



Journal of Applied Sciences

ISSN 1812-5654

science
alert

ANSI*net*
an open access publisher
<http://ansinet.com>

Amplitude of Wave Formation by Vortex Shedding in Open Channels

¹Mehdi Ghomeshi, ²Saied Ahmad Mortazavi-Dorcheh and ²Roger Falconer

¹Water Science Engineering College, Shahid Chamran University, Ahvaz, Iran

²Hydro-Environmental Research Center, Cardiff School of Engineering, Cardiff University, Cardiff, UK

Abstract: In this study, attention is focused on the transverse waves generated in an idealized open channel where the flow passes through a cluster of rigid vertical cylinders. The experiments are conducted in a laboratory flume with 10 m length and 1.2 m width. In the experimental setup the rigid rods were installed in the bed of flume with fourteen different situations. Analysing the data shows when the dimensionless distance between the rods in a single row (T/D), decreases, the amplitude of the generated wave will increase. This characteristic is reliable when T/D is not less than 5. When this parameter is less than the critical value of 5, the amplitude of the transverse wave decreases with decrease of T/D . Also, analysing the dimensionless distance between the rows (P/D), shows when P/D decrease; the amplitude of the generated wave will increase. In the range of the experimental work of this study ($P/D \geq 4.3$), analysing (P/D) does not show any critical point. Finally, the dimensional analysis was employed to find a relationship between the dimensionless amplitude and the other significant parameters and two equations are proposed to estimate the wave amplitude.

Key words: Vortex shedding, wave formation, wave amplitude, vegetation, transverse waves, flow vibration, water displacement

INTRODUCTION

When a solid body obstruct a flowing fluid, vortex shedding may occur and therefore, it can cause vibration in the flow. There are many situations, when the steady flow of water passes around a cluster of rigid vertical cylinders. These rigid cylinders may represent piers of bridge, jetty, or vegetation on floodplain of rivers. This situation of rigid cylinders may produce transverse waves created by vortex shedding. The frequency of the vortex shedding (f_s) can be calculated by following equation (Strouhal, 1878).

$$f_s = \frac{SU}{D} \quad (1)$$

Where:

S = Strouhal number

U = Free stream velocity (the average velocity of fluid before crossing the tubes bundle)

D = Cylinder diameter

For a cluster of tubes arranged in staggered rows, or in in-line rows and when the fluid is a gas the Strouhal number is based on the tube diameter and the distance between the cylinders in a row (T) and the distance between the cylinders rows (P). Fitz-hugh presented two

maps for these cases and these results are shown in Fig. 1 and 2 (Blevins, 1977). Zukauskas *et al.* (1988) presented the following two equations for Strouhal number estimation for air or water:

For staggered bundles

$$S_t = 0.2 + \exp \left[-0.44 \left(\frac{T}{D} \right)^{1.8} \right] \quad \text{when } \frac{T}{D} \geq 1.15 \quad (2)$$

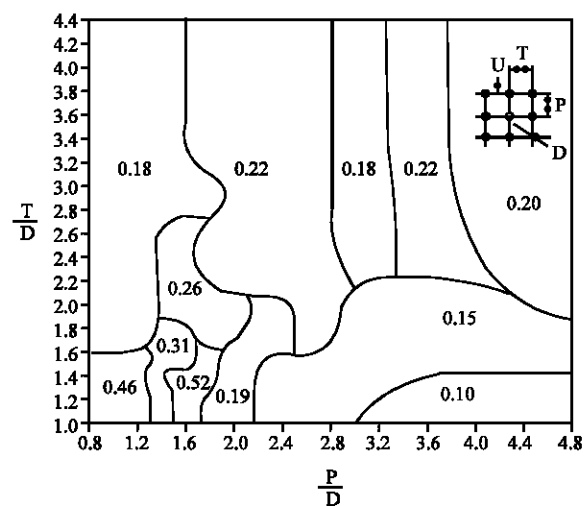


Fig. 1: Fitz-hugh's Strouhal number map for in-line cylinders

For in-line bundles

$$S_t = 0.2 + \exp \left[-1.2 \left(\frac{T}{D} \right)^{1.8} \right] \quad \text{when } \frac{T}{D} \geq 1.15 \quad (3)$$

Many researches can be found in the literature about vortex shedding and related phenomena. Fundamental studies on vortex shedding over the last two decades are summarized by Williamson and Govardhan (2004). But most of researches are focused on acoustic resonant as described by Fitz-Hugh (1973), Blevins (1985), Zukauskas and Katinas (1979), Weaver *et al.* (1986), Ziada *et al.* (1989), Hamakawa *et al.* (2001) and Naudascher and Rockwell (1980).

There are few researches on the vortex shedding and the related phenomena, when water is the medium. For the first time Crasse (1939) reported the wave oscillations in open channels. Clay and Tison (1968) observed oscillations in a canal with only one bridge pier in the cross section. Assi *et al.* (2006) investigated flow-induced vibration interference between two circular cylinders. To and Lam (2007) studied vibration of a flexibly mounted circular cylinder in the proximity of a large cylinder downstream with the aid of flow visualizations. Zima and Ackermann (2002) two flumes with 150 and 450 m widths and two sizes of rods with the diameters equal to 12.7 and 25.4 mm. They proposed a formula for simulating the maximum values of dimensionless displacement (A/h) that may occur in a canal. Their formula is presented as follow:

$$\frac{A}{h} = \frac{N D}{S_t^2 P} \left(\frac{D}{L} \right)^2 k \quad (4)$$

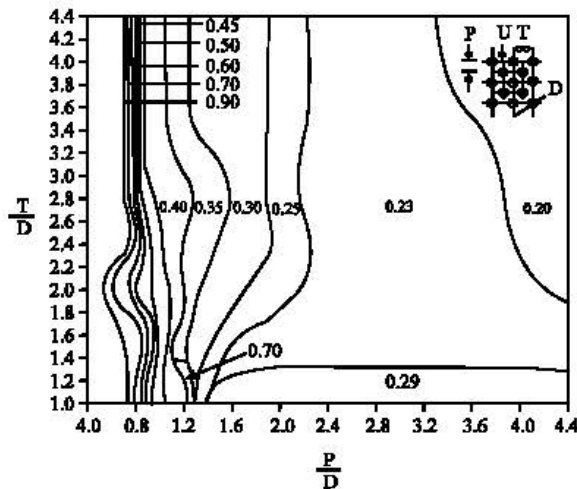


Fig. 2: Fitz-hugh's Strouhal number map for staggered cylinders

Where:

k = Constant equal to 2.255

A = Maximum amplitude

h = Depth of flow

S_t = Strouhal number

D = Rods diameter

P = Distance between rows of the rods

L = Width of channel

N = No. of rods in a single row

MATERIALS AND METHODS

A wide flume with 1200 mm in width and 10 m in length and 360 mm in height was used in this study (Fig. 3). A wooden pallet was employed to separate the flume into two different wide (800 and 400 mm) canals (Fig. 4). The slope of the flume was constant and equal to 0.001. The rods were wooden cylinders with 24 mm in diameter and the height was equal to 300 mm. The rigid rods were screwed in the bed of flume in different distances (different T and P) (Fig. 4) with different arrangement (staggered and in-line).

Wave displacements were measured in three stations located 2.70, 5 and 7.40 m from outlet (Fig. 4) and mean depth of flow was measured in centre part of the flume. Both parameters were determined visually with a ruler graduated in millimetres and placed in the sides and centre of the flume.

With the aid of the wooden pallet the flume was separated into two different wide canals and therefore three sizes of flumes were created. The widths of the canals were 1200, 800 and 400 mm. The flow discharge to the main flume was constant and equal to 18.33 l/s and in the case of flume separation this value divided to two



Fig. 3: The flume under experiment

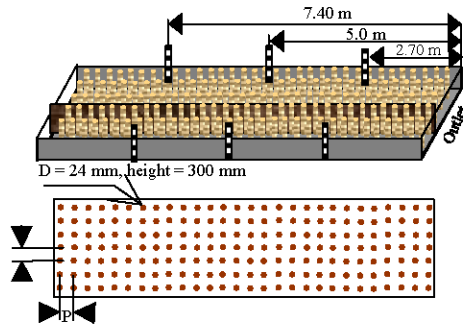


Fig. 4: Schematic view of the experimental set-up

portions equal to 12.22 and 6.11 l/s. There was a controllable wire gate in the outlet of the flume and this weir was used to make different conditions in the experimental works. The gate height was changing between 280 and 30 mm with an interval of 5 mm.

Table 1: Summary of the parameters of the experimental works

T (mm)	P (mm)	Arrangement	l (mm)	Q (L/s)	N (No. of rods)	Exp. No.
60	105	In-line	1200	18.33	20.0	1
			800	12.22	13.0	2
			400	6.11	7.0	3
	210	In-line	1200	18.33	20.0	4
			800	12.22	13.0	5
			400	6.11	7.0	6
	315	In-line	1200	18.33	20.0	7
			800	12.22	13.0	8
			400	6.11	7.0	9
	420	In-line	1200	18.33	20.0	10
			800	12.22	13.0	11
			400	6.11	7.0	12
120	105	In-line	1200	18.33	10.0	13
			800	12.22	6.0	14
			400	6.11	3.0	15
	420	In-line	1200	18.33	10.0	16
			800	12.22	6.0	17
			400	6.11	3.0	18
	240	In-line	1200	18.33	5.0	19
			800	12.22	3.0	20
			400	6.11	2.0	21
	105	In-line	1200	18.33	5.0	22
			800	12.22	3.0	23
			400	6.11	1.5	24
240	105	Staggered	1200	18.33	10.0	25
			800	12.22	6.5	26
			400	6.11	3.0	27
	315	Staggered	1200	18.33	10.0	28
			800	12.22	6.5	29
			400	6.11	3.0	30
	210	Staggered	1200	18.33	5.0	31
			800	12.22	3.0	32
			400	6.11	1.5	33
	105	Staggered	1200	18.33	3.5	34
			800	12.22	2.0	35
			400	6.11	1.0	36
360	315	Staggered	1200	18.33	3.5	37
			800	12.22	2.0	38

The details of the experimental works are shown in Table 1. The water temperature was constant and equal to 17°C. In each experiment the value of five parameters were saved; mode of waves, average depth of flow and water displacement in the side of the flume in three determined stations.

RESULTS AND DISCUSSION

In the experiments, the height of the outlet weir changed from highest to lowest level. Up to four modes of vibration are appeared in the experiments. These modes are shown in Fig. 5 and introduced by $n = 1, 2, 3$ and 4. The amplitude of the waves (A) is defined as the maximum displacement of water levels in the side of the channel.

As mentioned before, the amplitude is measured in three stations of the flume. In each experiment the differences between the measured amplitude in these three stations were small. The average of these three measured values of amplitude is used for the data analyses.

In Fig. 6, the relationship between relative wave amplitude, the transverse waves modes and depth of flow are shown for six sets of experiments with different channel widths. As it can be seen, in most experimental situations the modes of vibration are appeared in sequence and the maximum relative amplitudes (A/h) increase when the modes are increased. Only in experiments number 1 and 2 (Table 1), when T and P both are at their minimum values, the sequence was slightly different. In these experiments after first mode of waves ($n = 1$) the third mode of waves ($n = 3$) are appeared (Fig. 7).

The maximum relative amplitudes in each mode of the experiments are separated and used for further analyses. These data are shown in Table 2.

The Reynolds numbers of the experiments were in the range from 1000 to 4400 and the maximum wave amplitudes observed in the experiments was 40% of the mean flow depth.

From the collected data, points with highest amplitude in each mode are selected and used for further analysis. The number of data extracted in this way was 103 and can be shown in Table 2.

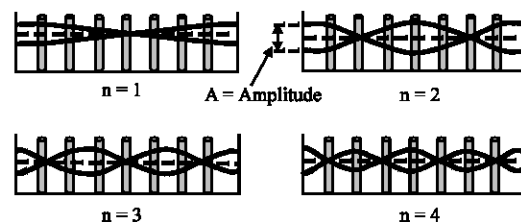


Fig. 5: Modes of waves appear in the laboratory flume

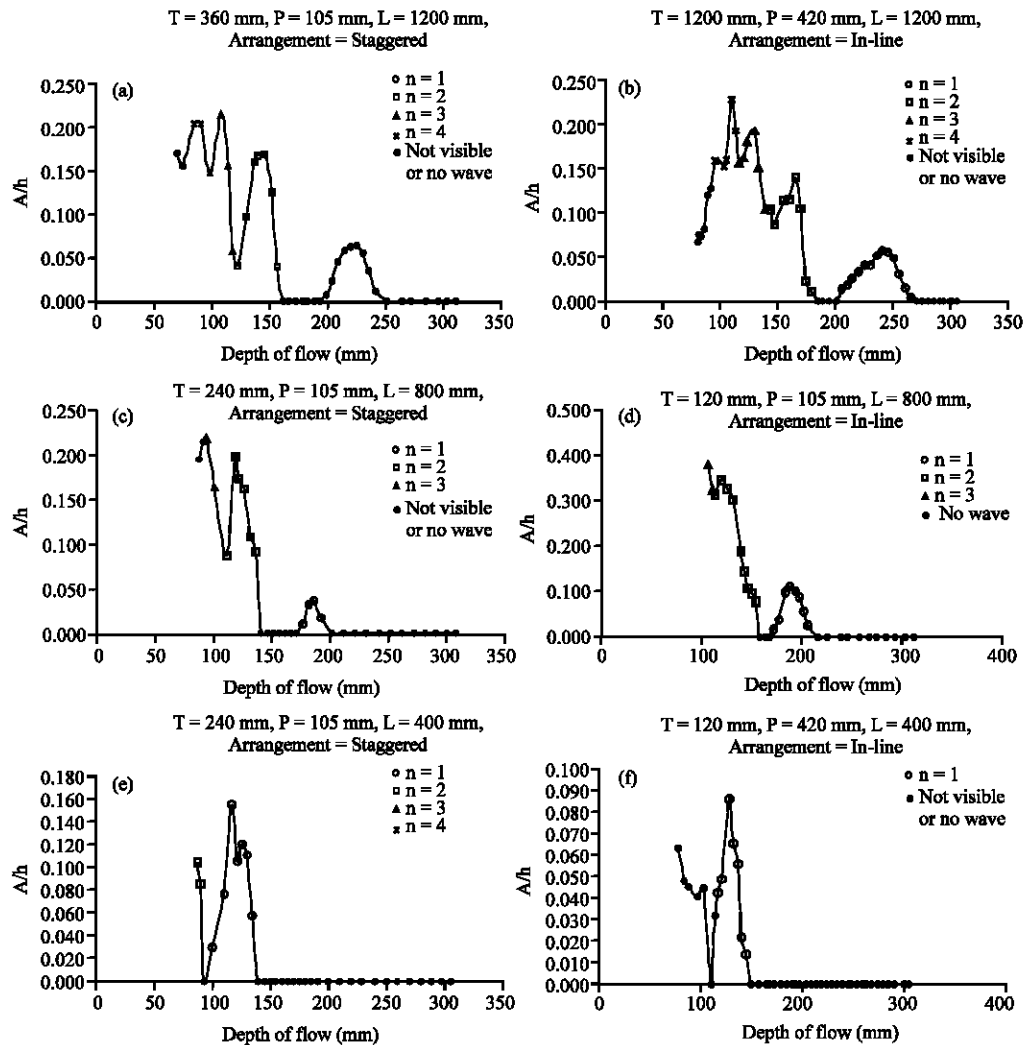


Fig. 6: Relationship between water amplitude, the transverse waves modes and the depth of flow for six sets of experiments

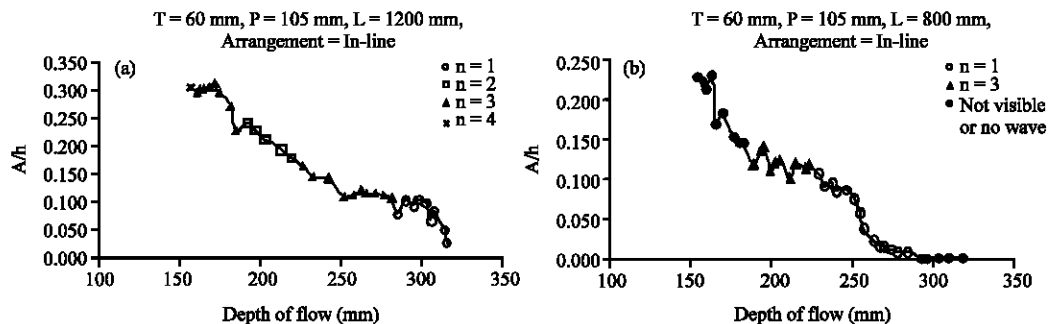


Fig. 7: Relationship between water amplitude, the transverse wave modes and the depth of flow for the experiments number 1 and 2

The Zima and Ackermann equation (Eq. 4) is used to estimate relative amplitude. In Fig. 8 the estimated A/h from Eq. 1 and the measured data are shown and can be

comparing together. As it can be seen the estimated values do not show good agreement with the line of perfect agreement.

Table 2: Measured parameters related to the points of maximum relative amplitude of the experimental works

Exp. No.	h (mm)	n	A (mm)	R_a	f_a
22	204	1	1.3	1661	0.62
16	240	1	13.7	1412	0.53
19	199	1	15.7	1703	0.64
1	299	1	30.3	1133	0.43
13	242	1	26.0	1400	0.53
10	304	1	19.0	1115	0.42
7	291	1	18.0	1164	0.44
4	309	1	23.7	1096	0.42
22	140	2	12.0	2420	0.91
16	165	2	23.0	2053	0.77
19	143	2	27.3	2369	0.89
1	193	2	46.0	1756	0.66
13	166	2	37.0	2041	0.77
10	234	2	31.0	1448	0.55
7	204	2	29.7	1661	0.63
4	259	2	34.7	1308	0.50
22	109	3	8.7	3108	1.17
16	130	3	25.0	2606	0.98
19	120	3	22.7	2823	1.06
1	264	3	30.7	1283	0.48
1	173	3	52.7	1958	0.74
13	130	3	40.7	2606	0.98
10	190	3	31.3	1783	0.68
7	175	3	36.7	1936	0.73
4	205	3	41.0	1653	0.63
22	93	4	9.0	3643	1.37
16	110	4	25.0	3080	1.16
19	100	4	24.0	3388	1.27
1	158	4	47.7	2144	0.81
13	119	4	47.3	2847	1.07
10	155	4	31.7	2186	0.83
7	139	4	32.7	2438	0.92
4	175	4	34.0	1936	0.73
2	238	1	22.7	1424	0.53
14	187	1	21.3	1812	0.68
11	230	1	2.0	1473	0.56
8	230	1	5.7	1473	0.56
5	251	1	10.0	1350	0.51
17	128	2	10.0	2647	0.99
14	120	2	42.7	2823	1.06
11	175	2	28.3	1936	0.73
8	144	2	29.0	2353	0.89
5	203	2	24.7	1669	0.63
20	110	2	17.3	3080	1.16
17	104	3	9.0	3258	1.22
2	196	3	27.3	1729	0.65
14	106	3	41.3	3196	1.20
11	150	3	15.7	2259	0.86
8	127	3	23.3	2668	1.01
5	160	3	23.3	2118	0.80
17	128	1	11.0	2647	0.99
20	100	1	13.7	3388	1.27
3	158	1	22.0	2144	0.81
15	114	1	31.0	2972	1.12
12	170	1	19.0	1993	0.76
9	148	1	16.0	2289	0.87
6	173	1	17.3	1958	0.74
3	196	2	10.7	1729	0.65
12	133	2	6.3	2547	0.97
9	123	2	9.0	2755	1.04
6	155	2	19.3	2186	0.83
12	119	3	3.0	2847	1.08
32	244	1	14.0	1389	0.52

Table 2: Continued

Exp. No.	h (mm)	n	A (mm)	R_a	f_a
23	245	1	21.3	1383	0.52
26	249	1	11.3	1361	0.51
29	277	1	19.3	1223	0.46
38	219	1	1.7	1442	0.54
35	225	1	15.0	1547	0.58
32	164	2	24.3	1506	0.57
26	170	2	22.0	2079	0.78
29	158	2	28.7	1993	0.75
38	140	2	5.7	2144	0.81
35	150	2	26.0	2420	0.91
32	125	3	28.3	2259	0.85
23	134	3	32.0	2711	1.02
26	129	3	21.7	2528	0.95
29	120	3	31.7	2626	0.99
38	118	3	8.3	2823	1.06
35	114	3	25.3	2871	1.08
32	81	4	10.0	2972	1.12
23	115	4	41.7	4183	1.57
26	95	4	18.0	2946	1.11
29	95	4	28.3	3566	1.34
35	92	4	19.3	3566	1.34
24	193	1	18.3	3683	1.38
27	210	1	3.7	1756	0.66
23	163	2	34.0	2066	0.78
30	185	1	6.7	1613	0.61
36	111	1	20.3	1831	0.69
24	130	2	30.0	3052	1.15
27	133	2	22.3	2606	0.98
30	117	2	23.3	2547	0.96
24	114	3	31.7	2896	1.09
27	108	3	11.0	2972	1.12
30	93	3	20.3	3137	1.18
25	130	1	18.3	3643	1.37
28	133	1	10.7	2606	0.98
31	117	1	18.0	2547	0.96
37	111	2	17.0	2896	1.09
25	114	2	21.7	3052	1.15
28	90	2	7.7	2972	1.12
31	87	2	9.0	3765	1.42
37	77	3	8.0	3894	1.46

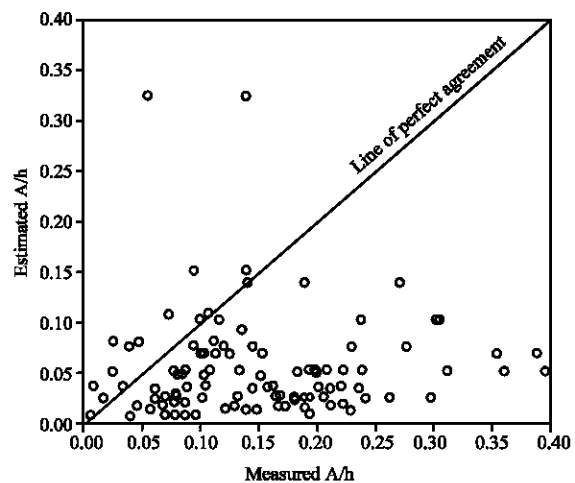


Fig. 8: Comparison between estimated A/h from the Zima and Ackermann equation (2002) and the measured data

To find a relationship between A/h and the significant variables, dimensional analysis was employed. A functional relationship that includes the parameters and may govern the wave amplitude is shown in Eq. 5.

$$\Phi(A, h, T, D, P, L, v, U, N, n, S_i) = 0 \quad (5)$$

In this equation, U is the velocity of water and calculated by dividing the discharge to the area of the cross section of the flume (the velocity of the flow without the space occupied by the rods diameters) and v is kinematics viscosity. The parameters can be expressed in terms of non-dimensional variable (Eq. 6).

$$\frac{A}{h} = \Phi\left(\frac{T}{D}, \frac{L}{D}, \frac{P}{D}, S_i, R_e, N, n\right) \quad (6)$$

Where:

A/h = Relative amplitude

T/D = Ratio distance between rods in a single row over rods diameter

L/D = Ratio width of channel over rods diameter

P/D = Ratio distance between rods rows over cylinder diameter

R_e = The Reynolds number (UD/v) and calculated with respect to the rods diameter and kinematics viscosity of water

To make sure the existing relationship between each of non-dimensional variables and A/h , the data of one variable are separated and analysed individually when the other variables are constant or nearly constant.

In Fig. 9a, the relationship between P/D and maximum of A/h and the power regression curve are shown. It is clear that when P/D is increased, A/h decreases. In Fig. 9b this relationship is shown for different widths of the flume and same result can be found.

In Fig. 10, the relationship between T/D and A/h is shown for different widths of the flume. In Fig. 10a, the variations when the values of T/D are equal or greater than 5 are shown for different widths of the flume. As it can be seen when T/D is decreased, A/h increases. In Fig. 10b, this relationship is shown for a situation when the values of T/D reach to 2.5. It can be seen when the value of T/D decrease from 5, the relationship would change in reverse order. Its mean, in this part of the graph when T/D decreases, A/h decreases as well. Therefore, it seems the point of $T/D = 5$ is a critical point and the direction of the relationship changes in this point. The same analysis applied to find the possible relation between R_e and A/h , but authors could not find any relationship. Also, comparison between staggered and in-line arrangements has been made and no significant differences were distinguished.

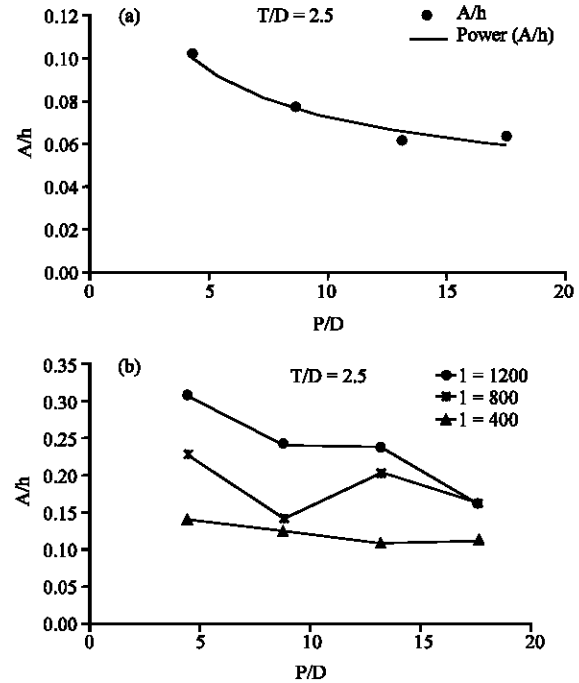


Fig. 9: Relationship between P/D and maximum A/h

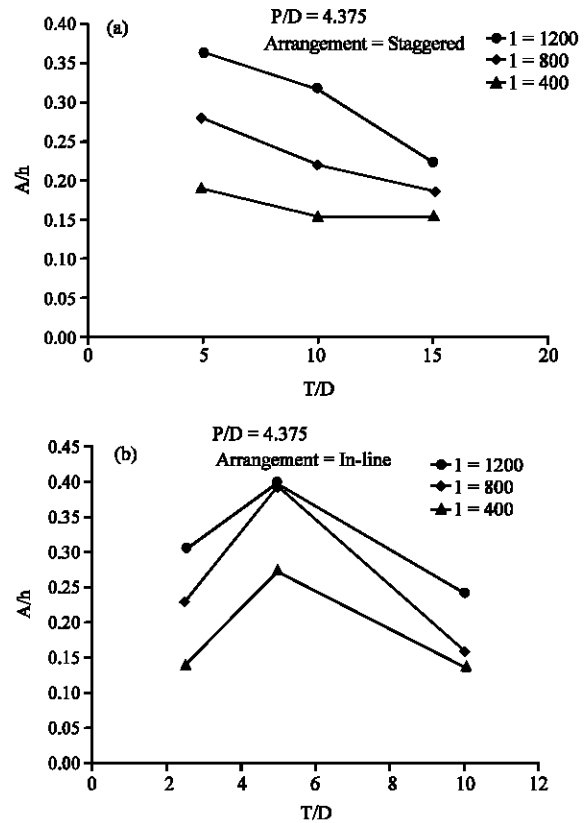


Fig. 10: Relationship between T/D and A/h

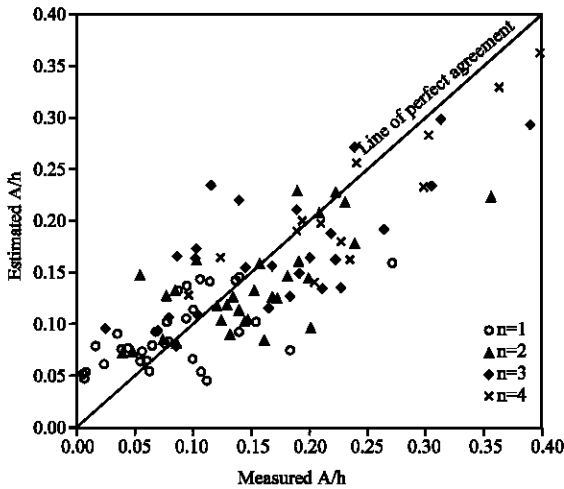


Fig. 11: Comparison between the relative amplitude (A/h) calculated from the proposed equation and the measured values

Therefore, for final analysis, the data were separated to two groups. The first group was contained the data with T/D equal or greater than 5 and the second group was contained the data with T/D equal or less than to 5. The effects of parameter T/D exists in parameters L/D and N together, so there is no need to include T/D in the data analysis. The Fitz-hugh maps are used to calculate the Strouhal numbers when T/D and P/D are located in the ranges of the maps and for the other data the Zukauskas *et al.* (1988) equations are employed. Statistical analyses are used to find the best relationship between the parameters. The result of the analysis can be fitted to and extracted using the following formulas.

For $T/D \geq 5$

$$\frac{A}{h} = 1.41 N^{\frac{2}{3}} \left(\frac{D}{L} \right) \left(\frac{D}{P} \right)^{0.5} S_t^{-\frac{1}{2}} n^{\frac{2}{3}} \quad (7)$$

For $T/D < 5$

$$\frac{A}{h} = 0.078 N^{\frac{2}{3}} \left(\frac{D}{L} \right)^{-\frac{1}{3}} \left(\frac{D}{P} \right)^{0.5} S_t^{-\frac{1}{2}} n^{\frac{2}{3}} \quad (8)$$

In Fig. 11, the relative amplitude (A/h) estimated by Eq. 7 and 8 are compared with the measured values. Also in this figure the line of perfect agreement is shown. It can be seen that the relative amplitude (A/h) calculated with the new equations are significant.

In Fig. 12a, comparison are shown between the estimated A/h with equations 4 and 8 and the measured data for specific part of the data ($T/D < 5$ and $n = 1$). This

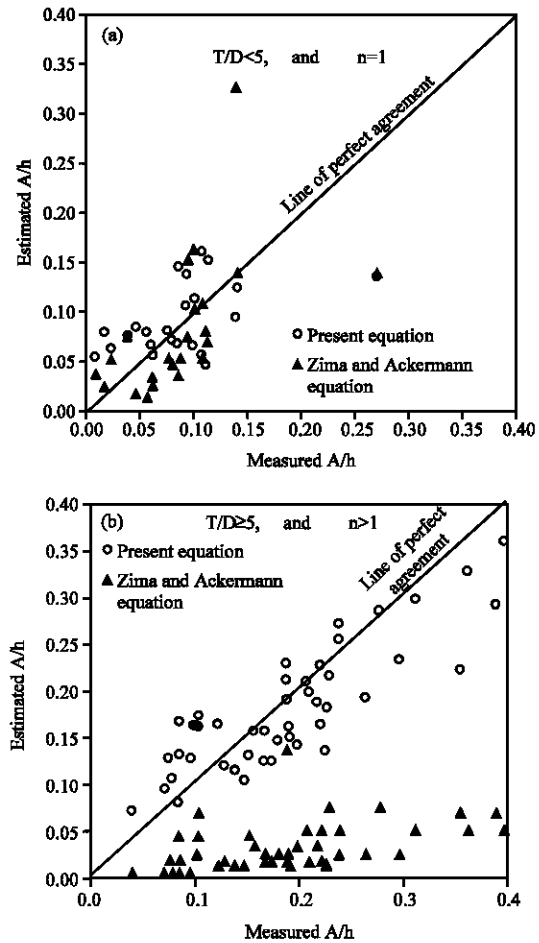


Fig. 12: Comparison between the relative amplitude (A/h) calculated from the proposed equations 1 and 6 and the measured values

part of the data is same as the situation of the Zima and Ackermann (2002) study. As it can be seen, however the new equation shows good agreement with the measured values but Eq. 4 also shows nearly reasonable agreement. For the other parts of the data that a sample is shown in Fig. 12b, Eq. 4 does not show good agreement. Therefore, It could be concluded that Eq. 4 can be used for specific situation when $T/D < 5$ and $n = 1$.

CONCLUSION

In many conditions, the steady flow of water passes around a cluster of rigid vertical cylinders. This situation of rigid cylinders may produce transverse waves created by vortex shedding. In this study the relative amplitude of vortex shedding when the flowing fluid is water was experimentally investigated.

Thirty four experimental situations were conducted in a rectangular laboratory flume. The Reynolds number in experiments was between 1000 and 4400 and the rods diameter was constant and equal to 24 mm and the rods arrangements were in-line and staggered. As a result, the following conclusions were obtained.

- The maximum wave amplitudes observed in the experiments was 40% of the mean flow depth.
- When the value of T/D is less than 5, the relationship between relative amplitude and T/D is in reverse order.
- A new formulation, as given in Eq. 7 and 8 were obtained from dimensional analyses, to estimate the amplitude of the transverse wave generated by vortex shedding.

REFERENCES

- Assi, G.R.S., J.R. Meneghini, J.A.P. Aranha, P.W. Bearman and E. Casaprima, 2006. Experimental investigation of flow-induced vibration interference between two circular cylinders. *J. Fluids Struct.*, 22 (6-7): 819-827.
- Blevins, R.D., 1977. *Flow-Induced Vibrations*. Van Nostrand Reinhold, New York.
- Blevins, R.D., 1985. The effect of sound on vortex shedding from cylinders. *J. Fluid Mech.*, 161: 217-237.
- Clay, D. and G. Tison, 1968. Vortex-induced oscillations at low-head weirs. *J. Hydraulic Div. Am. Soc. Civil Eng.*, 94 (4): 1160.
- Crasse, 1939. About Oscillation phenomenon on water surface Part 1: Flow around obstacle form piles of bridge. *Coptes Rendus de Séances de l'Academie de Sciences*, 209 (In French).
- Fitz-Hugh, J.S., 1973. Flow induced vibration in heat exchangers. In: *Proceeding UKAEA/NPL International Symposium on Vibration Problems in Industry*, Keswick, England, 427: 1-17.
- Hamakawa, H., T. Fukano, E. Nishida and M. Aragaki, 2001. Vortex shedding from a circular cylinder with fin. *Proceeding of 7th AIAA/CEAS Aeroacoustics Conference*, AIAA-2215.
- Naudascher, E. and D. Rockwell, 1980. *Practical Experiences with Flow Induced Vibrations*. Springer, New York, pp: 849.
- Strouhal, V., 1878. Über eine besondere art der tonerregung. *Ann. Physik (Leipzig)*, 5 (10): 216-251.
- To, A.P. and K.M. Lam, 2007. Flow-Induced vibration of flexible mounted circular cylinder in the proximity of a larger cylinder downstream. *J. Fluids Struct.*, 23 (3): 523-528.
- Weaver, D.S., J.A. Fitzpatrick and M. Elakashlan, 1986. Strouhal numbers for heat exchanger tube arrays in cross flow. *Flow Induced Vibration. PVP.*, 104: 193-200.
- Williamson, C.H.K. and R. Govardhan, 2004. Vortex-induced vibrations. *Ann. Rev. Fluid Mech.*, 36: 413-455.
- Ziada, S., A. Oengören and E.T. Buhlmann, 1989. On acoustical resonance in tube arrays Part I: Experiments. *J. Fluids Struct.*, 3 (03): 293-314.
- Zima, L. and N.L. Ackermann, 2002. Wave generation in open channels by vortex shedding from channel obstructions. *J. Hydraulic Eng.*, 128 (6): 596-603.
- Zukauskas, A. and V. Katinas, 1980. *Flow-induced Vibration in Heat Exchanger Tube Banks, Practical Experiences with Flow Induced Vibrations*, Naudascher, E. and Rockwell (Eds.). Springer, New York, pp: 188-196.
- Zukauskas, A., R. Ulinskas and V. Katinas, 1988. *Fluid Dynamics and Flow-Induced Vibrations of Tube Banks*. Hemisphere, New York, pp: 290.