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The Effect of Change in Refractive Index on Wave Propagation Through (FeS_2) Thin Film

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Abstract: Wave propagation in inhomogenous feS_2 film was studied and the influence of the change in the refractive index introduced as a perturbation on EM wave propagating through the film. The solution of the scalar wave equation was obtained first using series expansion solution method of Green's function with appropriate boundary condition. The result was written as sources of two fields where the second term was considered to be correction term due to the perturbation or small change in the refractive index. The influence of large step in refractive index and surface impedance offered by the film medium to the propagating wave was discussed.

Key words: Refractive index propagation, perturbation, impedance, green's function, field

INTRODUCTION

Wave propagation in materials like waveguide, crystals such as thin film had been studied by many researchers. Since the original research of Feit and Fleek (1979a, b and 1980) significant move has been mode in the area of beam propagation method. Having assessed its applicability, this method has certainly been most widely used as modeling tools for integrated optics (Roey *et al*., 1981).

Electromagnetic wave incident normally or obliquely of the surface of a thin film in the (r) direction experiences attenuation as the field penetrate the film (Ugwu and Uduh, 2005). Further studies has been conducted recently on determination of the refractive index inside the thin film medium as wave propagates on it (Ugwu, 2005).

This study looked at the influence of refractive index on the wave propagating on the film and specially considered a case where the variations in refractive index are small with reference to scalar field in which a scalar wave equation can be derived for the Transverse Electric (TE) or Transversed Magnetic (TM) modes separately. The change the index introduced as a perturbation to the propagating wave in the film. The solution of the fields ϕ was obtained using series expansion. Solution of creen's function (Schiff, 1955) and the results written as sources of two fields with one due to the perturbation term. The result was assigned first order differential co-efficient $\partial\phi_1/\partial z$ and operators to decompose the fields ϕ (Roey *et al*., 1981). The second term in $\partial\phi_1/\partial z = A^+\phi_1 + B^+\phi_2$ was considered as a correction term due to the change on the refractive index.

Finally, the observation showed that the smoothly change in the refractive index indicated clearly the sensitivity to polarization being as a result of interface and hence the need to decompose the wave into TE and T.M, Green's function was used in conjunction with the appropriate boundary conditions to analyse the impedance imposed by the thin film medium on the propagating wave.

WAVE EQUATION AND PERTURBATION IN REFRACTIVE INDEX

We consider the propagation of a high frequency beam through an inhomogeneous FeS_2 medium. The beam propagation method was considered and the cases in which the variations in refractive index are small or in which a scalar wave equation can be derived for the TE and TM modes separately looked at.
We start from the wave equation:

$$\nabla^2\Phi + K^2n^2(r)\Phi = 0 \quad (1)$$

Where ϕ represents the scalar field, n (r) the refractive index and k, the wave number in vacuum, in this equation $n^2(r) = n^2(r) + \Delta n^2(r)$ where $n_0(r)$ represents the unperturbed part and $\Delta n^2(r)$ is the perturbed part. This equation 1 can be written as

$$\nabla^2\Phi + K^2n^2(r)\Phi = -k^2\Delta n^2(r) \quad (2)$$

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Where the right hand of Eq. (2) is considered to be a source function. The refractive index $n_o^2(r)$ is chosen in such a way that the wave equation

$$\nabla^2\Phi + K^2 n^2(r)\Phi = 0 \tag{3}$$

Together with the radiation conditions at infinity can be solved. If the solution ϕ for $Z = Z_O$ is shown, the field ϕ and its derivative $\partial\phi_1/\partial z$ can be obtained for all values of Z by means of an operator A^+

$$\frac{\partial\Phi_1}{\partial z} = A^+\Phi(x,y,z) \tag{4}$$

Where the operator A^+ acts with respect to transverse co-ordinates (x, y) only. The function G(r, r'), for our problem can be determined by direct construction of Green's function, i.e., by joining the solutions of the homogeneous problem in equation 3 at r = a after which we now write the random integral Eq. as

$$G(r,r') = G_o(r,r') + \int G_o(r_1,r')\Delta n^2(r)G_o(r_1,r')\partial r_1 \tag{5}$$

Where G_O is the free space Green's function which is the determinant function while the change in refractive index $\Delta n^2(r)$ and the surface Green's function $G(r,r^1)$ are the random function (Ishimaru *et al.*, 2000)

SERIES SOLUTION OF GREEN'S FUNCTION

The Green's can be obtained using different technique, but here we use series expansion technique The equation can be written

as
$$\left(\nabla^2 + \gamma^2\right)\Phi = f(r') \tag{6}$$

The Green's function becomes where. $f(r^1) = K^2\Delta n^2_{(r)}\phi$

$$\left(\nabla^2 + \gamma^2\right)G(r,r') = \partial(r,r') \tag{7}$$

Where $\gamma^2 = k^2 n_o(r)$

With boundary condition that

$$G(0,r) = G(z,r) = 0 \tag{8}$$

As G (r, r) vanishes at the end of the interval (0, z), we can expand the expression in a suitably chosen orthogonal functions such as fourier sine series.

$$G(r,r') = \sum_{m=1}^{\infty}\gamma_m(r')\sin\left(\frac{m\pi r}{z}\right) \tag{9}$$

Where the expansion co-efficient Υ_m depends on the parameter r'. Differentiating Eq. 9

$$\nabla G(r,r') = \sum_{m=1}^{\infty}\left(\frac{m\pi}{z}\right)\gamma_m(r')\cos\left(\frac{m\pi r}{z}\right) \tag{10}$$

$$\nabla^2 G(r,r') = \sum_{m=1}^{\infty}\left(-\frac{m^2\pi^2}{z^2}\right)\gamma_m(r')\sin\left(\frac{m\pi r}{z}\right) \tag{11}$$

$$\delta(r-r') = \sum_{m=1}^{\infty}\Delta m(r')\sin\left(\frac{m\pi r}{z}\right) \tag{12}$$

soloving for $\partial(r, r)$ and Δm (r), γ_m(r) can be obtained as

$$\gamma_m(r') = -\frac{2z}{\pi^2}\frac{1}{m^2}\sin\left(\frac{m\pi r'}{z}\right) \tag{13}$$

subitituting Eq. 13 into 9 for the value of γ_m (r) we obtain

$$G(r,r') = -\frac{2z}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi r'}{z}\right)\sin\left(\frac{m\pi r}{z}\right) \tag{14}$$

The solution of the inhomogeneous equation becomes (Butkov, 1968)

$$\phi(r) = \int_0^a G(r,r')f(r')dr' \tag{15}$$

Which leads to

$$\phi(r) = -\frac{2z}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi r}{z}\right)\int_0^a f(r')\sin\left(\frac{m\pi r'}{z}\right)dr' \tag{16}$$

By fourier series as we assume Δn to be periodic

$$\int_0^a f(r')\sin\left(\frac{m\pi r'}{z}\right)dr' = \rho_m \tag{17}$$

$$\phi(r) = -\frac{2z}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^2}\rho_m\sin\left(\frac{m\pi r}{z}\right) \tag{18}$$

Where

$$\rho_m = \int_0^a k^2\Delta n^2(r)\sin\left(\frac{m\pi r'}{z}\right)dr'$$

ANALYTICAL SOLUTION OF THE PROPAGATING WAVE WITH STEP-INDEX AND THE DISCUSSION

If we consider the beam propagating towards increasing Z with no assumed paraxiality, we split the field ϕ into a part ϕ_1 generated by the sources in the region where r<r and a part ϕ_2 assigned to the sources where r>r i.e., perturbated term, we can write

$$\phi' = \phi_1' + \phi_2' \qquad (19)$$

Again if the propagation in the unperturbed medium is assigned an operator A^+ and another operator B^+ defined on ϕ_2 with respect to the transverse co-ordinate (x,y) only, we can write Eq. 4 as

$$\frac{\partial \phi'}{\partial z} = A^+\phi_1' + B^+\phi_2' \qquad (20)$$

if we neglect the influence of the reflected field on ϕ_1 we could use $\phi_2$instead of $\phi_1$in the equation 20, then

$$\frac{\partial \phi'}{\partial z} = A^+\phi_1' + B^+\phi_1' \qquad (21)$$

Equation 21 is an important approximation, though it restricts the use of the beam propagation method in analyzing the structures of matters for which the influence of the reflected waves would have on the forward-propagating wave. However this excludes the use of the method in cases where the refractive index changes abruptly as a function of r or in which reflections add up coherently. According to Eq. 21, the propagation of the field $\phi_1$is given by the term describing the propagation in an unperturbed medium and the correction term representing the influence of $\Delta n^2(r)$. Equation 21 is also first order differential equation which made it easy for one to determine the field ϕ^I_1 for $z>z_0$ starting from the input beam on the plane $z = z_0$ as the transversal variations of $\Delta n^2(r)$. ϕ^I_1 are considered to be very slow, than the second term in Eq. 21 becomes

$$B^+\phi_1' = -\frac{jk}{2n}\Delta n^2 \phi_1' \qquad (22)$$

Eq. 21 becomes now

$$\frac{\partial \phi'}{\partial z} = A^+\phi_1' + B^+\phi_1' \qquad (23)$$

and by the boundary condition imposed this equation, $\partial\varepsilon$, if the field ϕ_1 is known at $z = z_0$ and if Δn^2 is zero, Eq. 21 reduces to

$$\frac{\partial \varepsilon}{\partial z} = A^+\varepsilon$$

where ε (x, y, z_O) = ϕ_1 (x, y, z_{O+}) is given and ε representing the field propagating in a medium with a refractive index $n_0(r)$.

As the beam is propagated through the thin film showing a large step in refractive index of an imperfectly homogeneous thin film (Fig. 1) this condition presents the enabling provisions for the use of a constant refractive index n_0 of the thin film. One can then choose arbitrarily two different refractive indices n_1 and n_2 at the two sides of the step so that:

$$n_0(x) = n_1;\ x < 0;$$

$$n_0(x) = n_2;\ x < 0;$$

With $$\frac{n(x) = n_0(x)}{n_0(x)} \gg 1$$

for all x with smoothly changing refractive index at both sides of the step (Fig. 1) we assume that the sensitivity to polarization is due mainly to interface and hence in propagating a field ϕ through such a medium, one has to decompose the field into TE and TM polarized field for which we neglect the coupling between the E and H fields due to the small index variation ($n-n_0$). when the interface conditions that ϕ_m and were $\frac{\partial \phi_m}{\partial x}$ continuous at x = 0 were satisfied, the TE field could be propagated by the virtue of these decomposition similarly, TM field were also propagated by considering that ϕ_m and Were $\frac{\partial \phi_m}{\partial x}$ continuous at x = 0. When we use a set or discrete mode,

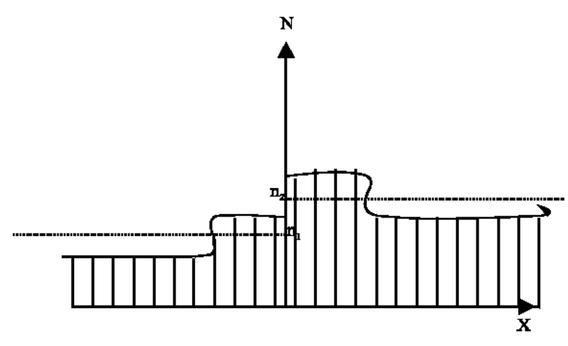

Fig. 1: Refractive index profile showing a step

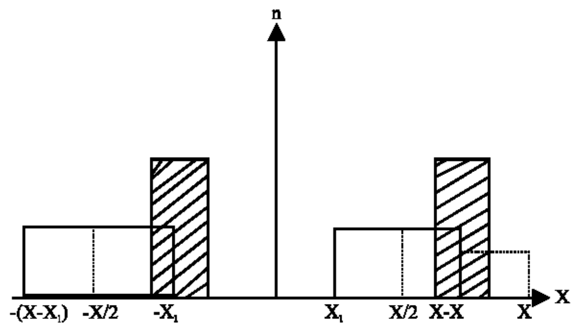

Fig. 2: Periodic extension of the refractive index profile of thin film used in this description (Roey *et al*., 1981)

different sets of ϕ_m can be obtained by the application of the discrete Fourier transform because of the periodic extension of the field in the Fig. 2 Also to obtain a square wave function for $n_0(x)$ as in Fig. 2 obtained by the application of the discrete Fourier transform because of the periodic extension of the field in the Fig 2. Also to obtain a square wave function for $n_0(x)$ as in Fig 2, n_o has to be considered period we were primarily interested.

In the field guided at the interface $x = x_1$. The field radiated away from the interface was assumed not influence the field in the adjacent region because of the presence of suitable absorber at $x = -x_1$ and $x = x - x_1$. The correction operator B^+ contains the perturbation term Δn^2 and as we considered it to be periodic function without any constant part as in Eq. 18. The phase variation of the correction term is the same such that the correction term provided a coupling between the two waves.

The Green's function as obtained in Eq. 17 satisfied

$$\left[\frac{\partial^2}{\partial x}+\frac{\partial^2}{\partial y}+\Delta n^2(y)\right]G(x,y)=\delta(x-x_0)\delta(y-y_0)$$

at the source point and satisfied the impedance boundary condition that.

$$G + B_0\frac{\partial G}{\partial n} = 0 \qquad (24)$$

where $$B_0 = -i\frac{z_s}{kz_0}$$

and $$z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Is the free space characteristic impedance and $\partial/\partial n$ is the normal derivative. The impedance z_s offered to the propagating wave by the thin film is given by

$$Z_s(1-K^2_s/K_0 n) = Z_0/n\,(1 - 2\pi\lambda^{-2}/\lambda^{-1}_0) \qquad (29)$$

From the same condition.

Where n is the average refractive index of the film
λ_s = wavelength of the wave in the thin film
λ_s = wavelength of the wave in free space from Eq. 29, for every given wave with a wavelength say λ_s in the film the appropriate Δn and the impedance Z_s of the film calculated.

CONCLUSIONS

In this study we analysed the effect of change in refractive index of FeS_2 thin film medium on propagating electromagnetic wave. With a change in refractive index, which is periodic, was identified as a perturbation. The propagating field ϕ used in the analysis was obtained by series expansion solution of the Green's function and splitted into two with the second term considered as a correction term factor due to change in the refractive index. We also observed that smoothly change in refractive index at both sides of the step according to the model is sensitive to polarization, which was assumed to be as a result of interface. However, the field was decomposed into Transverse Electric mode (TE) and Transverse Magnetic mode (TM) and with the refractive index being periodic, the phase variation of the correction terms were the same. Again, with the satisfaction of the boundary conditions at the some point and at impedance boundary condition, the expression for the calculation of the impedance of the film was given in Eq. 25.

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