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Unsteady Magnetohydrodynamic Stokes Flow of Viscous Fluid Between Two Parallel Porous Plates

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Abstract: The Unsteady stokes flow of an electrically conducting viscous, incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate is discussed. An exact solution is obtained for all values of R (Suction Reynolds number) and M (Hartmann number). Expressions for the velocity components and the pressure are obtained. The graphs of axial and radial velocity profiles have been drawn for different values of M.

Key words: Unsteady flow, parallel porous plates, transverse magnetic field, suction reynolds number, hartmann number, pressure drop

INTRODUCTION

The Unsteady Magnetohydrodynamic flow between two parallel porous plates is a classical problem whose solution has many applications in magnetohydrodynamic (MHD) power generators, cooling system, aerodynamics heating, polymer technology, petroleum industry, centrifugal separation of matter from fluid, purification of crude oil and fluid droplets sprays.

Hassanien and Mansour (1990) discussed the Unsteady magnetic flow through a porous medium between two infinite parallel plates.

Bagchi (1996) studied the problem of Unsteady flow of viscoelastic Maxwell fluid with transient pressure gradient through a rectangular channel.

Attia and Kotb (1996) studied the Steady, fully developed MHD flow and heat transfer between two parallel plates with temperature dependant viscosity.

Attia (1999) extended the problem to the transient state.

Ezzat *et al.* (1999) studied the problem of micropolar magnetohydrodynamic boundary layer flow.

Aboul-Hassan and Attia (2002) discussed the flow of a conducting Visco elastic fluid between two horizontal porous plates in the presence of a transverse-magnetic field.

Nabil *et al.* (2003) studied the MHD flow of Non-Newtonian visco-elastic fluid through a porous medium near an accelerated plate.

Attia (2004) has considered the Unsteady Hartmann flow with heat transfer of a viscoelastic fluid considering the Hall effect.

Hayat *et al.* (2004) studied the Hall effects on the Unsteady hydromagnetic oscillatory flow of a second-grade fluid.

Krishnambal and Ganesh (2004) discussed the Unsteady stokes flow of viscous fluid between two parallel porous plates.

Attia (2005a) studied the Unsteady laminar flow of an incompressible viscous fluid and heat transfer between two parallel plates in the presence of a uniform suction and injection considering variable properties.

Attia (2005b) studied the Unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity.

The objective of this study is to analyse the Unsteady Magnetohydrodynamic Stokes flow of viscous fluid between two parallel porous plates when the fluid is being withdrawn through both the walls of the channel at the same rate. The problem is reduced to a third order nonlinear differential equation which depends on a Suction. Reynolds number R and a Hartmann number M for which an exact solution is obtained.

MATHEMATICAL FORMULATION

The unsteady laminar flow of an incompressible viscous fluid between two parallel porous plates is

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considered in the presence of a transverse magnetic field Ho applied perpendicular to the walls. The origin is taken at the centre of the channel and let x and y be the coordinate axes parallel and perpendicular to the channel walls.

The length of the channel is assumed to be L and 2 h is the distance between the two plates. Let u and v be the velocity components in the x and y directions, respectively.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Equations of momentum are

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\sigma_e B_0{}^2 u \tag{2}$$

$$\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

Where σ is the electrical conductivity and $B_0 = \mu_e H_0$, μ_e being the magnetic permeability.

The boundary conditions are u (x , h) = 0, u (x, -h) = 0, v (x, h) = v_0 and v (x, -h) = - v_0 where v_0 is the velocity of suction at the walls of the channel.

Let $\eta = y/h$, $u = u(x,y)\, e^{i\omega t}$, $v = v(x,y)\, e^{i\omega t}$, $p = p(x,y)e^{i\omega t}$ and the Eq. 1-3 become

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \eta}=0 \tag{4}$$

$$\rho i\omega u e^{i\omega t}=-\frac{\partial p}{\partial x}e^{i\omega t}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)e^{i\omega t}-\sigma_e B_0^2 u e^{i\omega t}$$

$$\rho i\omega u=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right)-\sigma_e B_0^2 u \tag{5}$$

$$\rho i\omega v e^{i\omega t}=-\frac{\partial p}{\partial y}e^{i\omega t}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)e^{i\omega t}$$

$$\text{i.e. } \rho i w v=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}\right) \tag{6}$$

Let ν = μ/ρ = Kinematic Viscosity, ρ the density of the fluid, μ the coefficient of viscosity and p the pressure.

The boundary conditions are converted into

$$u(x,1) = 0,\ u(x,-1) = 0 \tag{7}$$

and

$$v(x,1) = v_0\ ,\ v(x,-1) = -v_0 \tag{8}$$

Let Ψ be the stream function such that

$$u=\frac{1}{h}\frac{\partial \psi}{\partial \eta} \tag{9}$$

$$v=-\frac{\partial \psi}{\partial x} \tag{10}$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi\,(x,\eta)=\frac{1}{h}\left(hu(0)-v_0 x\right)f(\eta) \tag{11}$$

Where u(0) is the average entrance velocity at x = 0 . From Eq. 11, the velocity components (9) and (10) are given by

$$u=\frac{1}{h}\left(hu(0)-v_0 x\right)f'(\eta) \tag{12}$$

$$v = v_0\, f(\eta) \tag{13}$$

where the prime denotes the differentiation with respect to the dimensionless variable η = y/h. Since the fluid is being withdrawn at constant rate from both the walls, v_0 is independent of x.

Using (12) and (13) in (5) and (6), the equations of momentum reduce to

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=\left(u(0)-\frac{v_0 x}{h}\right)\bullet\left(i\omega f'(\eta)-\frac{\nu}{h^2}f'''(\eta)+\frac{\sigma_e B_0^2}{\rho}f'(\eta)\right) \tag{14}$$

$$-\frac{1}{h\rho}\frac{\partial p}{\partial \eta}=i\omega v_0 f(\eta)-\frac{\nu\cdot v_0 f''(\eta)}{h^2} \tag{15}$$

Now differentiating (15) w.r.t.'x', we get

$$\frac{\partial^2 p}{\partial x\partial \eta}=0 \tag{16}$$

Differentiating (14) w.r.t. 'η' , we get

$$\frac{\partial^2 p}{\partial x\partial \eta}=\left(u(0)-\frac{v_0 x}{h}\right)\bullet\frac{d}{d\eta}\left(\begin{array}{l} i\omega f'(\eta)-\frac{\nu}{h^2}f'''(\eta)\\ +\frac{\sigma_e B_0^2}{\rho}f'(\eta)\end{array}\right) \tag{17}$$

From (16), Eq. 17 can be written as

$$\frac{d}{d\eta}\left(i\omega f'(\eta)-\frac{\nu}{h^2}f'''(\eta)+\frac{\sigma_e B_0^2}{\rho}f'(\eta)\right)=0 \quad (18)$$

which is true for all x.

Let R = Suction Reynolds number = hv_0/γ

M = Hartmann number = $B_0 h\left(\frac{\sigma_e}{\nu\rho}\right)^{\frac{1}{2}}$

Integrating (18) w.r.t. η and substituting the above expressions we get

$$f'''(\eta)-\alpha^2h^2f'(\eta)-a_1Rf'(\eta)=K \quad (19)$$

Where $R=\frac{hv_0}{\nu}$ and $a_1=\frac{\sigma_e hB_0^2}{\rho v_0}$

and K is an arbitrary constant.

Boundary conditions on f(η) are

$$f(1)=1,\ f(-1)=-1,\ f'(1)=0 \text{ and } f'(-1)=0 \quad (20)$$

Hence the solution of the equations of motion and continuity is given by a non linear third order differential Eq. (19) subject to the boundary conditions (20).

∴ Equation (19) can be rewritten as

$$f'''(\eta)-(\alpha^2h^2+M^2)f'(\eta)=K$$

where $a_1R=M^2$ and $\alpha^2=\frac{i\rho\omega}{\mu}$

i.e. $\left(D^3-(\alpha^2h^2+M^2)D\right)f(\eta)=K$

i.e. $D\left(D^2-(\alpha^2h^2+M^2)\right)f(\eta)=K$ where

$D=d/d\eta$ and $D^2=d^2/d\eta^2$

Solving the above equation, we get

$$f(\eta)=A+Be^{\sqrt{\alpha^2h^2+M^2}\eta}+Ce^{-\sqrt{\alpha^2h^2+M^2}\eta}-\frac{K\eta}{\alpha^2h^2+M^2} \quad (21)$$

Applying the boundary conditions on f(η) and solving the values of the arbitrary constants, we get

$$K=\frac{(\alpha^2h^2+M^2)^{3/2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}}{\left(1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)} \quad (22)$$

$$A=0 \quad (23)$$

$$B=\frac{1}{\left(2\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}\right)\left(1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)} \quad (24)$$

$$C=-\frac{1}{\left(2\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}\right)\left(1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)} \quad (25)$$

Substituting the values of the arbitrary constants in f(η), we get

$$f(\eta)=\frac{1}{\left(1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)}\bullet\left(\frac{\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}\eta}{\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}}-\eta\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right) \quad (26)$$

Hence the expressions for the velocity components are

$$u=\frac{1}{h}\left[hu(0)-v_0x\right]f'(\eta)e^{i\omega t}$$

$$=\left(u(0)-\frac{v_0x}{h}\right)\left(\frac{1}{\left(1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)}\right)\bullet\left(\frac{\sqrt{\alpha^2h^2+M^2}\operatorname{Cosh}\sqrt{\alpha^2h^2+M^2}\eta}{\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}}-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)e^{i\omega t}$$

$$=\left(u(0)-\frac{v_0x}{h}\right)\bullet\left(\frac{\sqrt{\alpha^2h^2+M^2}}{\left(1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)}\right)\bullet\left(\frac{\operatorname{Cosh}\sqrt{\alpha^2h^2+M^2}\eta}{\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}}-\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)e^{i\omega t} \quad (27)$$

$$v=v_0f(\eta)=v_0\left(\frac{1}{1-\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}}\right)\bullet\left(\frac{\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}\eta}{\operatorname{Sinh}\sqrt{\alpha^2h^2+M^2}}-\eta\sqrt{\alpha^2h^2+M^2}\operatorname{Coth}\sqrt{\alpha^2h^2+M^2}\right)e^{i\omega t} \quad (28)$$

Pressure distribution: From Eq (14), we have

$$\frac{h^2}{\rho\nu}\frac{\partial p}{\partial x}=\left(u(0)-\frac{v_0x}{h}\right)\left(f'''(\eta)-\frac{h^2}{\nu}\left(i\omega+\frac{\sigma_eB_0^2}{\rho}\right)f'(\eta)\right)$$

and since $f'''(\eta)-\alpha^2h^2f'(\eta)-a_1Rf'(\eta)=K$ (from 19)

i.e. $f'''(\eta)-\frac{h^2i\omega}{\nu}f'(\eta)-M^2f'(\eta)=K$

we have

$$\frac{\partial p}{\partial x} = \frac{K\rho\nu}{h^2}\left(u(0) - \frac{v_0 x}{h}\right)$$
$$= \frac{K\mu}{h^2}\left(u(0) - \frac{v_0 x}{h}\right) \qquad \because \left(\nu = \frac{\mu}{\rho}\right) \tag{29}$$

Now, from Eq. (15) we have

$$\frac{\partial p}{\partial \eta} = \frac{\mu v_0}{h} f''(\eta) - i\omega v_0 h\rho f(\eta) \tag{30}$$

$$\text{Since } dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$$
$$= \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial \eta}d\eta \qquad \because \left(\eta = \frac{y}{h}\right)$$

$$\Rightarrow dp = \frac{K\mu}{h^2}\left(u(0) - \frac{v_0 x}{h}\right)dx + \left(\begin{array}{c}\frac{\mu v_0}{h} f''(\eta) \\ -i\omega v_0 h\rho f(\eta)\end{array}\right)d\eta \tag{31}$$

Integrating (31), we get

$$p(x,\eta) = \frac{K\mu}{h^2}\left(u(0)x - \frac{v_0 x^2}{2h}\right) + \left(\begin{array}{c}\frac{\mu v_0}{h}\int f''(\eta)d\eta \\ -i\omega v_0 h\rho\int_0^\eta f(\eta)d\eta\end{array}\right) + K_1$$

$$p(x,\eta) = \frac{K\mu}{h^2}\left(u(0)x - \frac{v_0 x^2}{2h}\right) + \left(\begin{array}{c}\frac{\mu v_0}{h} f'(\eta) \\ -i\omega v_0 h\rho\int_0^\eta f(\eta)d\eta\end{array}\right) + K_1 \tag{32}$$

∴ The pressure drop is given by

$$\Rightarrow p(x,\eta) - p(0,0) = \frac{K\mu}{h^2}\left(u(0)x - \frac{v_0 x^2}{2h}\right)$$
$$+ \frac{\mu v_0}{h}\left(f'(\eta) - f(0)\right) - i\omega\rho v_0 h\int_0^\eta f(\eta)d\eta \tag{33}$$

DISCUSSION

The graphs of the axial velocity and radial velocity profiles have been drawn for different values of M.

Figures 1-4 represents the axial velocity profiles at different cross sections of the channel namely at x = 0, x = 3, x = 4 and x = 5 when the average entrance velocity is $u_0 = 0.5$ and h = 1.0. The magnitude of the axial velocity increases as x increases from x = 0 to x = 5 for different values of ωt, namely ωt = 0, π/4, π/2, 3π/4 and π, respectively.

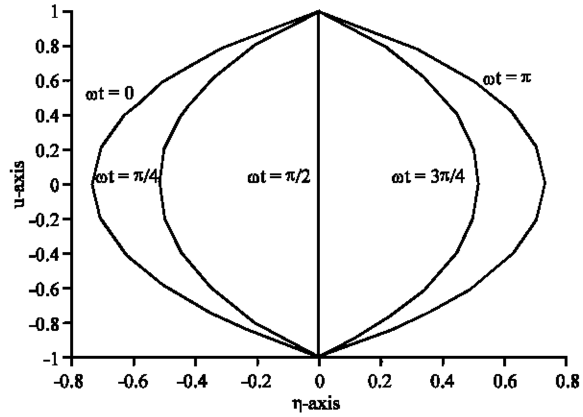

Fig. 1: Axial velocity profiles when u0 = 0.5, v0 = 0.5, h = 1.0, M = 1.0, α = 1.0, x = 0 and for different values of ωt

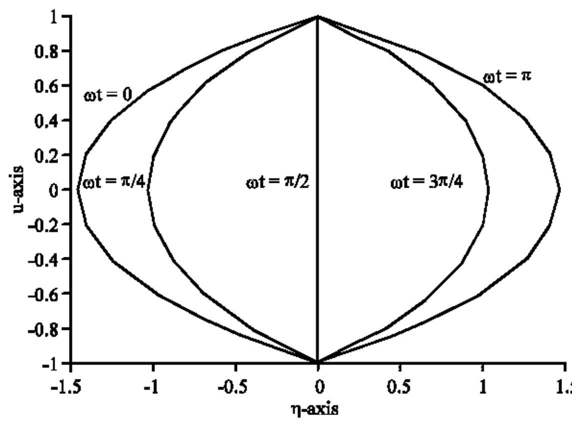

Fig. 2: Axial velocity profiles when u0 = 0.5, v0 = 0.5, h = 1.0, M = 1.0, α = 1.0, x = 3 and for different values of ωt

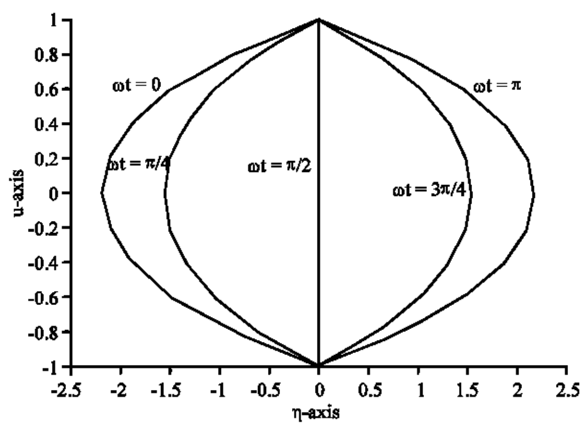

Fig. 3: Axial velocity profiles when u0 = 0.5, v0 = 0.5, h = 1.0, M = 1.0, α = 1.0, x = 4 and for different values of ωt

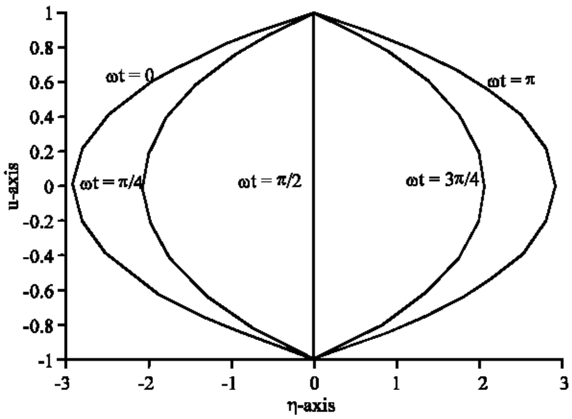

Fig. 4: Axial velocity profiles when u0 = 0.5, v0 = 0.5, h = 1.0, M = 1.0, α = 1.0, x = 5 and for different values of ωt

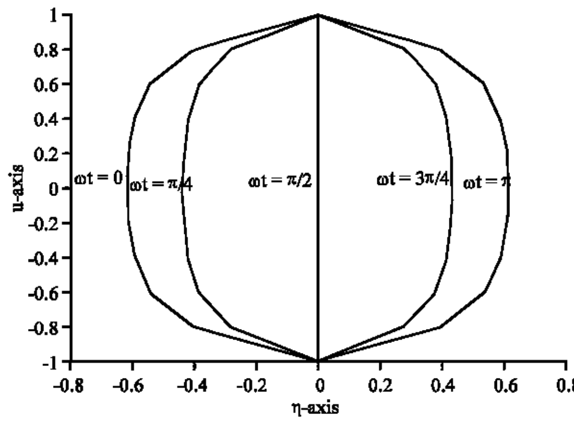

Fig. 7: Axial velocity profiles when u0 = 0.5, v0 = 0.5, h = 1.0, M = 5.0, α = 1.0, x = 0 and for different values of ωt

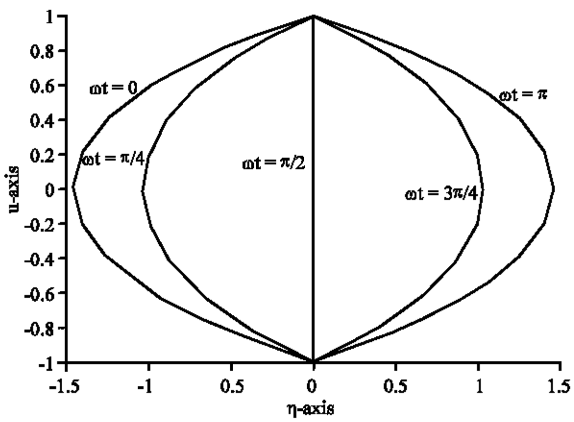

Fig. 5: Axial velocity profiles when u0 = 1.0, v0 = 0.5, h = 1.0, M = 1.0, α = 1.0, x = 0 and for different values of ωt

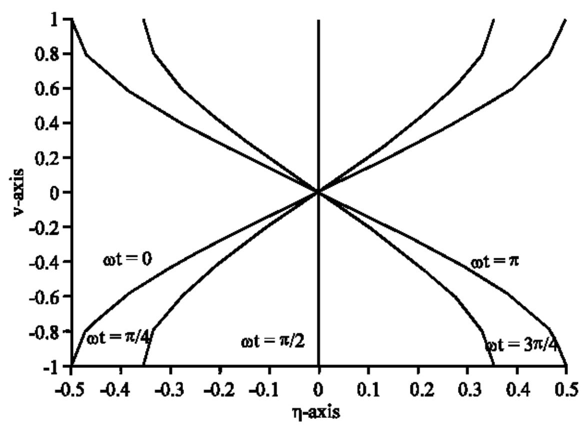

Fig. 8: Radial velocity profiles when v0 = 0.5, h = 1.0, M = 1.0, α = 1.0 and for different values of ωt

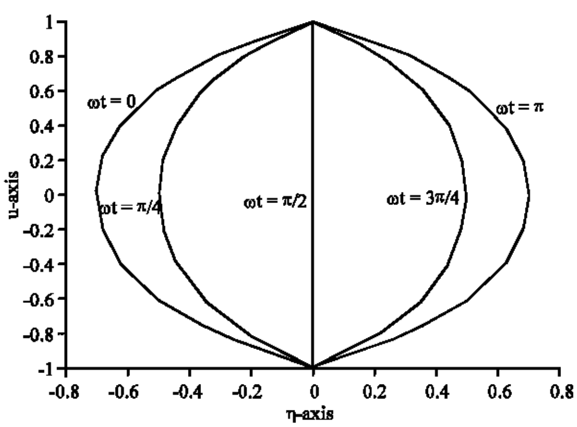

Fig. 6: Axial velocity profiles when u0 = 0.5, v0 = 0.5, h = 1.0, M = 2.0, α = 1.0, x = 0 and for different values of ωt

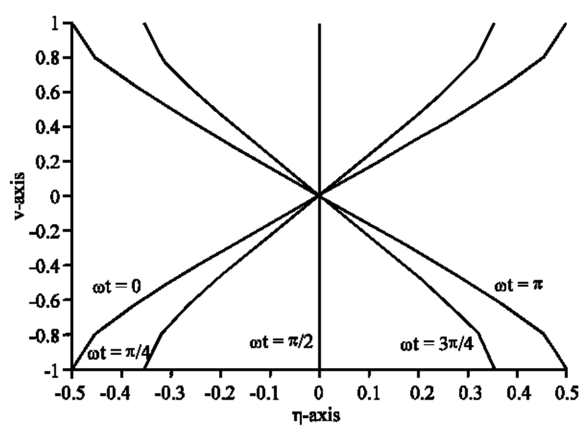

Fig. 9: Radial velocity profiles when v0 = 0.5, h = 1.0, M = 5.0, α = 1.0 and for different values of ωt

The Fig. 5 represents the axial velocity profiles of u at $x = 0$, $h = 1.0$ when the inlet velocity is increased to $u_0 = 1.0$ from $u_0 = 0.5$. It is clearly seen that the magnitudes of the axial velocity u are more when the values of x are increased and also the magnitudes of the axial velocity u are more when the inlet velocity is increased.

When the Hartmann number M is increased from $M = 1$ to $M = 2$ or $M = 5$, we see that the magnitude of the axial velocity profiles decreases as seen in Fig. 6 and 7.

The Fig. 8 and 9 represent the radial velocity profiles of v at $vo = 0.5$, $h = 1.0$, $\alpha = 1.0$ and for different values of M. As M increases from $M = 1.0$ to $M = 5.0$ we see that there is a marginal increase in the magnitude of the radial velocity. It is also seen from the Fig. 8 and 9 that the radial velocity vanishes for $\omega t = \pi/2$ and the radial velocity profiles are non linear for the other values of ωt.

Special case: If the distance between the two plates is assumed to be h, we get the solution as,

$$f(\eta) = \frac{-2\alpha h \mathrm{Sinh}\alpha h + 4\alpha h\eta \mathrm{Sinh}\alpha h - 2\cdot(1-e^{-\alpha h}) \cdot e^{\sqrt{\alpha^2h^2+M^2}\eta} + 2(e^{\alpha h}-1)e^{-\sqrt{\alpha^2h^2+M^2}\eta}}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)}$$

where the velocity components are

$$u = \frac{1}{h}(h\,u(0) - v_0 x) f'(\eta)$$

$$= \frac{1}{h}\begin{pmatrix} h\,u(0) \\ -v_0 x \end{pmatrix}\left(\frac{4\alpha h \mathrm{Sin}\alpha h - 2(1-e^{-\alpha h})\sqrt{\alpha^2h^2+M^2}\; e^{\sqrt{\alpha^2h^2+M^2}\eta} + 2(1-e^{\alpha h})\sqrt{\alpha^2h^2+M^2}\; e^{-\sqrt{\alpha^2h^2+M^2}\eta}}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)} \right)$$

and

$$v = v_0 f(\eta) = v_0\left(\frac{-2\alpha h \mathrm{Sinh}\alpha h + 4\alpha h\eta \mathrm{Sinh}\alpha h - 2\cdot(1-e^{-\alpha h})\cdot e^{\sqrt{\alpha^2h^2+M^2}\eta} + 2(e^{\alpha h}-1)e^{-\sqrt{\alpha^2h^2+M^2}\eta}}{4 + e^{\alpha h}(\alpha h - 2) - e^{-\alpha h}(\alpha h + 2)} \right)$$

The above result reduces to the result of (Krishnambal and Ganesh, 2004) when the Hartmann number is zero (i.e., when $M = 0$) and $a = 2$.
where $a = 1 - v_1/v_2$, $0 \le |v_1| \le |v_2|$
Here $a = 1 - v_0/-v_0, = 2$.

CONCLUSIONS

In the above analysis a class of solutions of unsteady magnetohydrodynamics stokes flow of viscous fluid between two parallel porous plates is presented, in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate.

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