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## Analysis of Two-Nucleon Transfer Cross Section Using Gobbi Optical Potential

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**Abstract:** In this study, we have studied the differential cross section of  ${}^{16}\text{O}({}^{6}\text{Li},\alpha){}^{18}\text{F}$  with two nucleon transfer of heavy ion reactions using distorted wave Born approximation (DWBA) calculation. The present study has used Gobbi optical potential in reproducing the large angle oscillatory structures of heavy-ion transfer reactions. The bound states of the transferred particle with the core nucleus, forming the projectile or target residual nuclei, are represented with Yukawa potential. The calculated angular distributions are found to be in a good agreement with the experimental data. The extracted spectroscopic factor is reasonable.

**Key words:** Gobbi Optical potential, transfer reactions

#### INTRODUCTION

Recently, extensive theoretical calculations have been done to explain the resonant structures, which usually observed in heavy ion transfer reaction, following the microscopic DWBA calculation (Farra, 2003a). The differential cross-section of two nucleon transfer of heavy ion reactions has been studied (Farra, 2003b) as a single direct-step process. The differential cross section (El-Azab and Hassanain, 2004) of <sup>6,7</sup>Li elastic and inelastic scattering <sup>12</sup>C, <sup>28</sup>Ni targets in the energy range 12-25 MeV were analyzed the coupled-channels mechanism. Nucleon pickup and striping reactions have been analyzed in terms of direct surface transfer reactions (Memaz et al., 1985) to continuum states. Multinucleon transfer reactions induced by heavy ions have been used in various contexts to study aspects of nuclear structure such as two particles (Tamura, 1974) and four particle (Lemaire, 1973) correlations. The angular distributions have been measured for <sup>7</sup>Li + <sup>54</sup>Fe at 48 MeV using finite-range DWBA calculations, employing real and imaginary Wood Saxon optical potential (Kemper et al., 1982) and have been reproduced reasonably well at energies below and above Coulomb barrier using the coulomb distorted Wave Born approximation (Lilley et al., 1987).

In the present study, the differential cross section of <sup>16</sup>O(<sup>6</sup>Li,α)<sup>18</sup>F heavy ion reactions with two-particle transfer have been calculated. The direct transfer reaction is investigated using the exact finite-range DWBA calculations as a single-step process. The optical potential is taken to have real and imaginary Gobbi potential (Kondo and Tamura, 1984) to generate the initial and final distorted waves.

#### FINITE-RANGE DIFFERENTIAL CROSS SECTION

The explicit transition matrix element of the T(A,C)R reaction with a transferred particle x is evaluated following the DWBA calculations. Therefore, the complete reaction transition  $T_{\rm f}$  (Farra, 2003c) is taken to have the following expression

$$\begin{split} T_{\mathbf{f}\mathbf{i}} &= \left\langle \psi_{XR}^{(-)}(\vec{K}_{X}, \vec{r}_{XR}) \phi_{X}^{J_{X}\mu_{X}} \phi_{R}^{J_{R}\mu_{R}}(\vec{r}_{AC}) \right. \\ &\left. \left| V_{cx}(\mathbf{r}_{cx}) + V_{cT}(\mathbf{r}_{ct}) - \tilde{V}_{cR}(\mathbf{r}_{cR}) \right| \right. \\ &\left. \psi_{AT}^{(+)}(\vec{K}_{A}, \vec{r}_{AT}) \phi_{A}^{J_{A}\mu_{X}} \phi_{T}^{J_{T}\mu_{T}}(\vec{r}_{XC}) \right\rangle \end{split} \tag{1} \end{split}$$

Where  $\psi_{AT}^{(+)}(\vec{K}_A,\vec{r}_{AT})$  and  $\psi_{XR}^{(-)}(\vec{K}_X,\vec{r}_{XR})$  are the ingoing and outgoing distorted wave functions,  $\varphi$ 's are the bound state wave functions and  $V_{ij}$  is the interaction potential between the particle i and j, while,  $\tilde{V}$  is the optical potential generating the distorted waves. The differential cross section for heavy ion reaction with particle transfer have been calculated in terms of one step DWBA calculations (Farra, 2003c) and described by clear form, which is given by

$$\frac{d\sigma}{d\Omega} = \frac{m_{\text{AT}} m_{\text{XR}}}{(2\pi\hbar^2)^2} \frac{K_f}{K_i} \frac{1}{(2J_{\text{A}} + 1)(2J_{\text{T}} + 1)} |T_{\text{fi}}|^2 \qquad (2)$$

Where  $m_{ij}$  is the reduced mass of the particles i and j,  $K_i$  and  $K_f$  are the wave vectors in the initial and final channels, respectively, while  $J_i$  and  $\mu_i$  are the respective spin angular momenta of the particle i and its magnetic projection on the z-component.

#### NUMERICAL CALCULATIONS AND RESULTS

Here, numerical calculations are carried out to find the angular distributions for  $^{16}\text{O}(^{6}\text{Li},\alpha)^{18}\text{F}$  heavy-ion reactions, which proceed via direct two- nucleon transfer processes at 34.0 and 48.0 MeV incident energies. In first set of calculations, the optical potentials describe the scattering of the heavy ions in both of the initial and final channels are taken to have Gobbi potential forms. These forms are used for the real and imaginary distorting potential in the initial and final channels together with a Coulomb potential. The nucleus-nucleus interaction is expressed as:

$$\begin{split} V(r) = & -V_{\circ} \quad 1 + \exp \frac{r - R_{V}}{a_{V}} \quad ^{-1} - i \ (W_{\circ} + W_{E} \ E_{c.m.}) \\ 1 + \exp \frac{r - R_{W}}{a_{W}} \quad ^{-1} + V^{c}(r) \end{split} \tag{3}$$

$$R_{v} = R_{w} = r_{o} (A_{x}^{1/3} + A_{R}^{1/3})$$

Where  $V_{\text{o}}$ ,  $R_{\text{V}}$  and  $a_{\text{v}}$  are the strength, radius and diffuseness of the real potential, while  $W_{\text{o}}$  and  $W_{\text{E}}$  describe an energy dependent absorptive potential,  $E_{\text{cm}}$  is the center of mass energy of the  $^{16}\text{O} + ^6\text{Li}$  channel,  $R_{\text{W}}$  and  $a_{\text{w}}$  describe the imaginary part and  $V^{\text{c}}(r)$  is the Coulomb potential due to a uniform sphere of radius  $R_{\text{C}} = r_{\text{C}}A^{1/3}$  and is given as

$$V^{c}(r) = \frac{Z_{i} Z_{j} e^{2} (3 - r^{2}/R_{c}^{2})/2R_{c}; r \leq R_{c}}{Z_{i} Z_{j} e^{2}/r}; r \geq R_{c}$$
(4)

where  $r_c = 1.35$  fm

The nuclear interactions describing the particlenucleus bound states are represented by Yukawa potentials (Ass'ad, 2002).

$$V_{ij}(r_{ij}) = V_{ij}^{0} 2 + \frac{r_{ij} - (R_i + R_j)}{a} e^{\frac{-r_{ij} + R_i + R_j}{a}}$$
(5)

Where  $R_i$  and  $R_j$  are the radii of the i and j nuclides given by  $r_o A^{1/3}$ , a is the diffuseness of the potential.  $V_{ij}^o$  represents the interaction strength  $V_{ij}^o$  represents the interaction strength given as:

$$V_{ij}^{o} = \left[C(i)C(j)\right]^{\frac{1}{2}} \frac{a R_{i} R_{j}}{r_{o}^{2}(R_{i} + R_{i})}$$
 (6)

The parameter C (i) which is appeared in Eq. 3 has the form

$$C(i) = A(i) 1 - K_s \frac{N_i - Z_i}{A}$$
 (7)

and another similar expression for C (j), while the bound state wavefunctions for both initial and final channels are expressed to have Morinigo wavefunctions (Ass'ad, 2002), with parameters determined to reproduce the particle-particle binding energies which is given by:

$$\varphi_{ij}(\vec{r}_{ij}) = N(l,...)e^{-\beta r_{ij}} \ r_{ij}^{\ell-1} \ Y_l^m \left(\hat{\vec{r}}_{ij}\right) \eqno(8)$$

Where

$$N(\ell,...) = \frac{(2\beta)^{2\ell+1}}{(2\ell)!}^{\frac{1}{2}}$$
 (9)

$$\beta^2 = \frac{2m_{ij}}{\hbar^2} \left| E_{ij}^{bin} \right|$$

 $E_{ij}^{bin}$  is the binding energy.

The different parameters of the interactions are:  $V_o = 159.0 \, \text{MeV}$ ,  $W_o = 8.6 \, \text{MeV}$ ,  $r_o = (1.18, 1.2, 1.25 \, \text{fm})$ ,  $a = 0.65 \, \text{fm}$  and the surface symmetry constant  $K_s = 3.0$  which are chosen to fit the static properties of nuclei. In general, the present spectroscopic factor is extracted from the reaction, that is:

$$S(\ell_{j}) = \frac{1}{N} \frac{(2j_{i} + 1)}{(2j + 1)} \frac{(d\sigma/d\Omega)_{exp}}{(d\sigma/d\Omega)_{theor}}$$
(10)

Where N is normalization factor for the reaction, i and j are the target spin and spin of the final state, respectively.

The parameters used above are found to reproduce the forward angle data reasonably well and fair at the large angle. Therefore, the present optical potential obtains the best fit to the data. The results obtained for the differential cross-sections at 0.34 MeV incident energy and  $r_{\circ}$  equal to 1.18 is shown in Fig. 1 by the solid with our calculations and the dash curve are compared with previous calculations (Farra, 2003b) who used real and imaginary Wood Saxon and J-dependent, respectively (WS+JD) optical potential (dashed) and experimental data dots (Cook *et al.*, 1984). In the same incident energy, the effect of  $r_{\circ}$  is shown in Fig. 2.

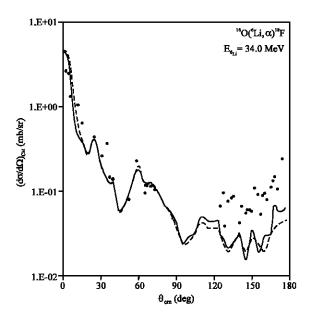


Fig. 1: The differential cross-section of the <sup>16</sup>O (<sup>6</sup>Li, α) <sup>18</sup>F two-nucleon transfer reaction (<sup>1+</sup>, g.s.) at 34.0 Mev incident energy. The solid curve is the present calculations using Gobbi potential, the dashed curve is the previous work taken from (Farra, 2003b) and the dots are the experimental data taken from (Cook *et al.*, 1984)

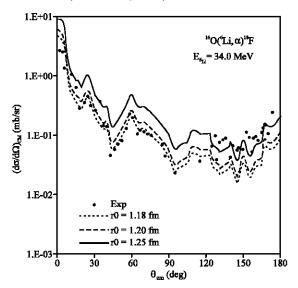


Fig. 2: The differential cross-section of the  $^{16}$ O ( $^6$ Li,  $\alpha$ )  $^{18}$ F two-nucleon transfer reaction ( $^{1+}$ , g.s.) at 34.0 Mev incident energy using Gobbi optical potential, for different  $r_{\circ}$ . The experimental data has been taken from (Cook *et al.*, 1984).

At incident energy 0.48 MeV is shown in Fig. 3, the differential cross-sections given by the solid and

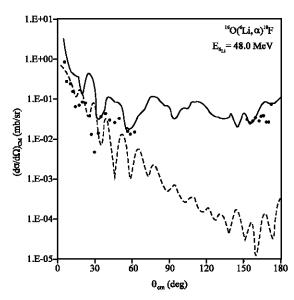


Fig. 3: The differential cross-section of the <sup>16</sup>O (<sup>6</sup>Li, α) <sup>18</sup>F two-nucleon transfer reaction (<sup>1+</sup>, g.s.) at 48.0 Mev incident energy. The solid curve is the present calculations using Gobbi optical potential, the dashed curve and the dots are previous work and the experimental data, respectively, are taken from (Cook et al., 1984)

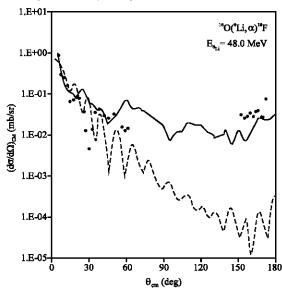


Fig. 4: The differential cross-section of the <sup>16</sup>O (<sup>6</sup>Li, α) <sup>18</sup>F two-nucleon transfer reaction (<sup>1+</sup>, g.s.) at 48.0 Mev incident energy. The solid curves is the present calculations using Gobbi potential (the differential cross-section have been multiplied by 0.4 to fit the forward and backward angles), the dashed curve and the dots are previous work and the experimental data, respectively, are taken from reference (Cook *et al.*, 1984)

dash curve are compared with previous calculations (Cook et al., 1984) who used real and imaginary Wood Saxon (WS+WS) optical potential (dashed) and experimental data dots (Cook et al., 1984). It is found our calculation is a little well at all the range of angles, to get very well results we multiply the differential cross section by the factor 0.40 as shown in Fig. 4.

#### DISCUSSION

In this study, the  $^{16}O(^6\text{Li},\alpha)^{18}\text{F}$  heavy ion reactions with two nucleon transfer have been studied using the DWBA calculations as a single step process. The numerical calculations are carried out to find the angular distributions of this reaction at incident energy 34.0 and 48.0 MeV. In Fig. 1, it seen that at large angle with Gobbi at 34.0 MeV was noticeably nearly good and significantly better than the previous work. It is clear that the present data are good against the different of the parameter  $r_o$ , we found the small values of  $r_o$  is better fit for the forward angles but the large values of  $r_o$  is better fit for backward angles as shown in Fig. 2. The spectroscopic factor in the previous work is 0.77 but in our calculation is 0.78.

The use of 0.48 MeV shown in Fig. 3, Wood-Saxoon potential is better than Gobbi potential at forward angles, but at back angles Gobbi potential gives better result than Wood-Saxoon potential. If we multiply the differential cross section by 0.4, the result is behaves very well for the whole range. Finally the spectroscopic factor for this reaction at 0.48 incident energy is 0.84. In conclusion, the use of Gobbi optical potential leads to a reasonable results which are better than (WS-WS) and (WS-JD) potentials at large angles.

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