Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 7 (14): 1936-1941, 2007
ISSN 1812-5654

A Control Chart Based on Ranked Data

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Abstract: In this study, a modified procedure of ranked set sampling is used to build the limits of X-bar chart. The performance of the X-bar chart based on ranked data is compared with the traditional limit specifications based on simple random samples. The simulation results showed that the control charts using the robust ranked set samples dominate the control charts based on other sampling techniques in terms of Average Run Length (ARL).

Key words: Average run length, X-bar chart, ranked set sampling

INTRODUCTION

The quality control charts have been the focus of lots of researchers in different areas of research including industrial, medical and engineering sectors (Champ and Woodall, 1987; Albers *et al*., 2006). The quality control charts using the SRS have been discussed in the literature for different cases (Montgomery, 2001). Suppose that X_{ij} represents the ith observation unit in the jth SRS of size n, where i = 1,2, ..., n and j =1, 2, ..., r. We assume that the underlying distribution is normal with mean μ and variance σ^2 and we intend to use the well-known x-bar control chart. It is clear that when the population mean and variance (μ, σ^2) are known, then the X-bar chart for the sample mean at the j^{th} cycle, $\bar{X}_j$ is given by:

$$\begin{aligned} LCL &= \mu - \frac{3\sigma}{\sqrt{n}} \\ CL &= \mu \\ UCL &= \mu + \frac{3\sigma}{\sqrt{n}} \end{aligned} \tag{1}$$

where, LCL, CL and UCL are the lower control limit, central limit and the upper control limit, respectively. On the other hand, we encounter the most common case where one or both population parameters are unknown in which we need to construct an estimated chart such that

$$\begin{aligned} LCL &= \hat{\mu} - 3\hat{\sigma}_{\bar{X}_{SRS}} \\ CL &= \hat{\mu} \\ UCL &= \hat{\mu} + 3\hat{\sigma}_{\bar{X}_{SRS}} \end{aligned} \tag{2}$$

where,

$$\hat{\mu} = \frac{1}{m}\sum_{j=1}^{r}\sum_{i=1}^{n} X_{ij} \quad \text{and}$$

$$\hat{\sigma} = \frac{\Gamma\left(\frac{n-1}{2}\right)}{r\sqrt{n}\left(\frac{2}{n-1}\right)^{2}\Gamma(n-2)}\sum_{j=1}^{r}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_{ij}-\bar{X}\right)^{2}}$$

In this study, we use the Average Run Length (ARL) to check the coverage of these control charts under the SRS mechanism and normal distribution assumption. It is known that the

$$ARL = \frac{1}{\alpha}$$

if the process is under control, otherwise

$$ARL = \frac{1}{\beta}$$

where α and β are the probability of type I and II error, respectively (Montgomery, 2001).

Eventually, sampling techniques are very essential in all statistical applications. This study will consider the sampling scheme of McIntyre (1952) which is known by the Ranked Set Sampling (RSS). McIntyre introduced the RSS methodology to reduce the cost and to boost the efficiency of the estimation process through monitoring the variance of the parameter estimate.

Following the footsteps of the noticeable work of McIntyre, numerous extensions and modifications of RSS have been introduced in the literature (Jesse, 2007; Modarres *et al*., 2006). However, Takahasi and Wakimoto (1968) were the first in providing the mathematical aspects of RSS that give the validation and support to the numerical computation concluded earlier. On the other hand, errors in ranking may sometimes appear in the process and as a result it is expected that the efficiency of the parameter estimates to be reduced. The need to solve this pitfall has been pointed out in many different occasions (Samawi *et al*., 1996). Control charts based on ranked set samples have been resported by Muttlak and Al-Sabah (2003).

RANKED SET SAMPLING

The main concept of RSS is based on a repeated systematic selection from a Simple Random Sample (SRS) each of size n. From the first SRS the smallest observation is selected. Then we choose the second ordered observation from the second SRS. The process is continued until we select the maximum observation from the last SRS. The selected observations are considered an RSS of size n denoted by:

$$X_{i[i:n],j},\ i = 1, 2, ..., n,$$

where, $X_{i[i:n]}$ is the ith ordered statistic obtained from the ith SRS of size n. Note that we actually need n^2 observations selected via SRS to obtain n RSS units which means that we have to, unfortunately, discard n(n-1)/2 observations. The RSS procedure may be repeated r times (cycles) to obtain an RSS of size m = n × r. The importance of RSS is pivotal in certain situations where the members of the random sample can be easily ordered via cheap means while the actual quantification of the observations is relatively expensive. The unbiased estimate of the population mean μ based on RSS is:

$$\bar{X}_{RSS} = \frac{1}{m}\sum_{j=1}^{r}\sum_{i=1}^{n} X_{i[i:n],j} \quad (3)$$

where, $X_{i[i:n],j}$ is the ith ordered statistic from the ith SRS in the ith cycle.

Various extensions of the RSS have appeared in the literature such as median RSS (Muttlak, 1997), paired RSS (Hossain and Muttlak, 1999) and selected RSS (Hossain and Muttlak, 2001).

ROBUST EXTENSIONS OF RSS

The robust ranked set sampling handles the deficiency of the RSS scheme in certain situations and as a result, it motivates researchers to look for alternative sampling procedures. The robust procedure proposed by Al-Nasser (2007) based on the idea of L statistic (LRSS) is used. The main idea of the LRSS is basically to discard the data in the tails of the data set permanently or sometimes it is recommended to replace the discarded observation with the next most extreme data. The LRSS scheme of size n is better described via the following steps:

- Select n random samples each of size n units then rank the units within each sample with respect to the variable of interest.
- Select the LRSS coefficient, k = [nα] such that 0 < α < 0.5, where [x] is the largest integer value less than or equal to x.
- For each of the first k+1 ranked samples; we select the $(k+1)^{th}$ unit for actual measurement while for each of the last k+1 ranked samples, we select the $(n-k)^{th}$ ordered observation for actual measurement.
- For j = k + 2, k + 3, ..., n -k-1, we select the ith ordered unit in the jth ranked sample for actual measurement.
- The cycle may be repeated r times to obtain the desired sample size m = n × r.

It should be noted that the above steps could be extended for robust extreme ranked set sampling (RERSS); Al-Nasser and Bani Mustafa (2007); by selecting the $(k+1)^{th}$ and the $(n-k)^{th}$ units for actual measurements from each of the first and last $[\frac{n}{2}]$ ranked samples, respectively. Moreover, if n is odd, then the RERSS scheme states that the

$$\left(\frac{n+1}{2}\right)^{th}$$

unit must be selected for the actual measurement.

The LRSS estimate of μ is given by:

$$\bar{X}_{LRSS} = \frac{1}{n}\left(\sum_{i=1}^{k} X_{i[k+1:n]} + \sum_{i=k+1}^{n-k} X_{i[i:n]} + \sum_{i=n-k+1}^{n} X_{i[n-k:n]}\right), \quad (4)$$

While its variance is given by:

$$Var(\bar{X}_{LRSS}) = \frac{1}{n^2}\left(\sum_{i=1}^{k} Var(X_{i[k+1:n]}) + \sum_{i=k+1}^{n-k} Var(X_{i[i:n]}) + \sum_{i=n-k+1}^{n} Var(X_{i[n-k:n]})\right), \quad (5)$$

Consequently, we intend to present a generalized estimate $(\bar{X}_{GF})$ of the population mean that subsumes the RSS estimate as well as most of its extensions can be formulated as:

$$\bar{X}_{GF}(k,k^*) = \frac{1}{nr}\left(\sum_{j=1}^{r}\sum_{i=1}^{k^*} X_{i[k+1:n],j} + c_1\sum_{j=1}^{r}\sum_{i=k^*+1}^{n-k^*} X_{i[i:n],j} + \sum_{j=1}^{r}\sum_{i=n-k^*+1}^{n} X_{i[n-k:n],j} + c_2 I\right), \quad (6)$$

where, c_1, c_2 and k* are known constant. Also

$$I = \begin{cases} \sum_{j=1}^{r} X_{n^*[n^*:n],j} & \text{if } n^* = \frac{n+1}{2} \text{ and n is odd} \\ 0 & \text{if n is even} \end{cases}$$

It is important to point out that c_1 and c_2 take values of either 0 or 1 which would be determined according to the sampling procedure. Had it been that the values of k, k* and c_2 are set to be zero and assuming that $c_1 = 1$, then the estimate (6) shall be reduced to the well known RSS estimate. LRSS also considered as a special case of (6) assuming that $c_1 = 1$, $c_2 = 0$, k* = k and r = 1. Also, setting

$$k^* = \left[\frac{n}{2}\right],\ c_1 = 0 \text{ and } c_2 = 1,$$

reduce the GF estimate of μ to its counterpart RERSS.

It is clear that $LRSS_k$ subsumes RSS technique while $RERSS_k$ unifies the median RSS and ERSS (Table 1). As a result the proposed generalized form (GF) subsumes all the previous procedures.

Theorem 1: Let $X_{i[i:n]j}$ be the ith order statistic from the ith SRS in the jth cycle defined in (3) and let c_1, c_2 and k* be fixed then:

- The estimate $(\bar{X}_{GF1})$ is unbiased estimator of μ.
- The estimate $(\bar{X}_{GF})$ is unbiased estimator of μ; assuming that the underlying distribution is symmetric for other formulations.
- The variance of the $(\bar{X}_{GF})$ is given by:

$$var(\bar{X}_{GF}) = \frac{1}{n^2 r}\left(\begin{array}{l} \sum_{i=1}^{k^*} \sigma^2_{i[k+1:n]} + c_1^2 \sum_{i=k^*+1}^{n-k^*} \sigma^2_{i[i:n]} + \\ \sum_{i=n-k^*+1}^{n} \sigma^2_{i[n-k:n]} + c_2^2 \sigma^2_{n^*[n^*:n]} \end{array} \right)$$

Where, $n^* = \frac{n+1}{2}$ n is the sample size and r is the number of cycles. The proof of the theorem is concluded directly using the ideas of ranked set sampling (Chen *et al*., 2004).

Quality control limits using variants of GF: As mentioned earlier, the quality control charts are determined via the lower and upper control limits as well as the central limit term. The estimates of the three parts are necessary when the population mean and variance are unknown. This leads us to present new set of estimates of (μ, σ^2) using RSS so that we may construct the quality control charts. Salazar and Sinha (1997) proposed the following:

$$\begin{aligned} LCL &= \mu - 3\sigma_{\bar{X}_{GF1}} \\ CL &= \mu \\ UCL &= \mu + 3\sigma_{\bar{X}_{GF1}} \end{aligned}$$

$$\sigma_{\bar{X}_{GF1}} = \sqrt{\frac{1}{n^2}\sum_{i=1}^{n} E\left(X_{i[i:n]} - E\left(X_{i[i:n]}\right)\right)^2}$$

is the standard deviation obtained via RSS for k = 0 (Table 1; Chen *et al*., 2004). The numerical value may be obtained via numerical computation of integration or using the table of order statistics for the standard normal distribution (Harter and Balakrishnan, 1996).

Table 1: Some of the well known variants of RSS and its extensions

Sampling method	k	k*	c_1	c_2	Unified Notation
RSS	0	0	1	0	GF_1 (0,0)
$LRSS_k$	k	k	1	0	GF_1 (k,k)
$RERSS_k$	k	$\left[\frac{n}{2}\right]$	0	1	$GF_2(k, \left[\frac{n}{2}\right])$
Median RSS	$\left[\frac{n}{2}\right]$	$\left[\frac{n}{2}\right]$	0	1	$GF_2(\left[\frac{n}{2}\right], \left[\frac{n}{2}\right])$
ERSS	0	$\left[\frac{n}{2}\right]$	0	1	$GF_2(0, \left[\frac{n}{2}\right])$

As another special case of (6), we adopt the work of Al-Nasser (2007) and use the robust ranked sampling (LRSS) and robust extreme ranked set sampling (RERSS) to construct the quality control limits. To accomplish this goal, we follow the same procedure used here and present the lower and upper control limits as well as the central limit term using the population mean and variance estimates. Consequently, we may present the quality control limits in the following manner:

$$\begin{aligned} LCL &= \mu - 3\sigma_{\bar{X}_{GF}} \\ CL &= \mu \\ UCL &= \mu + 3\sigma_{\bar{X}_{GF}} \end{aligned}$$

Comparing and summarizing the sampling methods presented in this study a simulation study is conducted using different quality control methods. The Average Run Length (ARL) is one the most common used method in the literature and it usually assumes that the process is under control with mean μ_0 and σ_0^2 variance. The process may fluctuate within the control limits and sometimes it may go out of control for many reasons. To set a line, we say that the mean μ_0 may get a shift of the amount:

$$\left(\frac{\delta\sigma_0}{\sqrt{n}}\right)$$

where, δ takes nonnegative values chosen to cover wide range of the shift in mean μ. Note that if δ = 0, then the process is in a state of control, otherwise it starts to get out of control as δ goes large (Montgomery, 2001).

Consequently, if δ = 0, then any point that falls outside the upper or lower control limit is considered as a false alarm. Here, we run a simulation study to accomplish this mission for different values of δ and different sample sizes to illustrate the performance of the various sampling schemes.

RESULTS AND DISCUSSION

Here, we use a simulation study to illustrate the quality control mechanism via different sampling approaches in order to find an alternative sampling method suitable in the research field and may compete with the well-known methodologies. The simulation study is conducted under the normality assumption with mean μ_0 and variance σ_0^2 assuming the ranking is perfect. Note that under the SRS procedure, the ARL of the $\bar{X}$ chart will be 370. This is the result of the reciprocal of the probability that a single point falls outside the control limits when the process is in fact under control. In other words, the out-of-control signal will flash every 370 observed samples even though the process is already under control.

We follow the procedure of Muttlak and Al-Sabah (2003) to simulate one million iterations for each value of δ and for all sampling methodologies. At each iteration, we simulate a sample of size n = 3, 4, 5, 6, 7 which are considered the most recommended sample size in the RSS ($GF_1(0,0)$) literature. As another simulation option, we set the shift-in-mean δ to vary between 0 and 3.4 to cover the under control process as well as the out of control process. The main criterion ARL is computed for all combinations of n, δ and the sampling method of interest (Champ and Woodall, 1987; Harter and Balakrishnan, 1996; Montgomery, 2001). Table 2 gives the ARL values when the sample size n = 3 for different values of δ. Comparing the results in Table 2 allow us to construct remarks on the effectiveness of the $GF_1(0,0)$ as well as the extensions made in this study.

- The process remains under control as long as we have δ = 0 even though we may still get some false alarms. The number of false alarms reaches a maximum of 370 using the SRS scheme while it has been reduced to 340.48 in the $GF_2(0,1)$ setup.
- It is important to mention that number of false alarms using all sampling procedures when δ = 0 are very comparable; consequently any of these plans should be sufficient to accomplish the mission under consideration.
- The significant role of the RSS procedure as well as the various extensions discussed earlier starts to change dramatically when the process gets out of control (i.e., δ gets larger than zero). In fact, the number of false alarms is cut into half or even more when we adopt alternative sampling schemes instead of the SRS.
- Although the ARL obtained via $GF_1(1,1)$ ($LRSS_1$) decreases in a slow pace, yet we clearly see the fast reduction in the ARL values compared to $GF_1(0,0)$ and RERSS ($GF_2(0,k)$, k = 1, 2) when $\delta \geq 0.3$.

Table 2: Average run length using SRS, as well as variants of GF when n = 3

δ	SRS	$GF_1(0,0)$	$GF_1(1,1)$	$GF_2(0,1)$
0.0	369.6858	340.5995	355.7453	340.4835
0.1	361.2717	321.8539	323.5199	319.1829
0.2	305.2503	254.7771	255.0370	247.5247
0.3	254.7122	185.1852	175.7778	184.2978
0.4	202.4701	128.5017	117.5641	133.7614
0.5	153.2332	93.7910	80.8669	91.2909
0.6	120.7146	65.0280	56.9282	65.6901
1.0	44.0393	18.8929	15.1423	18.8743
1.4	18.2282	6.9544	5.5339	6.9678
1.8	8.6675	3.2767	2.6505	3.2947
2.2	4.7417	1.9340	1.6329	1.9308
2.6	2.9033	1.3782	1.2329	1.3774
3.0	1.9985	1.1425	1.0751	1.1426
3.4	1.5246	1.0461	1.0192	1.0463

Table 3: Average run length using SRS, as well as variants of GF when n = 4

δ	SRS	$GF_1(0,0)$	$GF_1(1,1)$	$GF_2(0,2)$
0.0	369.4126	349.0401	355.7453	331.785
0.1	337.7238	312.3048	324.1491	304.5995
0.2	312.9890	229.4104	225.4283	243.2498
0.3	266.0990	166.7500	150.3986	179.6945
0.4	200.7226	115.9420	99.2654	126.3584
0.5	158.1778	76.7048	66.3306	88.1213
0.6	119.2890	52.7816	44.4030	60.2882
1.0	43.7101	14.1495	10.9786	17.4028
1.4	18.3006	5.1341	3.9591	6.3553
1.8	8.6781	2.4803	2.0055	3.0136
2.2	4.7293	1.5504	1.3371	1.8029
2.6	2.9022	1.1932	1.1011	1.3136
3.0	1.9999	1.0584	1.0233	1.1109
3.4	1.5244	1.0138	1.0040	1.0332

- Apparently the reduction in the ARL using the SRS is relatively small which could be considered as a pitfall holding back the SRS scheme from competing with the rest of methodologies in the literature.

Similarly, Table 3 that summarize the ARL values computed via all sampling procedures of interest when n = 4. In this respect, we intend to compare these results and draw some comments and remarks regarding the effectiveness of the LRSS and RERSS and hence the GF ranked set sampling estimates under the same simulation setup used earlier. Thus, we list the following conclusions based on the findings of Table 3-6:

- Assuming that the process is still under control (i.e., δ = 0), we clearly see that the number of false alarms dose not depend on the sample size for all sampling approaches in the sense that the ARL has no monotonic pattern when the sample size varies between 3 and 7.
- Deviating from the so-called under control state of the process, we can see the tremendous decline in the number of false alarms using all sampling procedures. Not only we notice this substantial and systematic decline in ARL values but also we see that the effect of increasing sample size escalates rapidly when the process gets out of control.

- The significant role of the $GF_1(0,0)$ procedure as well as the $GF_1(1,1)$ and RERSS ($GF_2(0,k)$, k = 1, 2) start to emerge when the process gets out of control gradually (i.e., δ gets larger than zero). Distinguishable differences between the $GF_1(0,0)$ and the new proposed sampling schemes arise more clearly while comparing results in Table 2 and 3.
- It is important to point out the numerical and a simulation result supports all findings in the literature. The ARL values using SRS, $GF_1(0,0)$ and $GF_1(1,1)$ matches the results of Muttlak and Al-Sabah (2003) where the GF is considered as a generalization of the median ranked set sampling methodology (Table 3-5).
- The performance of the LRSS ($GF_1(k,k)$; k = 1, 2, 3) as well as the RERSS ($GF_2(k,3)$; k = 0, 1, 2) via the ARL values dominate both SRS and $GF_1(0,0)$ for large sample sizes and certain values of k. In fact, Table 6 shows that ARL reaches a value 1 when the mean shift is 3.4. As a result, we may conclude that the process is already out-of-control and the signal for this purpose flashes every time we choose a sample. On the contrary, the SRS scheme produces an ARL around 1.52 when δ = 3.4 which allows us to conclude that the alarm flashes almost twice on the average every three samples.
- Despite the convincing results of Table 2-6, we may still get more supportive remarks. In fact, we clearly notice that the gap between the ARL values using the robust GF (LRSS and RERSS) compared to the regular SRS reaches its peak when δ is around 1.4 and it reaches more than nine manifolds comparing SRS with the robust GF (LRSS and RERSS).

Table 4: Average run length using SRS, as well as variants of GF when n = 5

δ	SRS	$GF_1(0,0)$	$GF_1(1,1)$	$GF_1(2,2)$	$GF_2(0,2)$
0.0	372.0238	356.7606	366.1662	368.4598	350.7541
0.1	346.2604	301.9324	309.5017	323.9391	298.8643
0.2	313.8732	225.8356	222.3210	214.7766	229.8322
0.3	249.4388	152.4623	139.3340	132.4153	164.4466
0.4	205.6767	98.4252	86.4977	81.7996	107.1352
0.5	157.3812	65.3339	55.1846	50.9243	74.0631
0.6	120.4094	44.0238	36.6569	33.0327	51.2453
1.0	43.9638	11.0552	8.5779	7.7966	13.5932
1.4	18.1831	3.9908	3.1417	2.8613	4.9395
1.8	8.6989	2.0078	1.6696	1.5583	2.3922
2.2	4.7118	1.3390	1.1974	1.1548	1.5117
2.6	2.9120	1.1008	1.0472	1.0328	1.1761
3.0	2.0007	1.0237	1.0079	1.0047	1.0514
3.4	1.5254	1.0040	1.0009	1.0004	1.0119

Table 5: Average run length using SRS, as well as variants of GF when n = 6

δ	SRS	$GF_1(0,0)$	$GF_1(1,1)$	$GF_1(2,2)$	$GF_2(0,3)$	$GF_2(1,3)$
0.0	375.5163	346.1405	363.6364	363.6364	331.1258	364.2987
0.1	349.4060	300.8423	310.7520	303.4901	309.8853	311.2356
0.2	309.0235	218.7705	203.4588	199.7204	232.6664	207.0822
0.3	247.0356	137.1178	125.1408	117.5226	158.4033	128.5017
0.4	196.8891	87.0019	75.6773	69.6670	110.6072	79.1202
0.5	154.9427	55.9503	48.0492	43.6472	75.0356	49.7562
0.6	120.8021	37.0508	30.3095	27.2020	51.0882	32.2799
1.0	43.5749	9.0035	7.0279	6.1607	13.6605	7.4843
1.4	18.2435	3.2467	2.6102	2.3262	4.9405	2.7638
1.8	8.6944	1.7118	1.4656	1.3609	2.3992	1.5206
2.2	4.7149	1.2144	1.1189	1.0818	1.5167	1.1391
2.6	2.9026	1.0530	1.0225	1.0127	1.1770	1.0287
3.0	1.9961	1.0095	1.0027	1.0012	1.0521	1.0038
3.4	1.5244	1.0012	1.0002	1.0001	1.0118	1.0003

Table 6: Average run length using SRS, as well as variants of GF when n = 7

δ	SRS	$GF_1(0,0)$	$GF_1(1,1)$	$GF_1(2,2)$	$GF_1(3,3)$	$GF_2(0,3)$	$GF_2(1,3)$	$GF_2(2,3)$
0.0	366.5680	339.9040	361.2710	361.6630	364.8300	327.6540	355.8710	369.0030
0.1	358.9370	295.4210	299.7600	302.2060	294.5500	302.3880	302.3880	295.5950
0.2	298.8640	204.1650	191.3140	181.6860	176.7720	224.8200	198.8860	187.3010
0.3	251.6990	127.9910	114.4290	105.5630	104.3730	150.1500	119.1180	107.6080
0.4	198.7670	78.6780	68.2270	61.7210	60.2260	99.1370	70.9370	62.2970
0.5	155.1830	49.1060	41.6700	37.2410	35.0860	66.4180	43.6300	36.9010
0.6	119.4740	31.9826	26.2010	23.2780	22.0440	43.8250	27.7550	23.3490
1.0	43.6472	7.4511	5.9216	5.1359	4.8445	11.2336	6.3097	5.1252
1.4	18.2625	2.7516	2.2534	2.0046	1.9207	4.0716	2.3789	2.0041
1.8	8.6794	1.5139	1.3309	1.2449	1.2160	2.0393	1.3784	1.2455
2.2	4.7193	1.1368	1.0725	1.0459	1.0376	1.3532	1.0880	1.0456
2.6	2.9039	1.0280	1.0103	1.0053	1.0038	1.1069	1.0143	1.0052
3.0	1.9988	1.0037	1.0009	1.0003	1.0002	1.0259	1.0014	1.0003
3.4	1.5258	1.0003	1	1	1	1.0046	1.0001	1

ACKNOWLEDGMENTS

This research has been supported by a grant from Yarmouk University. The authors would like to thank the managing and associated editors and the referees for their helpful comments that improved the presentation of the article.

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