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On Non-archimedean λ-Limited Spaces

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Abstract: In this research we study some types of limited sets and operators on non-archimedean locally convex spaces. We generate the concept of limited spaces into λ-limited spaces and study the relation between λ-limited spaces and λ-semiMontel spaces. We show that the non-archimedean locally convex space E is λ-limited space if and only if E is a space of type (S_λ).

Key words: Non-archimedean locally convex spaces, compactoid, limited sets, semi-montel spaces, kolmogrove diameters

INTRODUCTION

In this research we study locally convex spaces over a complete valued scalar field K that are not isomorphic to R or C.We treat some theorems about compact sets and operators in functional analysis over R or C and discussion whether or not they remain valid in non-archimedean functional analysis. We study some types of limited sets and operators which called λ-limited sets and operators in non- archimedean locally convex spaces and we use the Kolmogrove diameters to obtain results resembling previously known properties of limited sets. We generate the concept of limited spaces into λ-limited spaces and study the relation between λ-limited spaces and λ-semi-Montel spaces.

PRELIMINARIES

Let K be a field. A non-Archimedean valuation on K is a function $|.|:K\to[0,\infty)$ such that for all $\alpha,\beta\in K$ it satisfies: $|\alpha| = 0$ if and only if $\alpha = 0$; $|\alpha\beta| = |\alpha||\beta|$ and $|\alpha+\beta|\le\max\{|\alpha|,|\beta|\}$. Note that the last condition separates the absolute value on R or C from all other valuations. The mapping $(\lambda,\mu)\to|\lambda-\mu|$ is a metric on K making K into a topological field. We will call the valuation is dense if the set $|K|\backslash\{0\}$, where $|K| = \{|\lambda|: \lambda\in K\}$, is dense in $(0,\infty)$.

Let E be a vector space over the field K. A non-archimedean seminorm on E is a seminorm which verifies the strong triangle inequality: $||a+b||\le\max\{||a,b||\}$ for all $a,b\in E$. If in addition $||x|| = 0 \Leftrightarrow x = 0$, then we say that $||.||$ is a non-archimedean norm on E. The pair (E, $||.||$) is called a non-archimedean normed space.

Throughout this study K will stand for a complete non- archimedean valued field, whose valuation is non-trivial. The collection of all continuous non-archimedean seminorm on a vector space E over K will be denoted by cs (E). For $p\in cs$ (E) and $r>0$, $B_p(0, r)$ will be the set $\{x\in E: p(x)\le r\}$. L (E, F) will be the vector space of all continuous linear operators from E into F. The non-archimedean normed space E is said to be of countable type if, there exists a countable subset S of E, such that the subspace [S] spanned by S is dense in E.

For a continuous non-archimedean seminorms p on E we put E_p = E/ker p and denoted by π_p the canonical surjection $\pi_p: E\to E_p$. Then E_p is a non-archimedean normed space for the non-archimedean norm $|| ||_p$ defined by $||\pi_p(x)||_p = p(x)$, $x\in E$. By De Grande-De Kimpe and Perez-Garcia (1994) is a space of countable type.
The following sequence spaces will be need:

- $c_0(K) = \{(\lambda_n): \lambda_n\in K, \lim_n(\lambda_n) = 0\}$.
- $\iota_\infty(K) = \{(\lambda_n): \lambda_n\in K, (\lambda_n) \text{ is bounded}\}$.
- $\iota_1(K) = \{(\lambda_n\in K, \sum_{n=1}^{\infty}|\lambda_n|\le\infty\}$.
- $(S) = \left\{(\lambda_n): \lambda_n\in k \sup_n n^{\alpha}|\lambda_n|\le\infty\ \forall\ \alpha>0\right\}$.
- $\Lambda(\alpha) = \{(\alpha_n:\lambda_n\in K, \sup_n R^{\alpha_n}|\lambda_n|<\infty\ \forall R>0, \alpha_1\le, \le_2 ....\}$.
- $(R) = \{(\lambda_n): \lambda_n\in K, \lim_n \sqrt[n]{|\lambda_n|} = 0\}$.

Definition 1: A non-Archimedean sequence ideal λ on the valued field K is a subset of the space $\iota_\infty(K)$ satisfying the following conditions:

- $e_i\in\lambda$ where $e_i = (0,0,...,1,....)$ the one in the ith place.
- If $x_1, x_2\in\lambda$, then $x_1+x_2\in\lambda$.
- If $y\in\iota_\infty(K)$ and $x\in\lambda$, then $x.y\in\lambda$.
- If the sequence $x = (x_0,x_1,.....)\in\lambda$, then $(x_0,x_0, x_1,x_1,...)\in\lambda$.

Note that the sequence spaces, $\iota_\infty(K)$, $c_0(K)$, (S), (R) and $\Lambda(\alpha)$ are examples of sequence ideals (Pietch, 1980).

For a bounded subset B of a locally convex space E over K, a $p\in cs$ (E) and a non-negative integer n, the nth

Kolmogrove diameter $\delta_{n,p}(B)$ (or $\delta_n(B, B_p(0,1))$) of B with respect to p is the infimum of all $|\mu|$, $\mu \in K$, for which there exists a subspace F of E with dim $(F) \le n$, such that $A \subset F + \mu B_p(0,1)$ (Katasars and Perez-Garcia, 1997). These nth Kolmogrove diameters satisfy the following properties:

Proposition 1:

- $\delta_{0,p}(B) \ge \delta_{1,p}(B) \ge \delta_{2,q}(B) \ge ...... \ge 0.$ for all $p \in cs(E)$.
- If $B_1 \subseteq B$ and $p \le q$, then $\delta_{n,p}(B_1) \le \delta_{n,q}(B)$.
- If $T \in L(E, F)$, then for all $p \in cs(F)$ there exists $q \in cs(E)$ such that $\delta_{n,p}(T(B)) \le \delta_{n,q}(B)$.
- If $p' \ge p$, then $\delta_n(\pi_p(B_q(0,1)), \pi_p(B_{p'}(0,1))) = \delta_n(B_q(0,1), B_p(0,1))$ (Dubinsky, 1979; Jarchow, 1981; Safi, 2006).

λ-LIMITED SETS

Definition 2: Let E, F be locally convex spaces over K, then

- A subset B of E is called compactoid if for every zero-neighborhood U in E there exists a finite set $A \subset E$ such that $B \subset co(A) + U$, where co (A) is the absolutely convex hull of A.

An operator $T \in L(E, F)$ is called compactoid if there exists a zero-neighborhood U in E such that T (U) is compactoid in F (De Grande-De Kimpe *et al.*, 1995). Katasars and C. Perez-Garcia (1997) used the Kolmogrove diameters to give the following equivalent definition:

- The bounded subset B of a locally convex space E over K is called compactoid if and only if $(S_{n,p}(B)_{n=0}^{\infty}) \in c_0(K)$ for all $p \in cs(E)$. (1)
- A bounded subset B of E is called limited in E if and only if for each continuous linear map T from E to c_0 (k), T (B) is compactoid in c_o (K).

An operator $T \in L(E, F)$ is called limited if there exists a zero-neighborhood E in U such that T (U) is limited in F. We will denote by lim (E, F) the vector space of all limited operators from E to F ((De Grande-De Kimpe *et al.*, 1995). Parallel to this definitions we define the following:

Definition 3: Let E, F be locally convex spaces over K, then

- A subset B of E is called λ-compactoid if we replace c_o (K) in (1)
- by the sequence ideal.

An operator $T \in L(E, F)$ is called λ-compactoid if there exists a zero-neighborhood U in E such that T (U) is λ-compactoid in F. We will denoted by λ-C (E,F) the space of all λ-compactoid operators from E into F.

- A bounded subset B of E is called λ-limited in E if and only if for each continuous linear map T from E to c_o (K), T (B) is λ-compactoid in c_o (K).

An operator $T \in L(E,F)$ is called λ-limited if there exists a zero-neighborhood U in E such that T (U) is λ-limited in F. We will denoted by (λ)-lim (E, F) the space of all λ-limited operators from E into F.

Notes:

- If dim (E) = n, then every bounded subset of E is λ-compactoid.
- If

$$D = \left\{(\lambda_n) : \lambda_n \in K, \sum_{n=1}^{\infty} |\lambda_n| 2^n \le 1\right\} \text{ and}$$

$$B = \left\{(\lambda_n) : \lambda_n \in K, \sum_{n=1}^{\infty} |\lambda_n| n \le 1\right\}$$

are two subsets of ι_1 (K), then according to Pietch (1972) we have

$$\delta_n(D, B_{l_1}) = \frac{1}{2^n}$$

and

$$\delta_n(B, B_{l_1}) = \frac{1}{n},$$

where $B_{\iota 1}$ is the closed unit ball in ι_1. Hence D is c_0 (K)-compactoid and (S)-compactoid, but not (R)-compactoid and B is c_0 (K)-compactoid but not (S)-compactoid.

Proposition 2: Let E,F be locally convex spaces over K, then

i) Every λ-compactoid subset of E is λ-limited in E.
ii) If B is λ-limited in E and $T \in L(E, F)$, then T (B) is λ-limited in F.
iii) If B is λ-limited in E and $D \subset B$, then D is λ-limited in E.
iv) If A is λ-limited, then $\bar{A}$ is λ-limited.
v) If A, $B \subset E$ are λ-limited in E, then A+B is λ-limited in E.
vi) The product of any finite number of λ-limited sets is λ-limited.
vii) Let M be a subspace of E and $B \subset M$. If B is λ-limited in M, then B is λ-limited in E (De-Grande-De Kimpe *et al.*, 1995).

Proof:

- Let B be any λ-compactoid subset of E and let $T\in L$ (E, c_0(K)). It follows from property (iii) of proposition (1) that for all $p\in cs(F)$ there exists $q\in cs$ (E) such that $\delta_{n,p}(T(B))\leq\delta_{n,q}(B)$ and so T (B) is λ-compactoid in c_0 (K). Therefore B is λ-limited in E.
- Suppose B is λ-limited in E and $T\in L$ (E, F). Let $G\in L(F, c_0(K))$, then $G_0T\in L$ (E, c_0(K)). It follows that G (T(B)) is λ-compactoid in c_0(K) and so T(B) is λ-limited in E.
- Let $D\subset B$ and let $T\in L$ (E, F). Since $T(D)\subset T(B)$, then by property (ii), (iii) of proposition (1) it follows that $\delta_{n,p}(T(D))\leq\delta_{n,p}$ for all $p\in cs$ (F). Since B is λ-limited in E, then T (D) is λ-compactoid in c_0 (K) and this complete the proof.
- From definition of $\delta_{n,p}$ (A), if $\varepsilon>0$ there exist a subspace F of E with $\dim(F)\leq n$ and $\mu\in K$ such that $|\mu|\leq\delta_{n,p}(A)+\varepsilon$, $A\subseteq\mu B_p(0,1)+F$. It follows that $\bar{A}\subseteq\mu B_p(0,1)+F$ and so $\delta_{n,p}(A)\leq|\mu|\leq\delta_{n,p}(A)+\varepsilon$. Since $\varepsilon>0$ is an arbitrary, we deduce that $\delta_{n,p}(\bar{A})\leq\delta_{n,p}(A)$. That is, if A is λ-compactoid in E, then $\bar{A}$ is also λ-compactoid in E. Now, let $T\in L(E, c_0(K))$. Since A is λ-limited it follows that T(A) is λ-compactoid and hence $\overline{T(A)}$ is λ-compactoid. Since, $T(\bar{A})\subseteq\overline{T(A)}$ it follows that $T(\bar{A})$ is λ-compactoid.
- Let $T\in L(E,c_0)$ Since A, B are λ-limited, then T(A), T(B) are λ-compactoid. Since, $T(A+B)\subseteq T(A)+T(B)$, it follows by Safi (2006) that T (A+B) is λ-compactoid in c_0(K) and so A+B is λ-limited in E.
- Let D_i be any λ-limited set in E_i, i = 1,2,......, n and let $E = E_1\times E_2\times.....\times E_n$, $T\in L(E, c_0(K))$. Now If π_i: $E_i\rightarrow E$ is conical inclusion, then the operator $T_i = T\circ\pi_i\in L(E_i,c_0(K))$ and so T_i (D_i) is λ-compactoid in c_0(K). Since,

$$T(\prod_{i=1}^{n} D_i = T_1(D_1)+T_2(D_2)+........+T_n(D_n),$$

then

$$T(\prod_{i=1}^{n} D_i)$$

is λ-compactiod (Safi, 2006). Therefore

$$\prod_{i=1}^{n} D_i$$

is λ-limited (proposition 2.i).

- Let M be a subspace of E and let B be λ-limited M. If $T\in L$ (E, F), then the restriction operator $T|M\in L$ (M, c_0(K)). Since $T|M\in L(M,c_0(K))$ is λ-compactoid in c_0 (K) it follows that B is λ-limited in E.

Note: If λ = c_0 (K), then the unit ball ℓ_∞ of is λ-limited, but not λ-compactoid (De Grande-Dekimpe and Perez-Garcia, 1994).

Definition 4: A locally convex space over K is called λ-Gelfand-Philips space (λ-GP-space in short) if every λ-limited set in E is λ-compactoid. (De Grande-Dekimpe and Perez-Garcia, 1994).

Remark : c_0 (K) is λ-Gp space, for any sequence ideal λ (and hence any non-archimedean normed space of countable type (Van Rooij, 1978).

To see that let A be any λ-limited set in c_0 (K). Since the identity operator $I\in L$ (c_0(K), c_0(K)), then I(A) = A is λ-compactoid.

λ-LIMITED SPACES

De Ggrande-de Kimpe and Perez-Garia (1994) give the following definition:

The locally convex space E over K is called limited space if L (E, F) = lim (E, F) for all non-archimedean normed space F.

Definition 5: We say that the locally convex space E over K is λ-limited space if L (E, F) = λ-lim (E,F) for all non-archimedean normed spaces F.

Notes:

- If λ = c_0(K), then the concepts of λ-limited space coincide with the limited spaces and if the valuation K is dense. Then ℓ_∞(K) is λ-limited spaces. Since L(c_0 (K), c_0 (K))≠λ-C(c_0 (K), c_0 (K))⊆λ-lim (c_0 (K), c_0 (K)), then c_0 (K) is not λ-limited spaces (De Grande-De Kimpe *et al.*, 1995).
- If E is a non-archimedean normed space, then the closed unit ball of E, B_E is λ-limited if L (E, c_0 (K)) = λ-C (E, c_0 (K)).

Theorem 1: If L (E, F) = λ-lim (E, F) for any locally convex spaces E, F over K and M is a closed subspace of E then. L (E/M, F) = λ-lim (E/M, F).

Proof: Let M be a closed subspace of E and $T\in L$ (E/M, F). If π: E→E/M is the quotient map, then $T\circ\pi\in L$ (E, F). Since L (E, F) = λ-lim (E, F), there exists a zero-neighbourhood U in E such that $(T\circ\pi(U) = T(\pi(U))$ is λ-limited. Since π (U) is a zero-neighbourhood in E/M, then $T\in$λ-lim (E/M, F).

Proposition 3: Let F, E_1, E_2, ... be any locally convex spaces over K:

- If L (E_i, F) = λ-lim (E_i, F) for each i∈N, then.

$$L(\prod_{i=1}^{\infty} E_i, F) = \lambda - \lim(\prod_{i=1}^{\infty} E_i, F)$$

- If L (F, E_i) = λ-lim (F, E_i) for each i∈I, I is finite, then

$$L(F, \prod_{i \in I} E_i) = \lambda - \lim(F, \prod_{i \in I} E_i)$$

- If E_i is λ-GP-space and L (F, E_i) = λ-lim (F, E_i) for each i∈N, then

$$L(F, \prod_{i=1}^{\infty} E_i) = \lambda - \lim(F, \prod_{i=1}^{\infty} E_i)$$

(Van Rooij, 1978).

Proof:

- Let

$$E = \prod_{i=1}^{\infty} E_i$$

and let T∈(E, F). Then T is bounded on some zero-neighborhood W of E. This neighborhood can be taken in the form

$$W = \prod_{i=1}^{\infty} U_i$$

where U_i is a zero-neighborhood in E_i and the set J = {i∈N: $U_i \neq E_i$} is finite. So we can assume that $E = E_1 \times E_2 \times ..... \times E_n$ for some n∈N. Now for i = 1, 2, ..., n, let π_i: $E_i \rightarrow E$ be the conical inclusion. Since the operator $T_i = T \circ \pi_i \in L$ (E_i, F) and L (E_i, F) = λ-lim (E_i, F), then there exists a zero- neighborhood V_i in E_i such that T_i (V_i) is λ-limited set in F, then $V = V \times V_2 \times ..... \times V_n$ is zero-neighborhood in E for which $T(V) = T_1(V_1)+T_2(V_2)+.....+T_n(V_n)$ is λ-limited set in F (proposition (2.v)). So. T∈λ-lim (E, F).

- Let

$$T \in L(F, \prod_{i \in I} E_i)$$

I, is finite, and let P_i: $E \rightarrow E_i$ be the canonical operator, then $P_i \circ T \in L$ (F, E_i). Since L (F, E_i) = λ-lim (F,E_i), then $P_i \circ T$ is λ-limited operator. Thus, there exists a zero-neighborhood U in F such that $P_i \circ T$ (U) = W_i is λ-limited set in E_i. It follows by proposition (2.vi)

$$\prod_{i \in I} W_i = T(U)$$

is λ-limited set in

$$\prod_{i \in I} E_i$$

and so T is λ-limited operator.

- Let

$$T \in L(F, \prod_{i=1}^{\infty} E_i)$$

then like in part (ii) we can find a zero-neighborhood U in F such that $P_i \circ T$ (U) = W_i is λ-limited set in E_i for all i∈N. Since E_i is λ-GP-space, then W_i is λ-compactiod set in E_i. Now by Safi (2006)

$$\prod_{i=1}^{\infty} W_i = T(U)$$

is λ-compactod set and by proposition (2.i) T(U) is λ-limited set in

$$\prod_{i=1}^{\infty} E_i$$

Therefore

$$L(F, \prod_{i=1}^{\infty} E_i) = \lambda - \lim(F, \prod_{i=1}^{\infty} E_i)$$

Definition 6: A locally convex space E over K is said to be of type (S_λ) if for each P∈cs(E) there exists q∈cs(E) such that

$$(\delta_{n,q'}(B_q(0,1),)_{n=0}^{\infty} \in \lambda$$

for each q′≥p) (Zahriuita, 1973).

Proposition 4: The space E is of type (S_λ) if and only if E is λ-limited space.

Proof: Sufficiency, let E be λ-limited space and let p∈cs(E). Since E_p = E/Ker p is a non-archimedean normed space and the canonical surjection

$$\pi_p : E \to E_p$$

is continuous, then π_p is λ- limited operator, so there exists a neighborhood B_q (0, 1) in E such that π_p (B_q (0, 1) is λ-limited in E_p. Now since E_p is a non-archimedean normed space of countable type, then E_p is λ-Gp-space and so π_p (B_q (0, 1) is λ-compactoid set in E_p, hence

$$(\delta_n(\pi_p(B_q(0,1)), \pi_p(B_h(0,1)))_{n=0}^{\infty} \in \lambda$$

for each h∈cs (E). Now if p′≥p, then

$$(\delta_n(\pi_p(B_q(0,1)), \pi_p(B_{p'}(0,1)))_{n=0}^{\infty} \in \lambda$$

and by proposition (1. iv) it follows that

$$(\delta_n(B_q(0,1), B_{p'}(0,1))_{n=0}^{\infty} \in \lambda$$

thus E is a space of type (S_λ).

Necessity: Let E be a space of type (S_λ), F be an arbitrary non-archimedean normed space and T∈L (E, F). Now for the closed unit ball B_F, there exists p∈cs (E) such that T (B_P (0,1))⊂B_F. Since E is a space of type (S_λ), there exists q∈cs (E) such that

$$(\delta_n(B_q(0,1), B_{p'}(0,1)))_{n=0}^{\infty} \in \lambda$$

for all p′≥q. It follows by proposition (1. (iii)) that

$$(\delta_n(T(B_q(0,1)), T(B_{p'}(0,1))))_{n=0}^{\infty} \in \lambda$$

for all p′≥p. Now since,T ($B_{P'}$ (0,1))⊂T (B_q (0,1))≤ BF)) then ($\boldsymbol{\delta}_n$(T(B_q(0,1)), B_F)) ≤$\boldsymbol{\delta}$ (T(B_q(0,1)), T(B_q(0,1)), T ($B_{p'}$ (0,1)). Therefore ($\boldsymbol{\delta}_n$ (T (B_q (0,1)), B_F))∈λ and so T(B_q (0, 1) is λ-compactoid in F and by proposition (2.i) is λ-limited, Thus T is λ-limited operator.

Definition 7: A locally convex space E over K is called λ-semi-Montel, if every bounded subset D of E is λ-compactoid.

Notes:

- Every finite dimensional normed space is λ-semi-Montel.
- If E is λ-Gp space such that every bounded subset of E is λ-limited, then E is λ-semi-Montel space.
- If Eλ-semi-Montel space, then every bounded subset of E is λ-limited.

Proposition 5:

- If E is λ-limited space, then every bounded set in E is λ-limited.
- If F is a locally convex space over K and L (E, F) = λ-lim (E, F) for every non-archimedean normed space E, then every bounded set in F is λ-limited.
- If F is λ-semi-Montel space, then L (E, F) = λ-lim (E, F) for every non-archimedean normed space E.
- If F is λ-Gp space and L (E, F) = λ-lim (E, F) for every non-archimedean normed space E, then F is λ-semi-Montel space.

Proof:

- Let A be any bounded subset of E and let T∈L (E, c_0 (k)). Since L (E,c_o (K)) = λ-lim (E, c_o (K)), then there exists a zero-neighborhood U in E such that T(U) is λ-limited in c_0(K). Since A is bounded, then there exists r∈K, |r|>0, such that A⊂rU and so T(A)⊂rT(U). It follows by proposition (2.iii) T(A) is λ-limited in c_0(K). Since c_0(K) is λ-Gp space, then T(A) is λ-compactoid and so A is λ-limited set in E.
- Suppose L (E, F) = λ-lim (E, F) for every non-archimedean normed space E. We shall show that every bounded set A in F is λ-limited. Since A is bounded set in F, then for each p∈cs (F) there exists m(p)∈K, |m(p)|>0 such that A⊂m(p) B_p (0, 1). Now let,

$$q(y) = \sup\{\frac{1}{|m(p)|} p(y) : p \in cs(F)\} \leq 1, y \in A$$

Then q is a non-archimedean seminorm on A. If q(y) = 0, then p (y) = 0 for all p∈s(F) and so y = 0. Thus q is a non-archimedean norm. Now by E, we shall take the non-archimedean normed space of all y∈F with q (y)<∞. If B_E = {y∈F: q(y)≤1} is the closed unit ball of E, then A⊆B_E and if the operator T equal to the identity imbedding of E into F, then T∈L (E, F). Since L (E, F) = λ-lim (E, F), then T is λ-limited operator. Thus T (B_E) = B_E is λ-limited set in F and by proposition (2. iii) A is λ-limited set in F.

- Let F be any λ-semi-Montel space, E be a non-archimedean normed space and T∈L (E, F). Since T is bounded, then T maps the unit ball B_E into E a bounded set T(B_E) in F and so T (B_E) is λ-compactoid set in F, hence T(B_E) is λ-limited set in F. Therefore T is λ-limited operator.

- It follows from part (2) and the fact that the space F is λ-Gp spaces.

Theorem 2: Let F, E be any locally convex spaces over K and let L (E, F) = λ-lim (E, F), then L (E_0, F_0) = λ-lim (E_0, F_0) for a complement linear subspace E_0 of E and subspace F_0 of F.

Proof: Let $T_0 \in L$ (E_0, F_0) and let $T \in L$ (E, F) defined by $T(x) = T_0$ where $x = x_0+x_1$, $x_0 \in E_0$. Since L (E, F) = λ-lim (E, F), then T is λ-limited operator and so there exists a zero-neighborhood U in E such that T(U) is λ-limited set. Since $U \cap E_0$ is zero-neighborhood in E_0, then applying proposition (2.iii) we deduce that T ($U \cap E_0$) = T_0 ($U \cap E_0$) is λ-limited and therefore T_0 is λ-limited operator.

Note: If the valuation on K is dense and $\lambda = c_0$ (K), then L (ℓ_∞ (K), ℓ_∞(K)) = λ-lim (ℓ_∞ (K), ℓ_∞(K)) but L (c_0 (K), c_0(K)) $\neq$ λ-lim (c_0 (K), c_0 (K)) (De Grande-De Kimpe *et al.*, 1995, Example (2.6.iv)).

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