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Natural Convection in a Horizontal Wavy Enclosure

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Abstract: A numerical analysis is performed to study the influence of the amplitude b of the sinusoidal undulation, of bottom wall, of an horizontal enclosure on the heat transfers by two-dimensional natural laminar convection. The hot wavy bottom wall and the cold straight top wall are kept isothermal at temperature equal, respectively to T_h and T_c. And the two vertical straight walls (right and left) are considered symmetrical. Results are presented in the form of local and global Nusselt numbers distribution for a selected range of Raleigh (10^3-10^6) and for four values (0.00, 0.15, 0.25, 0.35) of aspect ratios A = 2.b/a. Air of Pr = 0.71 in an enclosure of non dimensional length As = L/H equal to 4 is chosen as the flow model to examine the influences of the aspect ratios at various Raleigh Number. The natural convection equations are discretised, using an implicit finite difference method, based on the finite volume approach. The SIMPLE algorithm, assumes the linkage between velocities and pressure fields. The variation of the Raleigh Number has allows the obtaining of several types of flows and several forks between these flows. The flow in enclosure is characterized with recirculation zones over summits and in hollows, where the local Nusselt numbers is always minimal. The numerical results show that the flow and the heat transfer are strongly affected by the amplitude b. In a general the transfers developed within an enclosure by not flat topography are lower than those obtained in a horizontal and uniform enclosure of the same length.

Key words: Natural convection, heat transfer, wavy wall, sinusoidal enclosure

INTRODUCTION

Natural convection heat transfer wall-fluid has many significant engineering applications experimentally and analytically; for example, double-wall thermal insulation, solar-collectors, electric machinery, cooling systems of micro-electronic devises, natural circulation in the atmosphere, etc. A considerable number of published articles are available that deal with flow characteristics, heat transfer, flow and heat transfer instability, design aspects, etc. On the other hand, studies dealing with convection problems inside more complex geometries have been rather limited. Complex geometry covers different types of geometries configurations, namely L-shaped cavities (Mahmud, 2002), trapezoïdal cavities (Peric, 1993), arc shaped enclosures (Chen and Cheng, 2002), wavy cavities (Mahmud *et al.*, 2002), etc. The complex fluid dynamic behaviour as well as their geometrical complexity and its effect on flow phenomenon have motivates many researchers to perform experimental or numerical work on this topic. In present study (Mahmud *et al.*, 2002), have shown the effect of surface waviness on natural convection heat transfer and fluid flow inside a vertical wavy walled enclosed for a range of wave ratio ($0.00 \le \lambda \le 0.4$) and aspect ratio ($1.0 \le A \le 2.0$). They observed that aspect ratio is the most important parameter for heat and fluid flows and higher heat transfer is obtained at a lower aspect ratio for a certain value of Grashof number. Das and Mahmud (2003) analyzed the free convection inside both the bottom and the ceiling, wavy and the isothermal enclosure. They indicated that, only at the lower Grashof number, the heat transfer rate rises when the amplitude wave length ratio changes from zero to other values. Mahmud and Islam (2003) solved the laminar free convection and entropy generation inside an inclined wavy enclosure using SIP (Strongly Implicit Procedure) solver on a non-staggered grid arrangement- They obtained that the inclination angle of cavity affects the entropy generation due to the heat transfer and fluid friction. Among them, Kumar (2000), presented the parametric results of flow and thermal fields inside a vertical wavy enclosure with porous media. Hadjadj and Kyal (1999) numerically examined the effect of sinusoidal protuberances on fluid flow and heat transfer inside an annular space using a non-orthogonal coordinate transformation. We have reported that both the local and average heat transfer increase with the increase of protuberance amplitude and Rayleigh number and decreasing Prandtl number. Kumar and Shalini (2003) investigated the effects of surface undulations on natural

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convection in porous enclosure with global cumulative heat flux boundary conditions for different undulation numbers and thermal stratification level. They indicated that the local Nusselt numbers are very sensitive to thermal stratification. Kumar *et al.* (1998) solved the free convection problem in an enclosure with bottom wave wall heated and cooled from the ceiling using finite element method. He indicated that separation zones starts around Ra = 50 in the single wave case and around Ra = 25 in the case with six waves per unit length. Murthy *et al.* (1997) analyzed the effect of the surface undulations on the natural convection heat transfer from an isothermal surface in a Darcian fluid-saturated porous enclosure by using the finite element method on a graded non-uniform mesh System. They found that the flow-driving Rayleigh number *Ra* together with geometrical parameters of wave amplitude, wave phase and the number of waves considered in the horizontal dimensions of the cavity influenced the flow and heat transfer process in the enclosure. Gao *et al.* (2000) solved the natural convection inside the wavy and inclined solar collector but they did not interest in flow behavior. Adjiout *et al.* (2002) performed and solved a similar problem with Gao and others, but in their case, the left vertical wall is flat and cold, whereas the right wall is wavy and hot. They indicated that the mean Nusselt number is decreased when it is compared with the square cavity. Dalal and Das (2005) made a numerical solution to investigate the inclined right wall wavy enclosure with spatially variable temperature boundary conditions. Oztop (2005) applied the elliptic grid generation to obtained sinusoidal duct geometry to enhance the forced convection heat transfer. Varol and Oztop (2006) investigated the effects of inclination angle on the laminar natural convection heat transfer and fluid flow in a wavy solar collector in steady state régime. They observed that the inclination angle is the most important and effective parameter on heal transfer which can be used to control the heat transfer inside the collector. Saidi *et al.* (1987) presented numerical and experimental results of flow over and heat transfer from, a sinusoidal cavity. They reported that the total heat exchange between the wavy wall of the cavity and the flowing fluid was reduced by the presence of vortex. The vortex plays the role of a thermal screen, which creates a large region of uniform temperature in the bottom of the cavity. Wang and Vanka (1995) presented heat transfer and flow characteristics inside a wavy walled charnel. Nishimura *et al.* (1984) investigated flow characteristics such as flow pattern, pressure drop and wall shear stress in a channel with symmetry; sinusoidal wavy wall. Asako and Faghri (1987) gave a finite-volume prediction of heat transfer and fluid flow characteristics inside a wavy walled duct and tube, respectively. Aydin *et al.* (1999), Eisherbiny (1996), Hamady *et al.* (1989) and Ozoe *et al.* (1975) presented results of heat transfer characteristics inside the rectangular enclosures at different aspect ratios and orientations without surface waviness.

The main purpose of the present study is to provide the effect of the amplitude of the sinusoidal oundulation, of bottom wall, of a shallow and horizontal wavy enclosure on the heat transfers by two-dimensional natural laminar convection natural. To the best of the authors' knowledge; free convection in the shallow wavy enclosure has not yet been investigated

The rate of heat transfer in terms of local and global Nusselt numbers are presented for different aspect ratios and Rayleigh numbers; respectively. Flow and thermal fields are analyzed by parametric presentation of streamlines and isothermal lines.

PROBLEM STATEMENT

Consider a two-dimensional wavy wall horizontal enclosure of length L and height H as shown in Fig. 1. The hot wavy bottom wall and the cold straight top wall are kept isothermal at temperature equal respectively to T_h and T_c and the two vertical straight walls (right and left) are considered symmetrical. The fluid is assumed to be of constant properties and the Boussinesq approximation is employed for the gravity terms. Stress-work are neglected. In the present study, a, a^2/α and α/a are used, respectively, as the length, time and velocity scales, where α denotes the thermal diffusivity. The dimension less temperature function is defined as $\theta = (T-T_r)/\Delta T$, in which $\Delta T = (T_h-T_c)$ stands for the temperature difference of the two isothermal walls and $T_r = T_c$ is the fluid temperature at reference state.

The wavy wall function and governing equations in dimension less form can be cast into the following form respectively.

$$F(X) = A.\left[1+\cos(2.\pi X)\right] \tag{1}$$

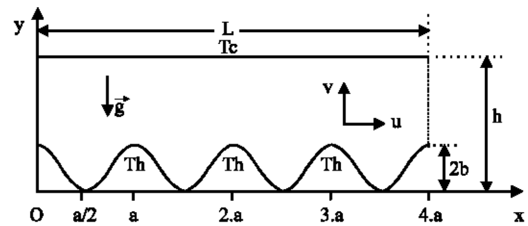

Fig. 1: Schematic representation of the computational geometry under consideration

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{2}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\Pr.\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{\partial P}{\partial X} \tag{3}$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\Pr.\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{\partial P}{\partial Y}+Ra.\Pr.\theta \tag{4}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{5}$$

where $A = (2.b/a)\nu$ is the aspect ratio of the enclosure; $Ra = g.\beta.a^3.(T_h-T_c)/\nu.\alpha$ is the Rayleigh number, $\Pr = \nu/\alpha$ is the Prandtl number, β is the thermal expansion coefficient and ν is the kinematics viscosity. The two opposite walls (bottom and top) are kept at the uniform temperatures T_h and T_c, respectively. The boundary conditions for the system of Eq. 2-5 are

$U = \partial V/\partial X = \partial\theta/\partial X = 0$; at two side walls
$U = V\ 0$, $\theta = 1$; at hot wavy bottom wall
$U = V = \theta = 0$; at cold straight top wall

HEAT TRANSFER

Heat transfer rate is measured by local (Nux) and average (Nu) Nusselt numbers. Following equations arc used to calculate the local and average Nusselt numbers:

$$Nux = -\left(\frac{\partial \theta}{\partial N}\right)_w \tag{6}$$

$$Nu = \int_0^4 Nux.dX \tag{7}$$

Where the gradient term in $(\partial\theta/\partial N)_w$ Eq. (6) is the temperature gradient normal to the hot wavy wall.

NUMERICAL PROCEDURE

The system of Eq. (2-5) with the boundary conditions stated above is solved by using finite volume method and the S1MPLE algorithm, Patanker (1980). Since the flow fields for the parameter range considered lie in the steady flow regime. The value of $\Delta\tau = 10^{-3}$ is used through the course of the computation. As. the maximum relative deviation of the mean Nusselt numbers between two successive time-steps is less than the value of 10^{-4} or the

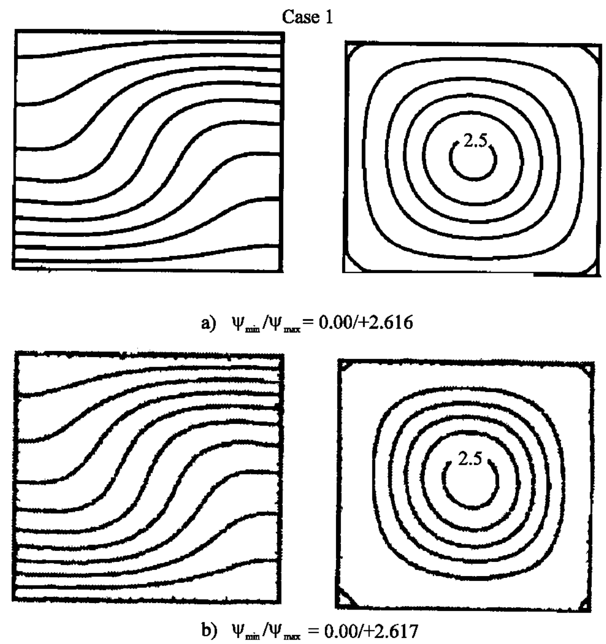

Fig. 2: Streamlines (right) and isotherms (left) for Ra=3800, Pr = 5580 and As = 1, a) Present prediction b) Soong *et al.* (1996) prediction

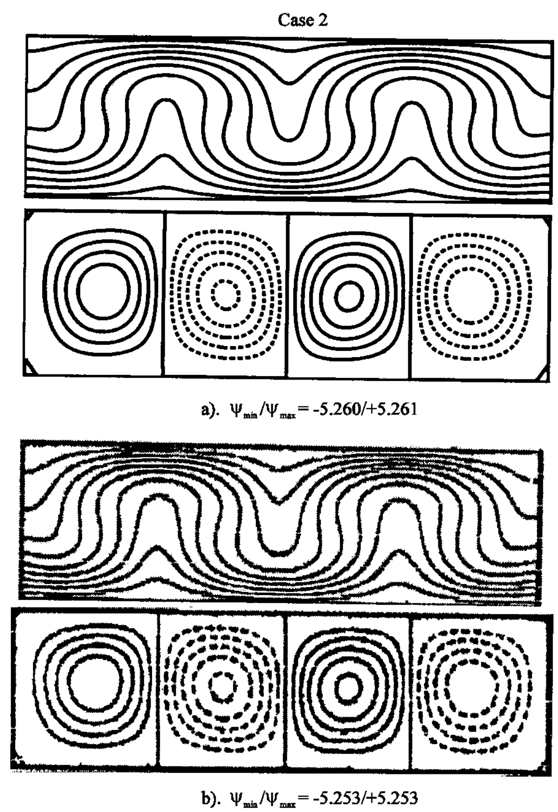

Fig. 3: Streamlines (bottom) and isotherms (top) for Ra = 5000, Pr = 0.71 and As = 4, a) Present prediction b) Soong *et al.* (1996) prediction

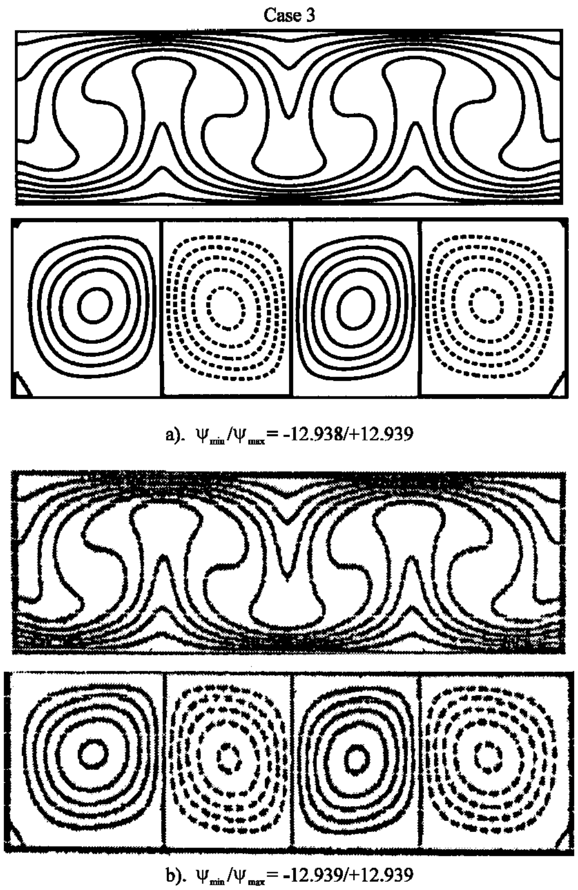

Fig. 4: Streamlines (bottom) and isotherms (top) for Ra = 20000, Pr = 0.71 and As = 4, a) Present prediction b) Soong *et al*. (1996) prediction

maximum value of the relative deviation of' the temperature less than 10^{-6} the procedure is regarded as converged.

To cheek the validity of the present numerical procedure, thermally driven flows in a square and rectangular cavity were solved.

For code validation; our numerical resultants (for aspect ratio of the enclosure A = 0) are compared with the resultants of Soong *et al*. (1996) solutions (for inclination angle γ = 0). Comparisons arc lines in Fig. 2-4. Present prediction shows a very good agreement with the result of Soong *et al*. (1996).

ACCURACY

In the present study, five combinations (30×113; 46×177; 62×241; 74×289 and 82×321) of control volumes are used to test the effect of grid size on the accuracy of the predicted results. Figure 5 shows the distribution of average Nusselt numbers of the hot wall as a function of

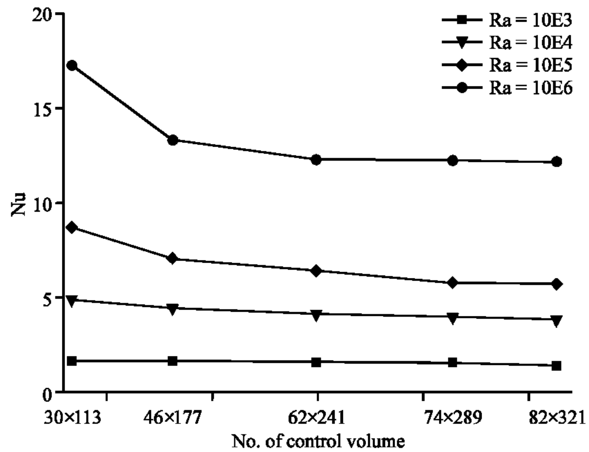

Fig. 5: Variation of average Nusselt number as a Function of control volume for A = 0.25 and As = 4

grid sizes for four different Rayleigh numbers. It is clear from the figure that at lower Rayleigh number, average Nusselt number is almost independent of grid sizes. At higher Rayleigh numbers, the two higher grid sizes (74×289) and 82×321) give almost the same result (Fig. 5). It is well known that the high mesh refinement always provides better result in the finite-volume method. The main disadvantage in taking higher mesh number is the increase in calculation time, which can be reduced by using a higher speed of Pentium processor. Our goal was to get better results. Thus; throughout this study, the results are presented for 74×289 CVs' for better accuracy.

RESULTS AND DISCUSSION

Flow and thermal field: The flow pattern inside the wavy enclosure and the temperature profile are presented in terms of streamlines and isothermal lines in Fig. 6(a-d) for A = 0.25; As = 4 at four selected Rayleigh numbers. At low Rayleigh number when convection current inside the cavity is comparatively weak fluid stream near the hot wall tends to move towards the centerline or crest of the cavity were two streams from the opposite direction mix and rise upwards.

Two symmetric counter-rotating vortices are observed at top and bottom of the wave wall on the flow field due to the uniform temperature gradient. At low Rayleigh number Fig. 6(a, b) circulation inside the cavity is very weak. Convection is less prominent at this Rayleigh number and heat transfer is mainly dominated by conduction. Isothermal lines are nearly parallel to each other and follow the geometry of the wavy surfaces. A further increase of Rayleigh numbers increase the circulation strength inside the enclosure. Here, uniform

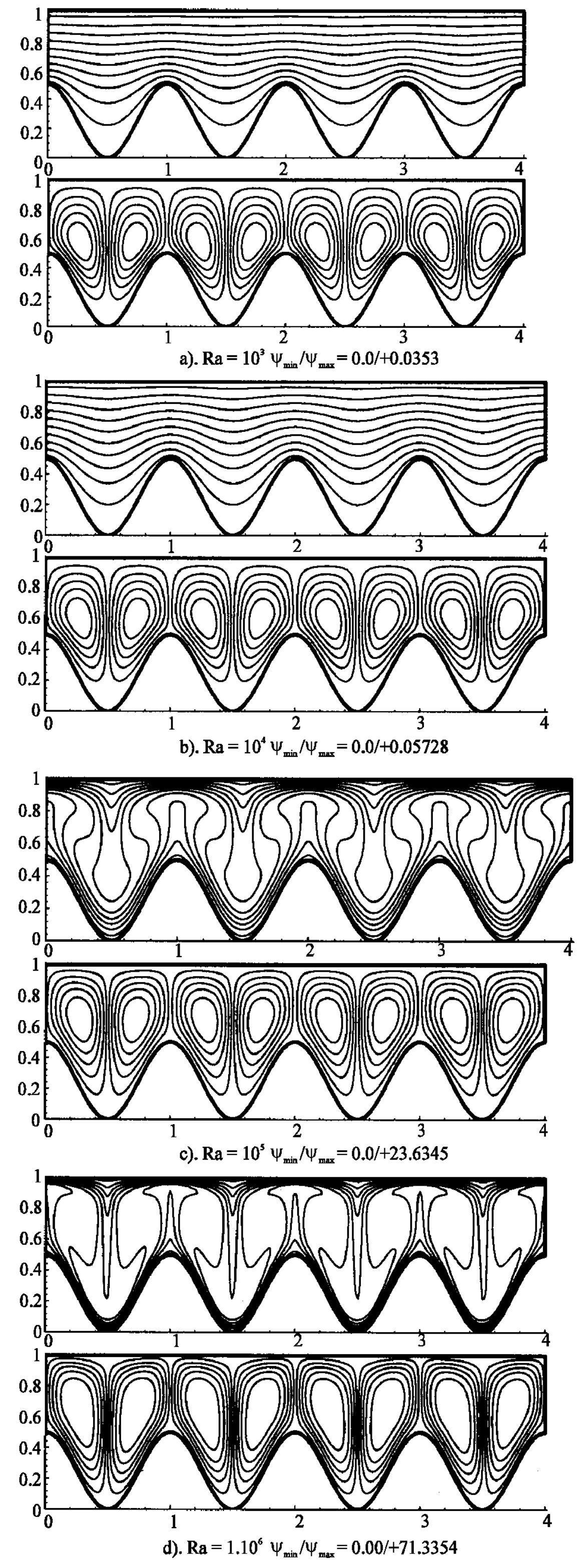

Fig. 6: Streamlines (bottom) and isotherms (top) for different Rayleight number at A = 0.25; As = 4 and Pr = 0.71

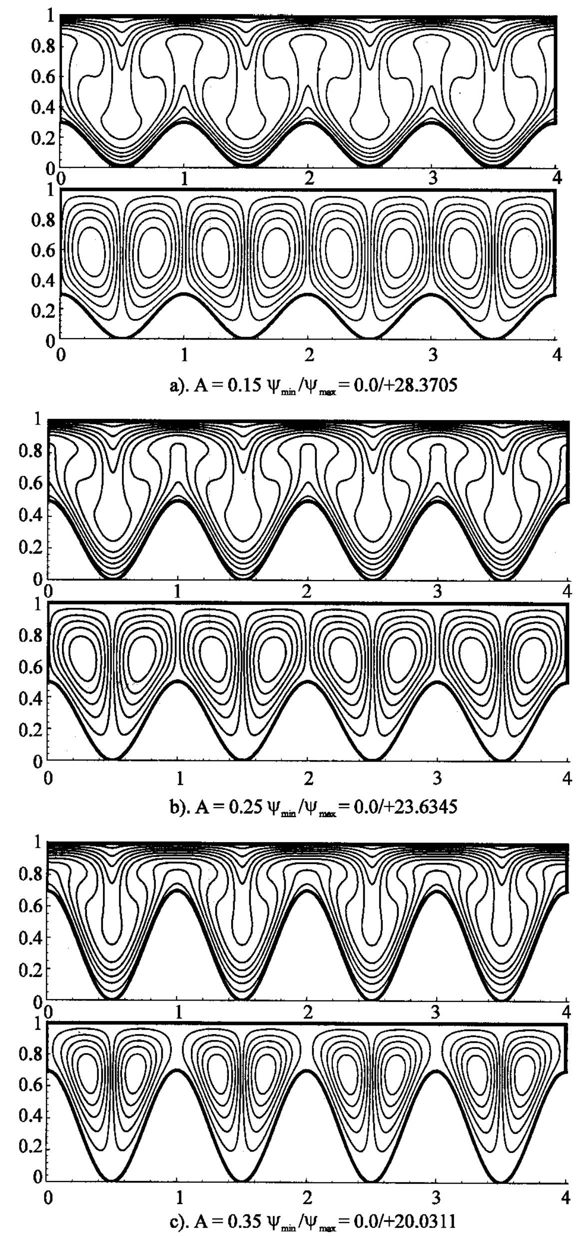

Fig. 7: Streamlines (bottom) and isotherms (top) for different aspect ratio at As = 4; Pr = 0.71 and Ra = 10^5

temperature profile is changed and three high gradient spot is observed at top and bottom of the hot wall due to rapid circulation of fluid inside the cavity. Isotherms turn up (convective distortion) towards die cold wall due to the strong influence of the convection current.

Figure 7 shows the effect of aspect ratio (A) on flow field at constant Rayleigh number (Ra = 1.10^5). Whatever the value of A, flow inside the cavity is characterized by same number of cell multi-cellular pattern. An increase in the value of A decreases the size and the maximum values of vortices respectively.

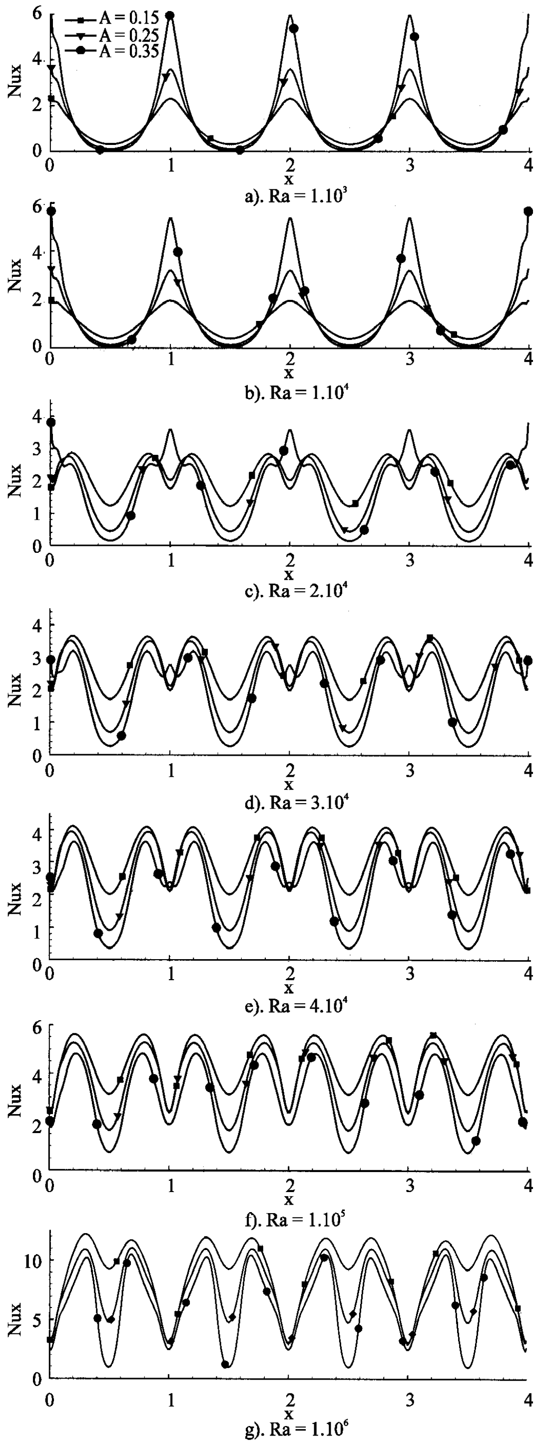

Fig. 8: Variation of local Nusselt number along the wave wall; for As = 4; Pr = 0.71 and for different aspect ratios at different Rayleigh number

Local heat transfer distribution: Local Nusselt number distribution at the bottom wall is shown in Fig. 8(a-g) for As = 4 and A = 0.15; 0.25 and 0.35 at seven selected Rayleigh numbers. The profile of the distribution of the local Nusselt number present a symmetry with regard to hollows and a periodicity with regard to summits.

For the low values of the Rayleigh number ($Ra \leq 10^4$) the thermal exchange makes essentially by conduction, the distribution of the local Nusselt number possesses maximums (peaks) at the level of summits (Fig. 8a, b). For $Ra > 10^4$, the thermal exchange by convection is dominant, the peaks observed at summits ease with the increase of the Rayleigh number (Fig. 8c-h).

Average heat transfer distribution: The effect of aspect ratio of the enclosure on average heat rate transfer is

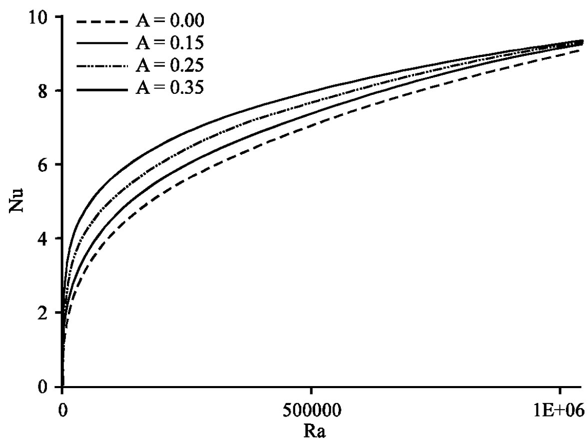

Fig. 9: Variation of average Nusselt number for As = 4; Pr = 0.71 and for different aspect ratios at different Rayleigh number

Table 1: Average Nusselt number and maximum values of ψ (ψ_{max}) at different Rayleigh number for different aspect ratios

$Ra = 10^3$ Aspect ratio (A)	Nu	ψ_{max}
0.00	1.0946	0.00043
0.15	1.2645	0.0298
0.25	1.6277	0.0353
0.35	2.3400	0.0512
$Ra = 10^4$		
0.00	1.0980	0.0050
0.15	1.2958	0.0825
0.25	1.6430	0.5725
0.35	2.3580	0.7414
$Ra = 10^5$		
0.00	4.4754	44.1528
0.15	4.6284	28.3705
0.25	4.8713	23.6345
0.35	5.0117	20.0310
$Ra = 10^6$		
0.00	9.5203	193.7096
0.15	9.7013	81.1707
0.25	9.8511	71.3354
0.35	10.0516	65.6711

shown in Table 1. At lower Rayleigh number the effect of aspect ratio is significant. However, at higher Rayleigh number this effect is very small. At higher aspect ratio heat transfer rate is higher at lower Rayleigh number when aspect ratio increase from zero to other values and after then, further increases of aspect ratio shows a negligible effect on average heat transfer rate (Fig. 9).

CONCLUSIONS

Laminar steady natural convection heat transfer and fluid flow in shallow wavy enclosure are investigated numerically. The main conclusions that are drawn the present study are provided below.

- Two circulation cells are obtained at the enclosure cavity of wave in different directions especially at the small aspect ratio.
- Flow and thermal fields are affected by geometrical parameters and Rayleigh number
- Heat transfer is increased with the increasing of aspect ratio.
- Local Nusselt numbers show wavy variation and maximum Nusselt number values are obtained on the top of the wave at low Rayleigh number ($Ra = 1.10^4$). For $Ra>10^4$, the peaks observed at summits ease with the increase of the Rayleigh number.

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NOMENCLATURE

A	aspect ratio, $A = 2.b/a$ (-)
a	wave length of the wavy wall; (m)
As	non dimensional length, $As = L/h$ (-)
b	amplitude height of the wavy wall; (m)
g	gravitational acceleration, (m sec^{-2})
H	height of the enclosure; (m)
h	heat transfer coefficient, (W m^{-2}.K)
L	length of the enclosure, (m)
Nu	mean Nusselt number
Nux	local Nusselt number
p, P	dimensional pressure and pressure (N m^{-2})
Pr	Prandtl number, $Pr = \nu/\alpha$
Ra	Rayieigh number, $Ra = g.\beta\ (T_p - T_o).h^3/\alpha.\nu$
t	time, (sec)
u, v	dimensional velocity components, (m sec^{-1})
U, V	dimensionless velocity components
x, y	cartesian coordinates; (m)
X, Y	dimensionless coordinates

Greek symbols

α	thermal diffusivity.(m^2 sec)
β	thermal expansion coefficient,(K^{-1})
λ	Thermal conductivity, (w.$m.^1$ $K.^1$)
ν	Kinemtic viscosity, (m^2 sec^{-1})
θ	Dimensionless temperature function $\theta = (T\text{-}T_c)/(T_h\text{-}T_c)$
ρ	Density of the fluid(kg m^{-3})
Ψ	Dimensionless stream function
τ	Dimensionless time, $\tau = \alpha.t/a^2$

Subscripts

h	Hot
c	Cold

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