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Characterization of Elliptically Polarized Light and Rotation Angle of PBS in Three-Longitudinal-Mode Laser Interferometer Using the Jones Matrices

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Abstract: In this study, the periodic nonlinearities resulting from ellipticity of polarized light and rotation angle of Polarizing Beam Splitter (PBS) with respect to the laser axis are presented. The nonlinearity in the three-longitudinal-mode laser interferometer is mathematically modeled using the Jones matrices. Based on the nonlinearity compensation for two-color interferometers, a new optical setup and signal conditioner circuit is also designed. Using the nonlinearity compensation setup, the peak-to-peak nonlinearity of 2.64 nm due to two-elliptically polarized modes in combination with rotation angle of polarizing beam splitter is successfully reached 26 pm.

Key words: Jones matrix, laser interferometer, nonlinearity, elliptically polarized light

INTRODUCTION

This study presents a new nonlinearity compensator for error reduction in three-longitudinal-mode interferometers. The nonlinearity reduction in the three-longitudinal-mode interferometer can effectively increase the nanometric displacement measurement accuracy. Laser interferometers are being widely used as instrument systems for small displacement measurement with sub-nanometric uncertainty. Two-color interferometers are commercially used for increasing the range of unambiguity. But the displacement measurement accuracy can be improved by replacing the two-color source by three-longitudinal-mode laser. A sub-nanometer heterodyne interferometric system using three-longitudinal-mode He-Ne laser is presented by Yokoyama *et al.* (2001) and then by utilizing a proper configuration, the maximum measurable velocity is enhanced (Yokoyama *et al.*, 2005).

The displacement accuracy is limited by many factors such as laser frequency instability, uncertainty of the refractive index determination, phase detection error and thermal instabilities (Yim *et al.*, 2000; Kim and Kim, 2004; Olyaee and Mohammad Nejad, 2007a). The optical setup can also effectively decrease the displacement accuracy. The optical deviations can be divided into two main groups' namely imperfect alignment and non-ideal polarized beams. These phenomena produce the periodic nonlinearity error that is modeled by Cosijns *et al.* (2002) for two-mode interferometer. However, imperfect alignment of optical components and non-ideal polarized light emerging from the laser head can limit the displacement measurement accuracy of the laser interferometers.

The modeling of polarization properties can be done by plane wave or matrix methods. Based on the Jones calculus, matrix methods are suited to modeling of polarization sensitive systems. In fact the Jones calculus is a powerful and simple method for modeling optical systems involving polarized modes.

In the present study, we characterized and mathematically modeled the most important optical deviations including ellipticity of polarized modes and rotation angle of polarizing beam splitter in three-longitudinal-mode heterodyne interferometer by using the Jones matrices of the optical components. In addition, we improved the system performance by designing an optical setup and a signal conditioner circuit in the measurement arm. The results confirm that the periodic nonlinearities are strongly reduced.

MATERIALS AND METHODS

A three-longitudinal-mode heterodyne interferometer similar to current commercial interferometric systems consists of two arms namely base and measurement (Olyaee and Mohammad Nejad, 2007b). Also measurement arm includes reference and target paths. The measurement beam is split into target and reference beams by the polarizing beam splitter. Ideally, the side and central

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modes are directed towards the reference and target corner cube prisms, respectively. When a rotation angle of polarizing beam splitter with respect to the laser head or elliptically polarized modes appears, there will be leakage of modes.

According to Fig. 1, the ellipticity of side and central modes are denoted by δ_{s_r} and δ_{s_t}, respectively. Therefore, the electrical fields of three optical frequencies are respectively given by:

$$\vec{E}_1 = \begin{bmatrix} \cos\delta_{s_r} \\ -j\sin\delta_{s_r} \end{bmatrix} e^{jx_1} \quad (1)$$

$$\vec{E}_2 = \begin{bmatrix} -j\sin\delta_{s_t} \\ \cos\delta_{s_t} \end{bmatrix} e^{jx_2} \quad (2)$$

$$\vec{E}_3 = \begin{bmatrix} \cos\delta_{s_r} \\ -j\sin\delta_{s_r} \end{bmatrix} e^{jx_3} \quad (3)$$

where, $x_i = 2\pi v_i t+\varphi_{0i}$, i = 1, 2, 3 are the optical frequencies and φ_{0i} are the initial phases. By Jones calculus, the Jones vector of the reference electrical field is obtained as

$$\vec{E}_r = PBS_r.CCP_r.PBS_r.\left(\sum_{i=1}^{3}\vec{E}_i\right) \quad (4)$$

where, PBS_r and CCP_r are the Jones matrices of the polarizing beam splitter and reference corner cube prism for reference beam, respectively, shown in Fig. 2. It is assumed that the optical axis of polarizing beam splitter is aligned properly compared to the laser head. These matrices can be described as:

$$PBS_r = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (5)$$

$$CCP_r = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad (6)$$

Similar to the reference path, the Jones vector of the target electrical field is given by:

$$\vec{E}_t = PBS_t.CCP_t.PBS_t.\left(\sum_{i=1}^{3}\vec{E}_i\right) \quad (7)$$

where:

$$PBS_t = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (8)$$

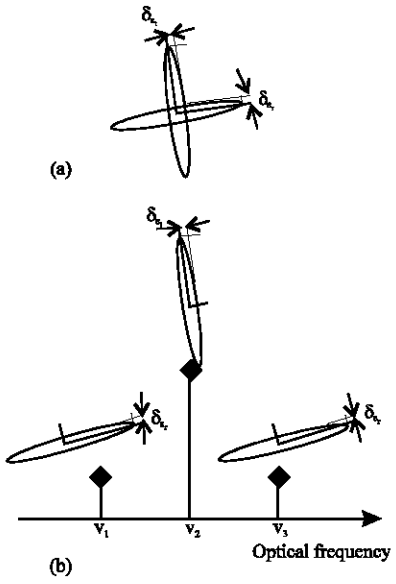

Fig. 1: Schematic representation of (a) the ellipticity of orthogonal polarized modes and (b) the position of three-longitudinal-mode in the gain profile

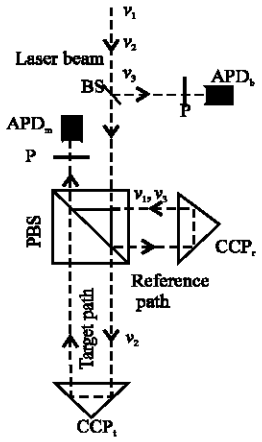

Fig. 2: The optical setup of the three-longitudinal-mode laser interferometer

$$CCP_t = \begin{bmatrix} -\exp(j\Delta\Phi) & 0 \\ 0 & -\exp(j\Delta\Phi) \end{bmatrix} \quad (9)$$

where, $\Delta\Phi = 4n\pi\Delta d/\lambda$, n is the refractive index, λ is the central wavelength and Δd is the optical path difference between two paths. The total electric field can be obtained by summing the reference and target fields.

$$\vec{E}_{BS} = \vec{E}_r + \vec{E}_t \quad (10)$$

The Jones vector of the electrical field incident on the ideal polarizer with 45° orientation is given as:

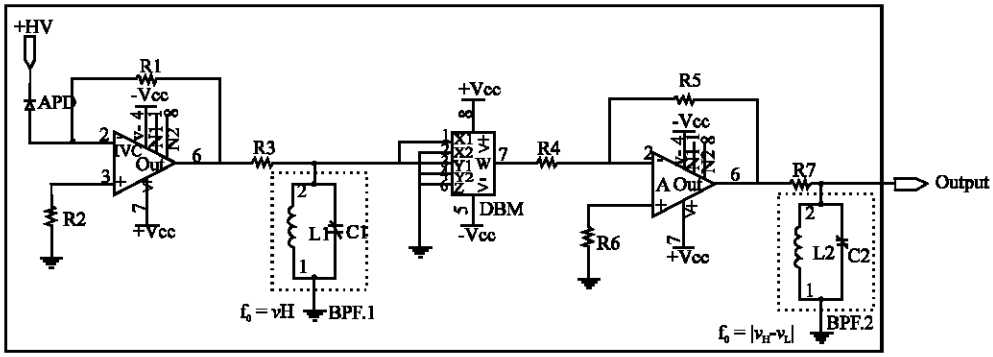

Fig. 3: The schematic diagram of analog signal conditioner for base and measurement arms

$$\vec{E}_P = P.\vec{E}_{BS} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\vec{E}_{BS} \tag{11}$$

The wave intensity can be expressed as the product of the electrical field and its complex conjugate. As a result, the photocurrent of the measurement avalanche photodiode, $I_{P_{APD_m}}$, is

$$I_{P_{APD_m}} = \vec{E}_P^{*}\vec{E}_P \tag{12}$$

where, $\vec{E}_P^{*}$ is the complex conjugate of the electrical field. Substitution of Eq. 11 into Eq. 12 gives a quite complex equation. But the optical frequencies are eliminated by the measurement avalanche photodiode (APDm). Furthermore, in the signal conditioner circuit a Band Pass Filter (BPF) is used, as shown in Fig. 3. DC component and high order frequencies (e.g., v_1+v_2, $2v_2$, v_2+2v_3) are eliminated and primary beat frequencies including v_2-v_1 and v_3-v_2 remain. The measurement photocurrent described in Eq. 12 is converted to voltage signal that can be simplified as:

$$\begin{aligned} V_{BPF_m} \propto\ & a_1(\cos(x_L - \Delta\Phi) + \cos(x_H + \Delta\Phi)) \\ & + a_2(\cos(x_L + \Delta\Phi) + \cos(x_H - \Delta\Phi)) \\ & + a_3(\sin x_L - \sin x_H) \end{aligned} \tag{13}$$

where, $x_L = 2\pi(v_2-v_1)t+(\varphi_{02}-\varphi_{01})$ and $x_H = 2\pi(v_3-v_2)t+(\varphi_{03}-\varphi_{02})$ are corresponding to the lower and higher intermode beat frequencies, respectively. The constant factors in Eq. 13 are obtained as

$$a_1 = \sin\delta_{e_r}\sin\delta_{e_t} \tag{14}$$

$$a_2 = \cos\delta_{e_r}\cos\delta_{e_t} \tag{15}$$

$$a_3 = \sin(\delta_{e_t} - \delta_{e_r}) \tag{16}$$

The signal is led to a Double-Balanced Mixer (DBM) and then is filtered. Therefore the secondary beat frequency, $x_0 = |x_H - x_L|$, is extracted. The normalized measurement signal is:

$$\begin{aligned} V_m = {} & (2a_1a_2 + a_3^2)\cos x_0 - 2a_1a_3\sin(x_0 - \Delta\Phi) \\ & - 2a_2a_3\sin(x_0 + \Delta\Phi) + a_1^2\cos(x_0 - 2\Delta\Phi) \\ & + a_2^2\cos(x_0 + 2\Delta\Phi) \end{aligned} \tag{17}$$

On the other hand, a similar signal conditioner is used in the base arm. Hence, the normalized base signal is given by:

$$V_b = A\cos x_0 \tag{18}$$

For extraction of the phase nonlinearity, Eq. 17 must be rewritten as:

$$V_m = A\cos(x_0 - 2\Delta\Phi + \Phi_{nl}) \tag{19}$$

where:

$$\Phi_{nl} = -\arctan\frac{\begin{aligned}&(2a_1a_2 - a_3^2)\sin 2\Delta\Phi + 2a_2a_3\cos\Delta\Phi \\ &+2a_1a_3\cos 3\Delta\Phi + a_1^2\sin 4\Delta\Phi\end{aligned}}{\begin{aligned}&-(2a_1a_2 - a_3^2)\cos 2\Delta\Phi + 2a_2a_3\sin\Delta\Phi \\ &+2a_1a_3\sin 3\Delta\Phi - a_2^2 - a_1^2\cos 4\Delta\Phi\end{aligned}} \tag{20}$$

Also interfering of three modes on the base avalanche photodiode produces offset phase error. This error can be obtained by substitution of zero for $\Delta\phi$ in Eq. 20 as:

$$\Phi_{off} = -\arctan\frac{2a_3(a_1 + a_2)}{(2a_1a_2 - a_3^2) + a_2^2 + a_1^2} \tag{21}$$

For characterization of rotation angle of polarizing beam splitter, we assume that the polarization of modes is perfectly linear, $\delta_{et} = \delta_{er} = 0$ therefore

$$\vec{E}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{jx_1} \quad (22)$$

$$\vec{E}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{jx_2} \quad (23)$$

$$\vec{E}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^{jx_3} \quad (24)$$

and instead of that a rotation angle of polarizing beam splitter with respect to the laser axis is considered and denoted by α. In this case the polarizing beam splitter matrices for target and reference beams are respectively taken as

$$PBS_t = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \quad (25)$$

$$PBS_r = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \quad (26)$$

and for the polarizer matrix,

$$P = \begin{bmatrix} \cos^2(\frac{\pi}{4}+\alpha) & \cos(\frac{\pi}{4}+\alpha)\sin(\frac{\pi}{4}+\alpha) \\ \cos(\frac{\pi}{4}+\alpha)\sin(\frac{\pi}{4}+\alpha) & \sin^2(\frac{\pi}{4}+\alpha) \end{bmatrix} \quad (27)$$

The reference and target electrical fields are calculated by substitution of above matrices in Eq. 4 and 7 as:

$$\vec{E}_r = \begin{bmatrix} -\cos^2\alpha \\ -\sin\alpha\cos\alpha \end{bmatrix} (e^{jx_1} + e^{jx_3}) + \begin{bmatrix} -\sin\alpha\cos\alpha \\ -\sin^2\alpha \end{bmatrix} e^{jx_2} \quad (28)$$

$$\vec{E}_t = \begin{bmatrix} -\sin^2\alpha \\ \sin\alpha\cos\alpha \end{bmatrix} (e^{j(x_1+\Delta\Phi)} + e^{j(x_3+\Delta\Phi)}) + \begin{bmatrix} \sin\alpha\cos\alpha \\ -\cos^2\alpha \end{bmatrix} e^{j(x_2+\Delta\Phi)}$$

Based on the comprehensive study on the nonlinearities which has been characterized and simulated in the Optoelectronic and Laser Laboratory of IUST, the phase nonlinearity resulting from rotation angle of PBS with respect to the laser head can be given by

$$\begin{aligned} \Phi_{nl} &= -\arctan\left(N/F\right) \\ N &= -\sin^4\alpha\sin 4\Delta\Phi + 2\sin^2\alpha\cos^2\alpha\sin 2\Delta\Phi \\ F &= \sin^4\alpha\cos 4\Delta\Phi + \cos^4\alpha - 2\sin^2\alpha\cos^2\alpha\cos 2\Delta\Phi \end{aligned} \quad (30)$$

RESULTS AND DISCUSSION

Earlier, the phase nonlinearity resulting from ellipticity of orthogonal polarized modes and rotation angle of PBS are modeled. Figure 4 shows the periodic nonlinearity due to the ellipticity of polarized modes in terms of the nanometric displacement. As shown in Fig. 4, the first order or second order nonlinearity can be

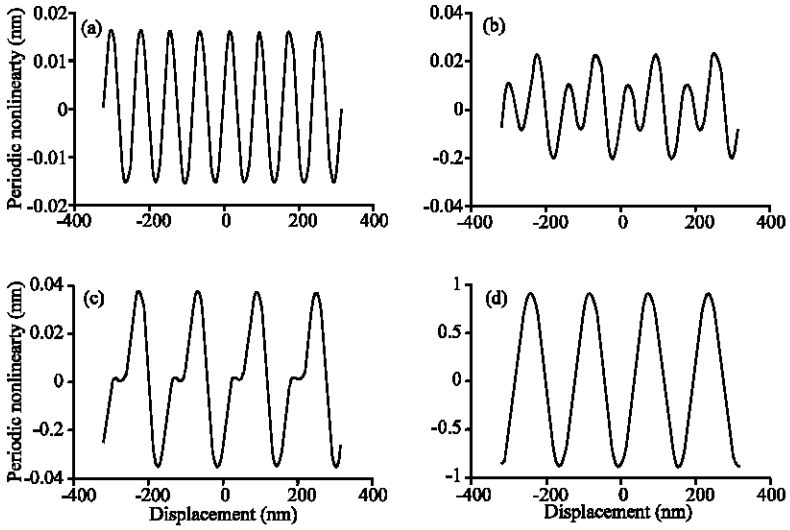

Fig. 4: The periodic nonlinearity resulting from ellipticity of polarized modes in terms of the nanometric displacement, $\delta_{\varepsilon_r} = 1°$, (a) $\delta_{\varepsilon_t} = 1°$, (b) $\delta_{\varepsilon_t} = 0.99°$, (c) $\delta_{\varepsilon_t} = 0.97°$ and (d) $\delta_{\varepsilon_t} = 0$

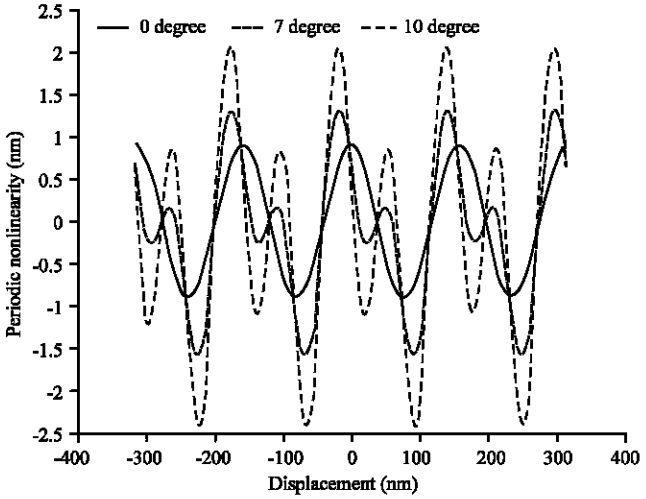

Fig. 5: The periodic nonlinearity resulting from one-elliptically polarized modes ($\delta_{\varepsilon_t} = 1^\circ$, $\delta_{\varepsilon_r} = 0$) in combination with the rotation angle of polarizing beam splitter

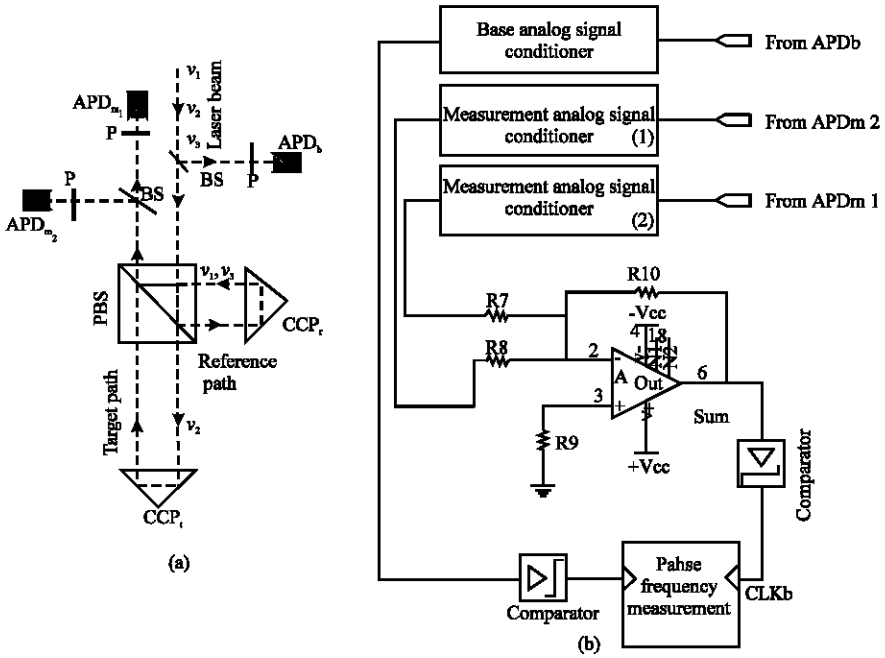

Fig. 6: The designed system for nonlinearity compensation. (a) The optical setup and (b) signal conditioner

produced. If $\delta_{\varepsilon_r} \neq \delta_{\varepsilon_t}$, the first order nonlinearity (Fig. 4d) and in other cases ($\delta_{\varepsilon_r} \cong \delta_{\varepsilon_t}$), the second order nonlinearity appears. The rotation angle of polarizing beam splitter effect in combination with the one-ellipticity of polarized modes is shown in Fig. 5. This deviation causes to increase the peak-to-peak of the second order periodic nonlinearity.

Based on the nonlinearity compensation for two-mode interferometer presented by Hou and Wilkening (1992), we design an optical setup and a signal conditioner for nonlinearity compensation, as shown in Fig. 6. In the measurement arm, two linear polarizers oriented at +45 and -45° and two avalanche photodiodes are used. The average signal of two photocurrents is produced by the signal conditioner circuit. As a result, the peak-to-peak nonlinearity is considerably decreased (Fig. 7). The peak-to-peak nonlinearity of 2.64 nm is result of the two photocurrents which by nonlinearity compensation, it is reached 26 pm.

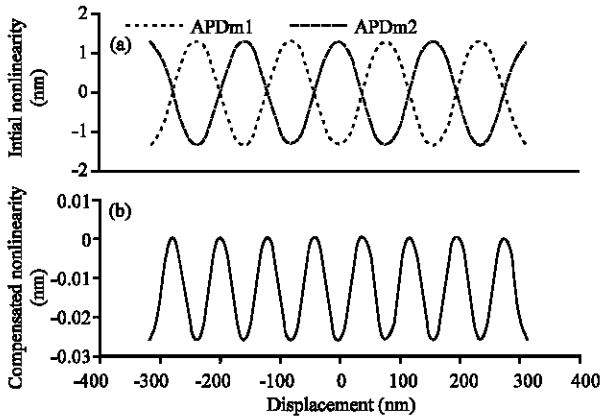

Fig. 7: The periodic nonlinearity in terms of the nanometric displacement, (a) initial nonlinearities and (b) compensated nonlinearity, $\alpha = 1.7°$, $\delta_{e_1} = 1°$, $\delta_{e_2} = -0.5°$

CONCLUSIONS

In this study, the periodic nonlinearities resulting from ellipticity of orthogonal polarized modes and rotation angle of polarizing beam splitter in the three-longitudinal-mode laser interferometer have been modeled. For nonlinearity modeling, the Jones matrices are used. The displacement error depends on the ellipticity and rotation angle can appear as first or second order nonlinearities. Also, an optical configuration and electronic signal conditioner are designed for nonlinearity compensation. Using this system, the periodic nonlinearity considerably decreased.

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