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EPQ Models under Permissible Payment Delay: An Algebraic Approach

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Abstract: The purpose of this research is to relax this assumption and establish the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time. Then, an algebraic approach is provided and an easy-to-use theorem is derived to efficiently determine the optimal cycle time. From the final numerical examples, result implies that the retailer will order less quantity to take the benefits of the permissible delay in payments more frequently when the larger the differences between the unit selling price per item and the unit purchasing price per item.

Key words: Inventory, optimization, noninstantaneous receipt, permissible delay in payments, algebraic method

INTRODUCTION

The traditional Economic Order Quantity (EOQ) model is one of the earliest and well-known theories in inventory control. Many researchers extend the basic EOQ model by relaxing various assumptions to develop the models to fit the real business situations. One of unrealistic assumptions is that the retailer's capital is unconstrained and the retailer must be paid for the items as soon as the items are received. In practice, the supplier frequently offers the retailer a fixed delay period, that is the trade credit period, in settling the accounts. Before the end of the permissible delay period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the permissible delay period. In real world, the supplier often makes use of this policy to promote his/her commodities. Goyal (1985) established a single-item inventory model under permissible delay in payments. Recently, Ouyang *et al*. (2004) assumed that the retailer would not consider paying payment until receiving all items to investigate the retailer's inventory problem with noninstantaneous receipt and permissible delay in payments. In this study, we want to relax this assumption to develop the retailer's inventory system. In addition, we try to use different method from Ouyang *et al*. (2004) for obtaining the optimal cycle time so that the annual total relevant cost is minimized. This research provides an algebraic approach to determine the optimal cycle time. In previous most published papers that have been derived using differential calculus to find the optimal solution and to prove optimality condition with second-order derivatives. In recent papers, Grubbström and Erdem (1999) and Cárdenas-Barrón (2001) showed that the formulae for the EOQ and EPQ with backlogging could be derived without differential calculus. Furthermore, this paper derives one theorem to efficiently determine the optimal cycle time. Finally, numerical examples are given to illustrate the results and obtained the managerial insights.

MODEL FORMULATION

Notation:

A	=	Cost of placing one order
c	=	Unit purchasing price per item
D	=	Demand rate per year
h	=	Unit stock holding cost per item per year excluding interest charges
I_e	=	Interest earned per $ per year
I_k	=	Interest charges per $ investment in inventory per year
M	=	The trade credit period in years
P	=	Replenishment rate per year, P>D
s	=	Unit selling price per item
T	=	The cycle time in years
ρ	=	$1 - \frac{D}{P} > 0$
TRC (T)	=	The total relevant cost per unit time when T > 0
T*	=	The optimal cycle time of TRC (T).

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Assumptions:

- Demand rate, D, is known and constant.
- Replenishment rate, P, is known and constant.
- Shortages are not allowed.
- Time horizon is infinite.
- $s \geq c$; $I_k \geq I_e$.
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When $T \geq M$, the account is settled at T = M, the retailer pays off all units sold and keeps his/her profits and starts paying for the higher interest charges on the items in stock. When $T \leq M$, the account is settled at T = M and we do not need to pay any interest charge.

Mathematical model:

The total annual relevant cost consists of the following elements.

(1) Annual ordering cost = A/T.
(2) Annual stock holding cost (excluding interest charges) = DThρ/2.
(3) For inventory interest charges per year, three cases are considered.

Case 1: $\underline{M < \frac{PM}{D} \leq T}$

$$\text{Annual interest payable} = cI_k[\frac{DT^2\rho}{2} - \frac{(P-D)M^2}{2}]/$$

$$T = cI_k\rho(\frac{DT^2}{2} - \frac{PM^2}{2})/T.$$

Case 2: $\underline{M \leq T \leq \frac{PM}{D}}$

$$\text{Annual interest payable} = cI_k[\frac{D(T-M)^2}{2}]/T$$

Case 3: $\underline{T \leq M}$

In this case, no interest charges are paid for the items.

(4) For interest earned per year, three cases are considered.

Case 1: $\underline{M < \frac{PM}{D} \leq T}$

$$\text{Annual interest earned} = sI_e(\frac{DM^2}{2})/T.$$

Case 2: $M \leq T \leq \frac{PM}{D}$

$$\text{Annual interest earned} = sI_e(\frac{DM^2}{2})/T.$$

Case 3: $\underline{T \leq M}$

$$\text{Annual interest earned} = sI_e[\frac{DT^2}{2} + DT(M-T)]/T.$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

TRC (T) = Ordering cost + stock-holding cost + interest payable - interest earned.

We show that the annual total relevant cost, TRC (T), is given by

$$TRC(T) = \begin{cases} TRC_1(T) & \text{if} \quad T \geq PM/D & (1a) \\ TRC_2(T) & \text{if} \quad M \leq T \leq PM/D & (1b) \\ TRC_3(T) & \text{if} \quad 0 < T \leq M & (1c) \end{cases}$$

where

$$TRC_1(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k\rho(\frac{DT^2}{2} - \frac{PM^2}{2})/T - sI_e(\frac{DM^2}{2})/T, \quad (2)$$

$$TRC_2(T) = \frac{A}{T} + \frac{DTh\rho}{2} + cI_k[\frac{D(T-M)^2}{2}]/T - sI_e(\frac{DM^2}{2})/T \quad (3)$$

and

$$TRC_3(T) = \frac{A}{T} + \frac{DTh\rho}{2} - sI_e[\frac{DT^2}{2} + DT(M-T)]/T \quad (4)$$

Since TRC_1 (PM/D) = TRC_2 (PM/D) and TRC_2 (M) = TRC_3 (M), TRC (T) is continuous and well-defined. All TRC_1 (T), TRC_2 (T), TRC_3 (T) and TRC (T) are defined on T > 0.
Then, we can rewrite

$$TRC_1(T) = \frac{D\rho(h+cI_k)}{2T}\left[T - \sqrt{\frac{2A + DM^2(cI_k - sI_e) - PM^2cI_k}{D\rho(h+cI_k)}}\right]^2$$

$$+\left\{\sqrt{D\rho(h+cI_k)[2A + DM^2(cI_k - sI_e) - PM^2cI_k]}\right\} \quad (5)$$

From Eq. (5) the minimum of TRC_1 (T) is obtained when the quadratic non-negative term, depending on *T*, is equal to zero. The optimum value T_1* is

$$T_1{}^* = \sqrt{\frac{2A + DM^2(cI_k - sI_e) - PM^2cI_k}{D\rho(h+cI_k)}} \text{ if}$$
$$2A + DM^2(cI_k - sI_e) - PM^2cI_k > 0. \quad (6)$$

Therefore

$$TRC_1\ (T_1\ *) = \left\{\sqrt{D\rho(h + cI_k)[2A + DM^2(cI_k - sI_e) - PM^2cI_k]}\right\} \quad (7)$$

Similarly, we can derive TRC_2 (T) without derivatives as follows:

$$TRC_2(T) = \frac{D(h\rho + cI_k)}{2T}\left[T - \sqrt{\frac{2A + DM^2(cI_k - sI_e)}{D(h\rho + cI_k)}}\right]^2$$

$$+\left\{\sqrt{D(h\rho + cI_k)[2A + DM^2(cI_k - sI_e)]} - cI_kDM\right\} \quad (8)$$

From Eq. (8) the minimum of TRC_2 (T) is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_2* is

$$T_2 * = \sqrt{\frac{2A + DM^2(cI_k - sI_e)}{D(h\rho + cI_k)}} \text{ if } 2A + DM^2(cI_k - sI_e) > 0. \quad (9)$$

Therefore

$$TRC_2\ (T_2 *) = \left\{\sqrt{D(h\rho + cI_k)[2A + DM^2(cI_k - sI_e)]} - cI_kDM\right\} \quad (10)$$

Likewise, we can derive TRC_3 (T) algebraically as follows.

$$TRC_3(T) = \frac{D(h\rho + sI_e)}{2T}\left[T - \sqrt{\frac{2A}{D(h\rho + sI_e)}}\right]^2$$

$$+\left\{\sqrt{2AD(h\rho + sI_e)} - sI_eDM\right\} \quad (11)$$

From Eq. (11) the minimum of TRC_3 (T) is obtained when the quadratic non-negative term, depending on T, is equal to zero. The optimum value T_3* is

$$T_3 * = \sqrt{\frac{2A}{D(h\rho + sI_e)}}. \quad (12)$$

Therefore

$$TRC_3\ (T_3 *) = \left\{\sqrt{2AD(h\rho + sI_e)} - sI_eDM\right\} \quad (13)$$

DECISION RULE OF THE OPTIMAL CYCLE TIME T*

From Eq. (6) the optimal value of T for the case of T ≥ PM/D is T_1* ≥ PM/D. We can substitute equation (6) into T_1* ≥ PM/D to obtain the optimal value of T

$$\text{if and only if} \geq A \geq \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2]. \quad (14)$$

Similarly, from Eq. (9) the optimal value of T for the case of M ≤ T ≤ PM/D is M ≤ T_2* ≤ PM/D. We can substitute Eq. (9) into M ≤ T_2* ≤ PM/D to obtain the optimal value of T

$$\text{if and only if } DM^2(h\rho + sI_e) \leq 2A \leq$$

$$\frac{M^2}{D}[P(P - D)h + cI_k(P^2 - D^2) + sI_eD^2]. \quad (15)$$

Finally, from Eq. (12) the optimal value of T for the case of T ≤ M is T_3* ≤ M. We can substitute Eq. (12) into T_3* ≤ M to obtain the optimal value of T

$$\text{if and only if } 2A \leq DM^2\ (h\rho + sI_e). \quad (16)$$

From above arguments, we can summarize following results.

Theorem 1:

(A) if $2A \geq \frac{M^2}{D}[P(P-D)h + cI_k(P^2 - D^2) + sI_eD^2]$, then $T^* = T_1\ *$.

(B) If $DM^2(h\rho + sI_e) \leq 2A \leq \frac{M^2}{D}[P(P - D)h + cI_k(P^2 - D^2) + sI_eD^2]$ then $T^* = T_2\ *$.

(C) If $2A \leq DM^2(h\rho + sI_e)$, then $T^* = T_3\ *$.

Furthermore, we let

$$\Delta_1 = -2A + \frac{M^2}{D}[P(P - D)h + cI_k(P^2 - D^2) + sI_eD^2]$$

and

$$\Delta_2 = -2A + DM^2(h\rho + sI_e)$$

We can easily obtain $\Delta_1 > \Delta_2$. Then, we can modify Theorem 1 to Theorem 2 as follows:

Theorem 2:

(A) If $\Delta_2 \geq 0$, then $T^* = T_3^*$.

(B) If $\Delta_1 \geq 0$ and $\Delta_2 < 0$, then $T^* = T_2^*$.

(C) If $\Delta_1 < 0$, then $T^* = T_1^*$.

Table 1: The optimal cycle time with various values of P and s

Let A = \$70/order, D = 2500 units/year, c = \$20/unit, h = \$2/unit/year, Ik = \$0.15/\$/year, Ie = \$0.1/\$/year and M = 0.1 year

	P = 3000 units/year			P = 4000 units/year			P = 5000 units/year		
s = \$/unit	Δ_1	Δ_2	T*	Δ_1	Δ_2	T*	Δ_1	Δ_2	T*
30	<0	<0	T_1^* = 0.154919	>0	<0	T_2^* = 0.122202	>0	<0	T_2^* = 0.118322
40	>0	<0	T_2^* = 0.117473	>0	<0	T_2^* = 0.110755	>0	<0	T_2^* = 0.107238
50	>0	<0	T_2^* = 0.103923	>0	>0	T_3^* = 0.098687	>0	>0	T_3^* = 0.096609

Theorem 2 is an effective procedure to find the optimal cycle time T* by easy judgments Δ_1 and Δ_2.

NUMERICAL EXAMPLES

To illustrate all results obtained in this study, let us apply the proposed method to efficiently solve the following numerical examples.

The following inferences can be made based on Table 1. When P is increasing, the optimal cycle time for the retailer are decreasing. So, the retailer will shorten the ordering time interval since the replenishment speed is faster. When s is increasing, the optimal cycle time for the retailer are decreasing. This result implies that the retailer will order less quantity to take the benefits of the permissible delay in payments more frequently.

CONCLUSIONS

This research is to relax the Ouyang *et al.*'s model (2004) using the algebraic method to determine the optimal inventory replenishment policy for the retailer under finite replenishment rate and permissible delay in payments. Then, we provide a very efficient solution procedure to determine the optimal cycle time T*. Theorem 2 helps the retailer accurately and quickly determining the optimal inventory policy under minimizing the annual total relevant cost. From the final numerical examples, we find a result that is interesting. This result implies that the retailer will order less quantity to take the benefits of the permissible delay in payments more frequently when the larger the differences between the unit selling price per item and the unit purchasing price per item.

A future study will further incorporate the proposed model into more realistic assumptions, such as probabilistic demand, deteriorating items and allowable shortages.

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