Fig. 2.

Figure 5 shows time-varying symmetrised Joe-Clayton copula for the portfolio. We can clearly argue that lower tail depencende is more volatilite than upper tail dependence and correlation varies over time.

Figure 6 presents a comparison for conditional symmetrized Joe-Clayton Copula and EWMA for Latin American equity portfolio. Symmetrized Joe-Clayton Copula have better performance on capturing the extremes rather than EWMA does.

Table 5 shows the results of Christoffersen (1988) backtests and number of exceptions for each value-at-risk models. As the backtests show time-varying Joe-Clayton Copula has better estimation performance than that of EWMA. In addition, number of exceptions is 43 for EWMA while it is 36 for time-varying Joe-Clayton Copula. The results display the fact that the Joe-Clayton Copula

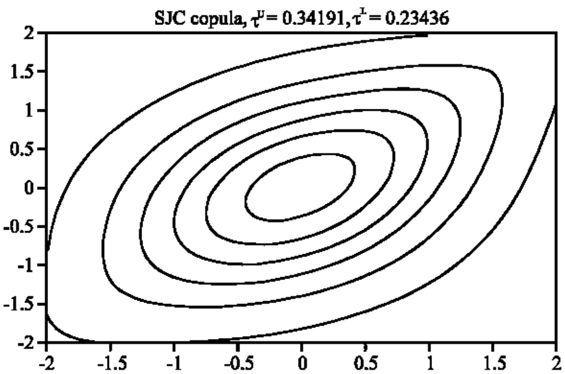

Fig. 4: SJC Copula Contour plot for Bovespa and IPS Mexico

Table 2: Main statistical properties of log differenced series

Statistical properties	LLBovespa	LLIPS Mexico
Mean	0.000746561	0.00108077
Standard Dev.	0.01817	0.0126084
Kurtosis	6.42424	6.02487
Skewness	-0.492586	-0.141769
Min	-0.145659	-0.0597751
Max	0.0733529	0.0711869
Jarque-Bera statistic	792.968	576.503
Linear correlation	0.487	

LL represents log-differencing. Kurtosis and skewness is 3 and 0 for normal (gaussian) distribution. Jargue-bera stat test whether the residuals have a normal distribution or not. Linear correlation is estimated with $\rho_{X,Y} = cov(X,Y)/\sigma_X\sigma_Y$ where X represents LLBovespa and Y represents LLIPS Mexico

Table 3: Comparison of copula models

Model	Akaike value
Normal copula	-382.75
Clayton copula	-346.13
Frank copula	Inf
Gumbel copula	-346.15
Student-t copula	-412.52
Symmetrised joe-clayton copula	-404.15
Conditional normal copula	-383.88
Conditional gumbel copula	-426.65
Conditional symmetric joe-clayton copula	-441.77

Table 4: Copula models

Parameters	Value
Symmetrised joe-clayton copula	
τ^U	0.34191 [0.1247]
τ^L	0.23436 [0.0787]
Copula Likelihood	202.08
Conditional symmetrised joe-clayton copula	
ω^U	-1.1943 [0.054]
α^U	-3.3279 [0.1401]
β^U	3.2087 [0.0367]
ω^L	-1.5913 [0.1283]
α^L	-1.1755 [1.6746]
β^L	3.4716 [0.0917]
Copula Likelihood	220.89

[ ] Standard errors

Table 5: Christoffersen Backtest Results*

Models	Test value*	No. of exceptions
EWMA portfolio var	0.855296	43
Conditional SJC copula var	0.193480	36

*99% confidence interval

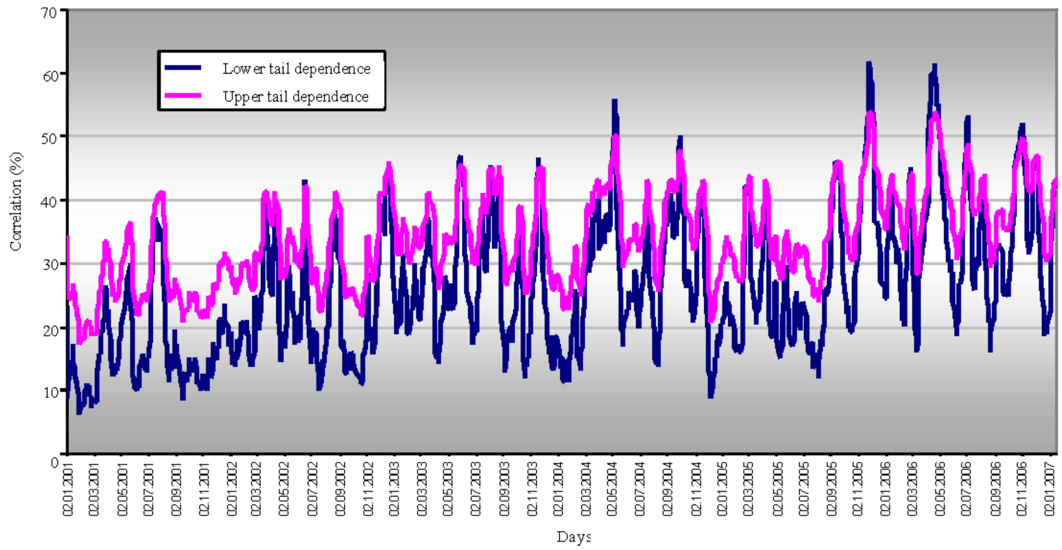

Fig. 5: Time varying correlation based on conditional SJC copula

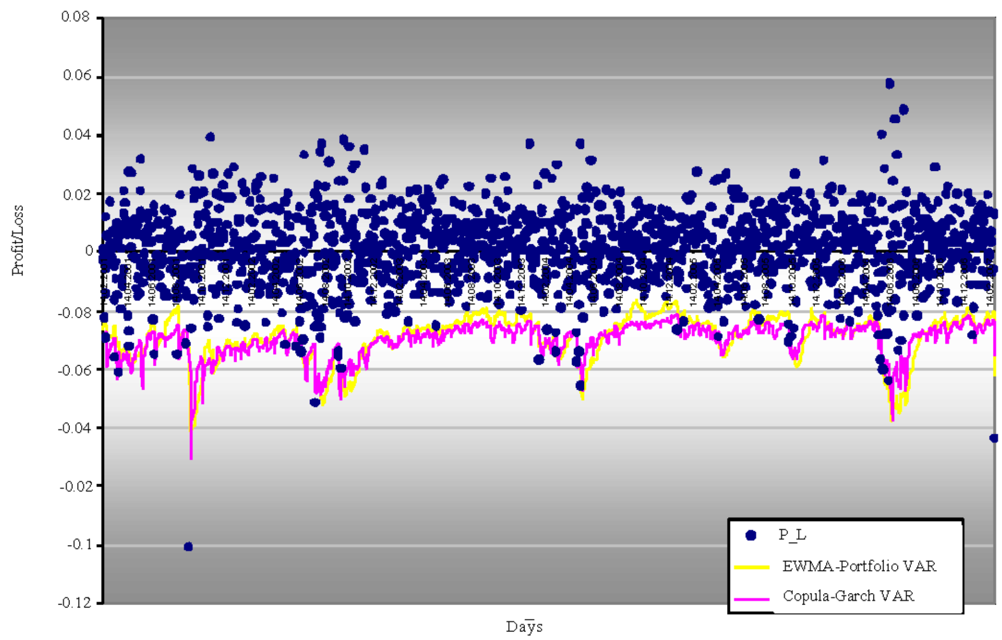

Fig. 6: Portfolio return and EWMA portfolio VAR

is both the most proper model among its alternatives examined in this research and also proper for regulatory capital requirement as it is on Basel II Accord.

CONCLUSIONS

The importance of model risk at value-at-risk calculation increases as the portfolio becomes multivariate. In this research we use copula to estimate the risk of portfolio that is constructed with Bovespa and IPC Mexico in equal and constant weights. In that perspective, the model estimates the risk of main Latin American equity markets in a basket.

Statistically significant model for the estimation of portfolio risk should both provide flexible joint distributions by splitting the marginality from the dependencies among the financial assets within the portfolio and also capture the non-linear behaviours and extremes in the returns arising from the distinguished features of the emerging markets. In this research we employ time-varying copula to estimate the value-at-risk of the Latin American equity portfolio.

EWMA is used as the benchmark against copula to evaluate the prediction performance. The results of Christoffersen (1988) back-test algorithm show that the copula model captures the extremes more successfully. Conditional Symmetrized Joe-Clayton Copula is reasonably weel to estimate the value-at-risk of the Latin American equity portfolio. While the exceptions number for EWMA is 43, it decreases into 36 for conditional symmetrized Joe-Clayton Copula. The back-test results display another fact that conditional symmetrized Joe-Clayton Copula satisfies regulatory capital requirement in accordance to Basel II Accord in terms of exception numbers.

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