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## Determining the Optogeometric Properties of Al<sub>2</sub>O<sub>3</sub> Thin Film Waveguides Using M-line Spectroscopy Results

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**Abstract:** Results obtained from m-line spectroscopy experiments were used to calculate optogeometric parameters (the refractive indices and thickness) of Al<sub>2</sub>O<sub>3</sub> thin film by solving the conventional prism waveguide coupler mode dispersion equations. These equations were solved using graphical and numerical methods. The numerical method is based on the Newton-Raphson algorithm. The solution to one mode of propagation was obtained by solving the zero order transverse electric (TE<sub>0</sub>) mode and transverse magnetic (TM<sub>0</sub>) mode. The values of 1.61±0.02 and 253.00±0.05 nm were obtained for the refractive index and thickness respectively from the graphical solution while the Newton-Raphson algorithm gave 1.6200±0.0007 for the refractive index and 240.1100±0.0005 nm for the film thickness. For two modes of propagation, two TE and two TM modes were considered. The graphical solution for the two TE modes gives a refractive index of 1.5995±0.0002 and a thickness of 772.526±0.001 nm with the Newton-Raphson algorithm giving 1.5995±0.0003 for the refractive index and 767.850±0.005 nm as film thickness. The two modes of propagation for the TM modes gives the graphical results for the refractive index and thickness as 1.6010±0.0002 and 774.671±0.004 nm respectively while the result of the Newton-Raphson algorithm is 1.6010±0.0002 and 773.336±0.003 nm respectively. In comparison with the experimental values obtained, the percentage uncertainty is less than ±5% for both parameters. The results generally, showed a higher uncertainty in determining the film thickness than the refractive index for both TE and TM polarization.

**Key words:** Waveguide, dielectric, refractive index, thickness, polarization

### INTRODUCTION

Thin film materials and devices have a wide range of optical, electronic, mechanical and other capabilities. They are required for application in communication systems, information storage, display system, sensors and other fields (Cooper, 1999). For dielectric thin films, their performance as a waveguide when coated on a substrate is a function of the refractive index and thickness Graugnard *et al.* (2006). Given the relevance of these parameters vis-à-vis their wide range of practical applications, this work sets to compute these parameters for dielectric thin films using a 2D step-index waveguide. This is based on results obtain from m-line spectroscopy by the prism film coupling technique following the pioneering work of Tien *et al.* (1969).

Optoelectronic and integrated optical technology is faced with very rapid development that poses an enormous challenge to the optical communication industry. The extreme miniaturization of a system, expected to minimize the effect of ambient conditions and

cutting the cost of production, constitutes the basis for this development (Tien and Ulrich, 1970). Optoelectronic and integrated optics concern themselves with the transmission and processing of a detectable transmitted energy that can be use to carry information (signals) at a frequency corresponding to visible light. This is some times in the neighboring Infra-Red (IR) Or ultra-Violet (UV) regions and the light source (e.g., a laser beam) is adapted for the production of the frequency carriers. There is an evident need to modulate the frequencies, the amplitudes and the phases of the waves propagating along the transmission line, which we shall refer to hereafter, as a waveguide.

The physics of thin films, particularly dielectric films reveal that the refractive index and thickness are the main parameters characterizing such films (Tien and Ulrich, 1970; Ulrich and Torge, 1973; Agan *et al.*, 2005). They account mainly for the properties of the film. In this study, these important parameters are determined theoretically using the m-line spectroscopy results where a graphical solution using plots of thickness against refractive

indexes are obtained. The numerical solution using the Newton-Raphson algorithm for particular synchronous angles of incidents and observed modes are thereafter presented.

These graphical and numerical approaches were used to solve the conventional prism waveguide coupler mode equations in order to verify the validity of the experimental results obtained. The Al<sub>2</sub>O<sub>3</sub> thin films used in this work were prepared by the sol gel technique and the experimental results were obtained using m-line spectroscopy measurements in the Laboratoire de Physico-Chimie des Matériaux Luminescent (LPCML), (UMR.5620), Université Claude Bernard Lyon 1, Villeurbanne, 69622 France.

**MATERIALS AND METHODS**

The film coupler system for the production of m-lines is shown in Fig. 1.

The analysis is based on the equations obtained for the prism-film coupler waveguide mode for TE and TM polarizations for homogenous dielectric waveguide with 2D step-index profile. The graphical and numerical computer program simulations for comparison with available experimental results were limited to the TE<sub>0</sub> and TM<sub>0</sub>; TE<sub>0</sub> and TE<sub>1</sub> and TM<sub>0</sub> and TM<sub>1</sub> modes. A P4 computer of 40 GHz was used for the program simulation to calculate the refractive index and thickness of thin dielectric film in the 2-D step-index waveguide.

**THEORETICAL FORMULATIONS**

Electromagnetic waves propagating in a planar dielectric waveguide obey Maxwell's equations, which consist of four equations with field vectors, E, D and H, B. These equations are expressed as

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1}$$

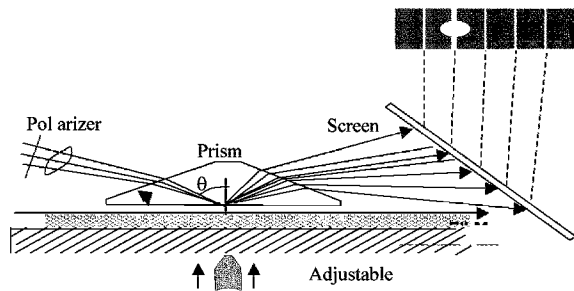


Fig. 1: The prism-film coupler showing the observation of mode spectrum

$$\nabla \times H = \frac{\partial D}{\partial t} + J \tag{2}$$

$$\nabla \cdot B = 0 \tag{3}$$

$$\nabla \cdot D = \rho \tag{4}$$

Manipulation of Eq. 1-4 for a linear, homogenous and non-conducting medium with no sources permit the derivation of wave equations describing TE and TM mode propagation. With the vacuum optical wavelength given by  $\lambda_0$  and  $k_0 = 2\pi/\lambda_0$ , the propagation of the TE and TM modes is described by Eq. 5 and 6, respectively.

$$\text{For the TE mode; } \frac{\partial^2 E_y}{\partial x^2} + (k_0^2 n_i^2 - \beta^2) E_y = 0 \tag{5}$$

$$\text{For the TM mode } \frac{\partial^2 H_y}{\partial x^2} + (k_0^2 n_i^2 - \beta^2) H_y = 0 \tag{6}$$

The solution for both cases involve plane waves of the form

$$E_y = \exp(ik_x) \tag{7}$$

$$H_y = \exp(ik_x) \tag{8}$$

$k_x$  is the propagation constant in the x-direction with an amplitude defined by the equation

$$k_x = \sqrt{k_0^2 n_i^2 - \beta^2} \tag{9}$$

Using the theory of total internal reflection and doing a dispersion analysis for guided wave modes, Cheo (1990) derived the prism-coupled mode dispersion equation for a planar dielectric waveguide with a step-index profile. Taking a cue from his results we could represent the mode equation as follows.

$$\xi_{\text{imp}}(N_m, n) = \left\{ \frac{\Phi_{\text{fc}}(N_m, n) + \Phi_{\text{is}}(N_m, n) + m\pi}{k_0 t} \right\} \tag{10}$$

Where m in both equations is the mode order, having values of 0, 1, 2 ...; t is the thickness and  $k_0 = \frac{2\pi}{\lambda_0}$  is the optical propagation constant of the waves in the dielectric medium.

The subscripts f, c, s refer to the film, cladding and substrate layers respectively. In Eq. 10, we define

$$\xi_{\text{imp}} = \sqrt{n_f^2 - N_m^2} \tag{11}$$

$$\Phi_f(N_m, n) = \arctan \left[ \left( \frac{n_f}{n_i} \right)^{2\rho} \sqrt{\frac{N_m^2 - n_i^2}{n_f^2 - N_m^2}} \right] \tag{12}$$

where  $N_m = n_f \sin \theta_m = n_p \sin \left\{ \gamma + \arcsin \left( \frac{\sin i_p}{n_p} \right) \right\}$  (13)

$\rho$  defines the polarization state and  $i = c, s,$ ,  $n_p$  is the refractive index of prism and  $\gamma$  is the refracting angle of prism.

Given that

$$\Psi_{\text{mp}}(N_m, n) = \Phi_{\text{ie}}(N_m, n) + \Phi_{\text{is}}(N_m, n) + m\pi \tag{14}$$

the relationships developed from Eq. 10 for use in the computation of the refractive index and thickness for two observed modes are therefore defined thus;

$$t_{\text{mp}} = \frac{\Psi_{\text{mp}}(N_m, n)}{k_0 \xi_{\text{mp}}(N_m, n)} \tag{15}$$

$$\Psi_{\text{mp}1}(N_m, n) \xi_{\text{mp}2}(N_m, n) - \Psi_{\text{mp}2}(N_m, n) \xi_{\text{mp}1}(N_m, n) = 0 \tag{16}$$

$$\sum_{i=1}^2 \left( \frac{\Psi_{\text{imp}}(N_m, n)}{\xi_{\text{imp}}(N_m, n)} \right) - \sum_{i=3}^q \left( \frac{\Psi_{\text{imp}}^q(N_m, n)}{\xi_{\text{imp}}(N_m, n)} \right) = 0 \tag{17}$$

$I = 1, 2, \dots, q$ . Hence,  $q$  is the total number of propagated modes in the film of the waveguide. Equations 15-17 were used for the evaluation of the refractive index and thickness for any two observe modes of the forms  $TE_0$  and  $TM_0$  or  $TE_0$  and  $TE_1$  or  $TM_0$  and  $TM_1$  with  $\rho = 0$  for TE polarization and  $\rho = 1$  for TM polarization.

The numerical approach employs Eq. 16 and 17, which takes the general form

$$f(n_q) = 0 \tag{18}$$

where the Newton-Raphson equation is of the form

$$n_{q+1} = n_q - \frac{f(n_q)}{f'(n_q)} \tag{19}$$

in which  $f'(n_q)$  is the derivative of the function. The choice of a good starting guess is achieved by linearizing Eq. 17 below,

$$n_f = \sqrt{\left[ \left( \frac{\Psi_{\text{mp}1}(N_m, n) \xi_{\text{mp}2}(N_m, n)}{\Psi_{\text{mp}2}(N_m, n)} \right)^2 + N_{\text{mp}}^2 \right]} \tag{20}$$

To compare the result of this study with experimental results; we used the percentage error  $E_r$  or the approximate degree of shift which is simply deduced from the relation given by

$$E_r = 100 \times \frac{\chi_e - \chi_t}{\chi_e} \tag{21}$$

where  $\chi_e$  is the experimental value  $\chi_t$  is the theoretical value. The values of the refractive indices or thickness of the thin film are used separately to obtain  $E_r$  which gives a measure of the uncertainty whose value determines the acceptability or rejection of the result within a reasonable range of tolerance.

### RESULTS

**Refractive index and thickness for  $TE_0$  and  $TM_0$ :** First we consider the propagation of a single transverse electric and transverse magnetic mode with observed m-line angles of  $i_p$  and  $(TE_0) = -23.77^\circ$  and  $i_p (TM_0) = -25.66^\circ$  at a wavelength of 543.5 nm in a dielectric waveguide fabricated with a substrate of refractive index 1.4607. The results obtained for the refractive index of the film ( $n_f$ ) and the thickness ( $t$ ) using the two approaches are shown in Table 1. The results show that for the zero order mode with mode angles of  $\theta_{TE0} = -23.77^\circ$  and  $\theta_{TM0} = -25.66^\circ$ , a refractive index and thickness of  $1.61 \pm 0.02$  and  $253 \pm 0.05 \text{ nm}$  were obtained respectively using a 543.5 nm laser beam. This result is reported in Table 1 together with the experimental results. The numerical method of Newton-Raphson algorithm on the other hand gave  $1.62 \pm 0.07$  for the refractive index and  $240.11 \pm 0.05 \text{ nm}$  for the film thickness.

**Refractive index and thickness for  $TE_0$  and  $TE_1$  modes:**

For a similar waveguide in which the zero order and first order for the TE polarization were observed to have modes angles of  $i_p (TE_0) = -18.673^\circ$  and  $i_p (TE_1) = -25.321^\circ$  when a 543.5 nm laser beam is used. The results obtained for  $n_f$  and  $t$  using both methods and the experimental results are presented in Table 2. The graph produced two points of intersection A and B that are suggestive of the solution. From the results for this

Table 1: Refractive index and thickness for the zero order modes

Method	Refractive index (n)	Thickness t (nm)
Graphical	1.61±0.02	253.00±0.05
Numerical	1.62±0.07	240.11±0.05
Experimental	1.64	240

Table 2: Refractive index and thickness for the  $TE_0$  and  $TE_1$  modes using a 543.5 nm laser beam

Method	Refractive index (n)	Thickness t (nm)
Graphical	A: 1.562±0.002 B: 1.5995±0.0002	1047.933±0.004 772.526±0.001
Numerical	1.5996±0.0003	767.8499±0.0005
Experimental	1.59912	771.57

Table 3: Refractive index and thickness for TM<sub>0</sub> and TM<sub>1</sub> modes using a 543.5 nm laser beam

Method	Refractive index (n)	Thickness t (nm)
Graphical	A: 1.5581±0.0003	1042.122±0.004
	B: 1.600998±0.000002	774.6707±0.0004
Numerical	1.600986±0.000002	773.3359±0.0003
Experimental	1.6007	771.26

analysis shown in Table 2, we observed a refractive index of 1.5995±0.0002 and a thickness of 772.526±0.001 nm at point B. The numerical method using a carefully chosen step-size of 0.025 gave 1.5995±0.0003 and 767.8499±0.0005 nm for the thin film refractive index and thickness respectively.

**Refractive index and thickness for TM<sub>0</sub> and TM<sub>1</sub> modes:**

When the film is made to sustain or support waves of transverse magnetic polarization only instead of the transverse electric polarization the mode angles of the fundamental and first order were found to be  $\alpha_0$  and  $\alpha_1$  at a wavelength of 543.5 nm. The results obtained for the refractive index and thickness with their uncertainties comparable to experimental results are shown in Table 3. From the observed mode angles at 543.5 nm laser wavelength, the result presented in Table 3, shows the possible solutions. The refractive index of 1.6009983±0.0002 and thickness of 774.6707±0.004 nm are the accepted solutions. The numerical solution gave an index value of 1.6009862±0.00002 and a thickness of 773.3359±0.003, which showed a better agreement than the results obtained graphically.

**DISCUSSION**

For the different mode combinations, the computed results were analyzed and compared to the experimental ones. The results that show relatively low percentage errors were accepted while those with relatively high percentage errors even below 5% were rejected. Another yardstick for the acceptance or rejection of the results is the degree of variance between the film’s refractive index and that of the substrate. The more significant the variation or difference the more accurate the result. These results are in close agreement with experiments, however, the uncertainties of 0.07 and 0.05% for index and thickness shows that the results obtain by the numerical approach relative to the graphical approach are in better agreement with experimental results.

In the graphical solution approach, there were two points of intersection A and B. At point B there is a low percentage error of the values of refractive index and thickness obtained compared to those of the values obtained at point A. The solutions of the point B are the accepted ones since they are in better agreement with experiments. The solutions at point A were rejected

because the refractive index obtained at that point is close to that of the substrate thereby introducing the relatively high percentage error.

In this research, we have been able to determine the refractive index and thickness of Al<sub>2</sub>O<sub>3</sub> thin film waveguides using m-lines Spectroscopy Results. The difference in the experimental values from those of the theoretical values is attributable to a number of factors; this may include factors such as stress (Agan *et al.*, 2005), which is quite probable, since the method does not work contact-less (Ulrich and Torge, 1973) Other factors include any deviations from the conditions for accurate measurements of these parameters such as (i) a good knowledge of the prism’s characteristic, (ii) accurate measurement of angles and (iii) taking into account the distance between the prism and guide i.e., the Air-Layer-Thickness (ALT) (Monneret *et al.*, 2000).

The model prism waveguide coupler system with variable air-gap thickness from 0 to 0.5 μm have been reported and is linked to the discrepancy between the effective indices values predicted by the conventional prism waveguide coupler and those of thin-film optics (Liu and Samuels 2004). The choice of the method to adopt for the computation of these parameters either by graphical or numerical means depends on interest. However, the numerical technique using the Newton-Raphson algorithm presented in this work is recommended since the results obtained using this method show a closer agreement with those obtain from experiments than those obtained graphically. This could be due to the difficulty in ascertaining the exact point of intersection of the curves, which are often found to lie close to each other over a range of distance, instead of crossing at an exact point.

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**REFERENCES**

Agan, S., A.F. Kocabas and A. Aydinli, 2005. Stress effects in prism coupling measurements of thin polymer films. *Applied Phys., A* 80: 341-345.  
 Cheo, P.K., 1990. *Fiber Optics and Optoelectronics*. 2nd Edn., New Jersey, Prentice-hall international, pp: 1-26.  
 Cooper, K.L., 1999. Electrostatic self-assembly of linear and non-linear optical thin film. Ph.D Thesis. Faculty of Virginia Polytechnic Institute and State University, Blacksburg, Virginia, pp: 107.

- Graugnard, G.E.D.P., S.N. Dunham, C.W. Neff, T. Yamashita and C.J. Summers, 2006. Photonic band tuning in two-dimensional photonic crystal slab waveguides by atomic layer deposition. *Applied Phys. Lett.*, 89: 181108-181108.
- Liu, T. and R.J. Samuels, 2004. Physically correct theoretical prism waveguide coupler mode. *J. Opt. Soc. Am.* 12: 1322-1332.
- Monneret, S., P. Huguet-Chantome and F. Flory, 2000. M-line technique: Prism film coupling measurements and discussions of accuracy for homogenous waveguides. *J. Opt. Pure. Applied Opt.*, 2: 188-195.
- Tien, P.K., R. Ulrich and R.J. Martin, 1969. Modes of propagating light waves in thin deposited semiconductor films. *Applied Phys. Lett.*, 14: 291-294.
- Tien, P.K and R. Ulrich, 1970. Theory of prism-film coupler and thin-film light guides. *J. Opt. Soc. Am.*, 60: 1325-1337.
- Ulrich, R. and R. Torge, 1973. Measurement of thin film parameters with a prism coupler. *Applied Opt.*, 12: 2901-2908.