



# Journal of Applied Sciences

ISSN 1812-5654

**science**  
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## Optimal Inventory Planning under Permissible Delay in Payments When a Larger Order Quantity

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**Abstract:** In the previous related studies, the inventory replenishment problems under permissible delay in payments are independent of the order quantity. In this study, the restrictive assumption of the trade credit independent of the order quantity is relaxed. This study discusses the inventory policies under permissible delay in payments when a larger order quantity.

**Key words:** EOQ, permissible delay in payments, trade credit, inventory

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### INTRODUCTION

In the classical EOQ model, it is tacitly assumed that the payment of an order is made on the receipt of items by the inventory system. In practice, however, this may not be true. Under certain conditions suppliers are known to offer their customers a delay in paying for an order of a particular commodity. Frequently, suppliers allow credit for some fixed time period for settling the payment for the goods and do not charge any interest from the buyer on the amount owed during this credit period. However, a higher interest is charged if the payment is not settled by the end of the credit period. The existence of credit period serves to reduce the cost of holding stock to the user, because it reduces the amount of capital invested in stock for the duration of the credit period. Recently, several researchers have developed analytical inventory models with consideration of permissible delay in payments.

Goyal (1985) established a single-item inventory model under permissible delay in payments. Khouja and Mehrez (1996) investigated the effect of four different supplier credit policies on the optimal order quantity within the EOQ framework. Chung (1998) developed an efficient decision procedure to determine the economic order quantity under condition of permissible delay in payments. Teng (2002) assumed that the selling price was not equal to the purchasing price to modify Goyal's model (1985). Chung and Huang (2003a) investigated this issue within EPQ (economic production quantity) framework and developed an efficient solving procedure to determine the optimal replenishment cycle for the retailer. Huang and

Chung (2003) investigated the inventory policy under cash discount and trade credit. Chung and Huang (2003b) adopted alternative payment rules to develop the inventory model and obtain different results. Huang (2004) adopted the payment rule discussed in Chung and Huang (2003b) and assumed finite replenishment rate, to investigate the buyer's inventory problem. Huang (2006) extended Huang (2003) to develop retailer's inventory policy under retailer's storage space limited. Recently, Huang (2007) incorporated Chung and Huang (2003a) and Huang (2003) to investigate retailer's ordering policy.

This research combines the above both studies by Goyal (1985) and Khouja and Mehrez (1996) to discuss the inventory policies under permissible delay in payments when a larger order quantity. Finally, numerical examples are used to illustrate all theorems in this study.

### MODEL FORMULATION

#### Notation:

- Q = Order quantity
- D = Annual demand
- W = Quantity at which the delay in payments is permitted
- A = Cost of placing one order
- c = Unit purchasing price
- h = Unit stock holding cost per year excluding interest charges
- $I_e$  = Interest which can be earned per \$ per year
- $I_p$  = Interest charges per \$ investment in inventory per year

- M = Trade credit period
- T = The cycle time
- TVC(T) = The total relevant cost function per unit time
- T\* = The optimal cycle time of TVC(T)

**Assumptions:**

- Demand rate is known and constant
- Shortages are not allowed
- Time period is infinite
- The lead time is zero
- If  $Q < W$ , i.e.,  $T < W/D$ , the delayed payment is not permitted. Otherwise, fixed trade credit period  $M$  is permitted. Hence, if  $Q < W$ , pay  $cQ$  when the order is received. If  $Q \geq W$ , pay  $cQ$   $M$  time periods after the order is received.
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and we start paying for the interest charges on the items in stock
- $I_p \geq I_e$

The annual total relevant cost consists of the following elements. There are two cases to occur:

- $M \geq W/D$
- $M < W/D$

**Case 1: Suppose that  $M \geq W/D$ .**

- Annual ordering cost =  $\frac{A}{T}$
- Annual stock holding cost (excluding interest charges) =  $\frac{DTh}{2}$
- There are three cases to occur in cost of interest charges for the items kept in stock per year.
  - $0 < T < W/D$   
 Cost of interest charges for the items kept in stock per cycle =  $\frac{cI_p DT^2}{2}$   
 Cost of interest charges for the items kept in stock per year =  $\frac{cI_p DT}{2}$
  - $W/D \leq T \leq M$   
 In this case, no interest charges are paid for the items kept in stock.
  - $M \leq T$   
 Cost of interest charges for the items kept in stock per cycle =  $\frac{cI_p D(T-M)^2}{2}$   
 Cost of interest charges for the items kept in stock per year =  $\frac{cI_p D(T-M)^2}{2T}$

- There are three cases to occur in interest earned per year.

- $0 < T < W/D$   
 In this case, no interest earned because the delayed payment is not permitted.
- $W/D \leq T \leq M$

$$\text{Interest earned per cycle} = cI_e \left[ \frac{DT^2}{2} + DT(M-T) \right] = DTcI_e \left( M - \frac{T}{2} \right)$$

$$\text{Interest earned per year} = DcI_e \left( M - \frac{T}{2} \right)$$

- $M \leq T$

$$\text{Interest earned per cycle} = cI_e \int_0^M Dtdt = \frac{DM^2 cI_e}{2}$$

$$\text{Interest earned per year} = \frac{DM^2 cI_e}{2T}$$

From the above arguments, the annual total relevant cost for the retailer can be expressed as:

$$\text{TVC}(T) = \text{ordering cost} + \text{stock-holding cost} + \text{interest payable} - \text{interest earned}$$

We show that the annual total relevant cost, TVC(T), is given by:

$$\text{TVC}(T) = \begin{cases} \text{TVC}_1(T) & \text{if } 0 < T < W/D & (1a) \\ \text{TVC}_2(T) & \text{if } W/D \leq T \leq M & (1b) \\ \text{TVC}_3(T) & \text{if } M \leq T & (1c) \end{cases}$$

Where:

$$\text{TVC}_1(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_p DT}{2} \quad (2)$$

$$\text{TVC}_2(T) = \frac{A}{T} + \frac{DTh}{2} - DcI_e \left( M - \frac{T}{2} \right) \quad (3)$$

and

$$\text{TVC}_3(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_p D(T-M)^2}{2T} - \frac{DM^2 cI_e}{2T} \quad (4)$$

Since  $\text{TVC}_1(W/D) > \text{TVC}_2(W/D)$ ,  $\text{TVC}_2(M) = \text{TVC}_3(M)$ , TVC(T) is continuous except  $T = W/D$ . Furthermore, we have  $\text{TVC}_1(T) > \text{TVC}_2(T)$  for all  $T > 0$ . Equation 2, 3 and 4 yield

$$\text{TVC}'_1(T) = \frac{-A}{T^2} + \frac{D(h + cI_p)}{2} \quad (5)$$

$$\text{TVC}''_1(T) = \frac{2A}{T^3} > 0 \quad (6)$$

$$TVC_2'(T) = \frac{-A}{T^2} + \frac{D(h + cI_e)}{2} \quad (7)$$

$$TVC_2''(T) = \frac{2A}{T^3} > 0 \quad (8)$$

$$TVC_3'(T) = \frac{-[2A + cDM^2(I_p - I_e)]}{2T^2} + \frac{D(h + cI_p)}{2} \quad (9)$$

and

$$TVC_3''(T) = \frac{2A + cDM^2(I_p - I_e)}{T^3} > 0 \quad (10)$$

Equation 6, 8 and 10 imply that  $TVC_1(T)$ ,  $TVC_2(T)$  and  $TVC_3(T)$  are convex on  $T > 0$ . Moreover, we have  $TVC_1'(W/D) \neq TVC_2'(W/D)$  and  $TVC_2'(M) = TVC_3'(M)$ .

**Case 2: Suppose that  $M < W/D$**

If  $M < W/D$ , Eq. 1a, b, c will be modified as:

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } 0 < T < W/D \\ TVC_3(T) & \text{if } W/D \leq T. \end{cases}$$

Since  $TVC_1(W/D) > TVC_3(W/D)$ ,  $TVC(T)$  is continuous except  $T = W/D$ . Equation 6 and 10 imply that both  $TVC_1(T)$  and  $TVC_3(T)$  are convex on  $T > 0$ .

**DECISION RULE OF THE OPTIMAL CYCLE TIME  $T^*$  When  $M \geq W/D$**

Recall

$$T_1^* = \sqrt{\frac{2A}{D(h + cI_p)}} \quad (11)$$

$$T_2^* = \sqrt{\frac{2A}{D(h + cI_e)}} \quad (12)$$

and

$$T_3^* = \sqrt{\frac{2A + cDM^2(I_p - I_e)}{D(h + cI_p)}} \quad (13)$$

Then

$$TVC_1'(T_1^*) = TVC_2'(T_2^*) = TVC_3'(T_3^*) = 0.$$

We have  $T_2^* \geq T_1^*$  and  $T_3^* \geq T_1^*$ . By the convexity of  $TVC_i(T)$  ( $i = 1, 2, 3$ ), we see

$$TVC_1'(T) \begin{cases} < 0 & \text{if } T < T_1^* \\ = 0 & \text{if } T = T_1^* \\ > 0 & \text{if } T > T_1^*, \end{cases} \quad (14a)$$

$$= 0 \quad (14b)$$

$$> 0 \quad (14c)$$

$$TVC_2'(T) \begin{cases} < 0 & \text{if } T < T_2^* \\ = 0 & \text{if } T = T_2^* \\ > 0 & \text{if } T > T_2^* \end{cases} \quad (15a)$$

$$= 0 \quad (15b)$$

$$> 0 \quad (15c)$$

and

$$TVC_3'(T) \begin{cases} < 0 & \text{if } T < T_3^* \\ = 0 & \text{if } T = T_3^* \\ > 0 & \text{if } T > T_3^* \end{cases} \quad (16a)$$

$$= 0 \quad (16b)$$

$$> 0 \quad (16c)$$

Equation 14a-c, 15a-c and 16a-c imply that  $TVC_i(T)$  is decreasing on  $(0, T_i^*]$  and increasing on  $[T_i^*, \infty)$  for all  $i = 1, 2, 3$ . Equation 5, 7 and 9 yield that:

$$TVC_1'\left(\frac{W}{D}\right) = \frac{-2A + \frac{W^2}{D}(h + cI_p)}{2\left(\frac{W}{D}\right)^2} \quad (17)$$

$$TVC_2'\left(\frac{W}{D}\right) = \frac{-2A + \frac{W^2}{D}(h + cI_e)}{2\left(\frac{W}{D}\right)^2} \quad (18)$$

and

$$TVC_2'(M) = TVC_3'(M) = \frac{-2A + DM^2(h + cI_e)}{2M^2} \quad (19)$$

Furthermore, we let

$$\Delta_1 = -2A + \frac{W^2}{D}(h + cI_p) \quad (20)$$

$$\Delta_2 = -2A + \frac{W^2}{D}(h + cI_e) \quad (21)$$

and

$$\Delta_3 = -2A + DM^2(h + cI_e) \quad (22)$$

Equation 20, 21 and 22 yield that  $\Delta_1 \geq \Delta_2$  and  $\Delta_3 \geq \Delta_2$ . Furthermore, we have

$$\Delta_1 > 0 \text{ if and only if } T_1^* < W/D \quad (23)$$

$$\Delta_2 > 0 \text{ if and only if } T_2^* < W/D \quad (24)$$

$$\Delta_3 > 0 \text{ if and only if } T_2^* < M \quad (25)$$

$$\Delta_3 > 0 \text{ if and only if } T_3^* < M \quad (26)$$

Therefore, the optimal cycle times can be obtained as follows:

Table 1: The optimal cycle time and optimal order quantity using Theorem 1

| A   | D    | W   | c   | I <sub>p</sub> | I <sub>e</sub> | h | M   | W/D    | Δ <sub>1</sub> | Δ <sub>2</sub> | Δ <sub>3</sub> | Other judgment                                                          | Theorem | Optimal cycle time, (T*)  | Optimal order quantity | TVC (T*) |
|-----|------|-----|-----|----------------|----------------|---|-----|--------|----------------|----------------|----------------|-------------------------------------------------------------------------|---------|---------------------------|------------------------|----------|
| 200 | 2000 | 180 | 170 | 0.15           | 0.12           | 5 | 0.1 | 0.0900 | >0             | >0             | >0             | TVC <sub>1</sub> (T <sub>1</sub> *)>TVC <sub>2</sub> (W/D)              | 1-(A)   | W/D = 0.09                | 180                    | 428.2    |
| 200 | 2000 | 180 | 140 | 0.15           | 0.12           | 5 | 0.1 | 0.0900 | >0             | <0             | >0             |                                                                         | 1-(B)   | T <sub>2</sub> * = 0.0958 | 192                    | 816.1    |
| 200 | 1700 | 160 | 150 | 0.15           | 0.12           | 5 | 0.1 | 0.0941 | >0             | <0             | <0             | TVC <sub>1</sub> (T <sub>1</sub> *)>TVC <sub>3</sub> (T <sub>3</sub> *) | 1-(C)   | T <sub>3</sub> * = 0.101  | 172                    | 894.8    |
| 300 | 3500 | 200 | 100 | 0.15           | 0.12           | 5 | 0.1 | 0.0667 | <0             | <0             | >0             |                                                                         | 1-(D)   | T <sub>2</sub> * = 0.0886 | 266                    | 916.6    |
| 200 | 2200 | 200 | 100 | 0.15           | 0.12           | 5 | 0.1 | 0.0909 | <0             | <0             | <0             |                                                                         | 1-(E)   | T <sub>3</sub> * = 0.1029 | 226                    | 1228.1   |

Table 2: The optimal cycle time and optimal order quantity using Theorem 2

| A   | D    | W   | c   | I <sub>p</sub> | I <sub>e</sub> | h | M   | W/D    | Δ <sub>1</sub> | Δ <sub>4</sub> | Other judgment                                                          | Theorem | Optimal cycle time, (T*)  | Optimal order quantity | TVC (T*) |
|-----|------|-----|-----|----------------|----------------|---|-----|--------|----------------|----------------|-------------------------------------------------------------------------|---------|---------------------------|------------------------|----------|
| 200 | 2000 | 600 | 100 | 0.20           | 0.05           | 5 | 0.1 | 0.3000 | >0             | >0             | TVC <sub>1</sub> (T <sub>1</sub> *)<TVC <sub>3</sub> (W/D)              | 2-(A)   | T <sub>1</sub> * = 0.0894 | 179                    | 4472.1   |
| 300 | 3500 | 400 | 50  | 0.15           | 0.05           | 5 | 0.1 | 0.1143 | <0             | <0             |                                                                         | 2-(B)   | T <sub>3</sub> * = 0.1331 | 466                    | 3197.9   |
| 200 | 3000 | 400 | 50  | 0.15           | 0.05           | 2 | 0.1 | 0.1333 | >0             | <0             | TVC <sub>1</sub> (T <sub>1</sub> *)>TVC <sub>3</sub> (T <sub>3</sub> *) | 2-(C)   | T <sub>3</sub> * = 0.1389 | 417                    | 1709.2   |

**Theorem 1:**

- If Δ<sub>1</sub>>0, Δ<sub>2</sub>>0 and Δ<sub>3</sub>>0, then TVC(T\*) = min {TVC<sub>1</sub>(T<sub>1</sub>\*), TVC<sub>2</sub>(W/D)}. Hence T\* is T<sub>1</sub>\* or W/D associated with the least cost.
- If Δ<sub>1</sub>>0, Δ<sub>2</sub>≤0 and Δ<sub>3</sub>>0, then TVC(T\*) = TVC<sub>2</sub>(T<sub>2</sub>\*) and T\* = T<sub>2</sub>\*.
- If Δ<sub>1</sub>>0, Δ<sub>2</sub>≤0 and Δ<sub>3</sub>≤0, then TVC(T\*) = min {TVC<sub>1</sub>(T<sub>1</sub>\*), TVC<sub>3</sub>(T<sub>3</sub>\*)}. Hence T\* is T<sub>1</sub>\* or T<sub>3</sub>\* associated with the least cost.
- If Δ<sub>1</sub>≤0, Δ<sub>2</sub>≤0 and Δ<sub>3</sub>>0, then TVC(T\*) = TVC<sub>2</sub>(T<sub>2</sub>\*) and T\* = T<sub>2</sub>\*.
- If Δ<sub>1</sub>≤0, Δ<sub>2</sub>≤0 and Δ<sub>3</sub>≤0, then TVC(T\*) = TVC<sub>3</sub>(T<sub>3</sub>\*) and T\* = T<sub>3</sub>\*.

**Proof:** Appendix.

**DECISION RULE OF THE OPTIMAL CYCLE TIME T\* When M<W/D**

In this section, we will discuss the condition of M<W/D. Equation 1a, b, c will be reduced to

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if } 0 < T < W/D \\ TVC_3(T) & \text{if } W/D \leq T \end{cases} \quad (27a)$$

Equation 5 and 9 yield that

$$TVC_1'(\frac{W}{D}) = \frac{-2A + \frac{W^2}{D}(h + cI_p)}{2(\frac{W}{D})^2} \quad (28)$$

and

$$TVC_3'(\frac{W}{D}) = \frac{[-2A + \frac{W^2}{D}(h + cI_p)] - cDM^2(I_p - I_e)}{2(\frac{W}{D})^2} \quad (29)$$

Furthermore, we let

$$\Delta_1 = -2A + \frac{W^2}{D}(h + cI_p) \quad (30)$$

and

$$\Delta_4 = [-2A + \frac{W^2}{D}(h + cI_p)] - cDM^2(I_p - I_e) \quad (31)$$

From Eq. 30 and 31, we can find that Δ<sub>1</sub> ≥ Δ<sub>4</sub>. Furthermore, we have

$$\Delta_1 > 0 \text{ if and only if } T_1^* < W/D \quad (32)$$

$$\Delta_4 > 0 \text{ if and only if } T_3^* < W/D \quad (33)$$

Therefore, the optimal cycle times can be obtained as follows:

**Theorem 2:**

- If Δ<sub>1</sub>>0 and Δ<sub>4</sub>≥0, then TVC(T\*) = min {TVC<sub>1</sub>(T<sub>1</sub>\*), TVC<sub>3</sub>(W/D)}. Hence T\* is T<sub>1</sub>\* or W/D associated with the least cost.
- If Δ<sub>1</sub>≤0 and Δ<sub>4</sub><0, then TVC(T\*) = TVC<sub>3</sub>(T<sub>3</sub>\*) and T\* = T<sub>3</sub>\*.
- If Δ<sub>1</sub>>0 and Δ<sub>4</sub><0, then TVC(T\*) = min {TVC<sub>1</sub>(T<sub>1</sub>\*), TVC<sub>3</sub>(T<sub>3</sub>\*)}. Hence T\* is T<sub>1</sub>\* or T<sub>3</sub>\* associated with the least cost.

**Proof:** Appendix.

**NUMERICAL EXAMPLES**

To illustrate all results, let us apply the proposed method to solve the following numerical examples. The optimal cycle times and optimal order quantity are shown in Table 1, 2, respectively.

**CONCLUSIONS**

The purpose of this paper is to investigate the effect of supplier credit policies depending on the order quantity

within the Economic Order Quantity (EOQ) framework. Our inventory model generalizes Goyal (1985) and Khouja and Mehrez (1996). Theorem 1 gives the decision rule of the optimal cycle time when  $M \geq W/D$ . However, Theorem 2 does the decision rule of the optimal cycle time when  $M < W/D$ . Finally, numerical examples are used to illustrate all results obtained by this paper.

**ACKNOWLEDGMENTS**

This study is supported by NSC Taiwan, No. NSC 96-2221-E-324-007-MY3 and CYUT.

**APPENDIX**

**Proof of Theorem 1:**

- If  $\Delta_1 > 0, \Delta_2 > 0$  and  $\Delta_3 > 0$ , then  $T_1^* < W/D, T_2^* < W/D, T_2^* < M$  and  $T_3^* < M$ . So, we have

$$TVC_1'(\frac{W}{D}) > 0, TVC_2'(\frac{W}{D}) > 0 \text{ and } TVC_2'(M) = TVC_3'(M) > 0.$$

Equation 14a-c, 15a-c and 16a-c imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is increasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i), (ii) and (iii), we have  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_2(W/D)\}$ . Hence,  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.

- If  $\Delta_1 > 0, \Delta_2 \leq 0$  and  $\Delta_3 > 0$ , then  $T_1^* < W/D, T_2^* \geq W/D, T_2^* < M$  and  $T_3^* < M$ . So, we have

$$TVC_1'(\frac{W}{D}) > 0, TVC_2'(\frac{W}{D}) \leq 0 \text{ and } TVC_2'(M) = TVC_3'(M) > 0.$$

Equation 14a-c, 15a-c and 16a-c imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, T_2^*]$  and increasing on  $[T_2^*, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i), (ii) and (iii), we have  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_2(T_2^*)\}$ . Since  $TVC_1(T) > TVC_2(T)$  for all  $T > 0$ , we obtain  $TVC_1(T_1^*) > TVC_2(T_2^*)$ . Hence  $T^* = T_2^*$ .

- If  $\Delta_1 > 0, \Delta_2 \leq 0$  and  $\Delta_3 \leq 0$ , then  $T_1^* < W/D, T_2^* \geq W/D, T_2^* \geq M$  and  $T_3^* \geq M$ . So, we have

$$TVC_1'(\frac{W}{D}) > 0, TVC_2'(\frac{W}{D}) \leq 0 \text{ and } TVC_2'(M) = TVC_3'(M) \leq 0$$

Equation 14a-c, 15a-c and 16a-c imply that

- (i)  $TVC_3(T)$  is decreasing on  $[M, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i), (ii) and (iii), we have  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_3(T_3^*)\}$ . Hence,  $T^*$  is  $T_1^*$  or  $T_3^*$  associated with the least cost.

- If  $\Delta_1 \leq 0, \Delta_2 \leq 0$  and  $\Delta_3 > 0$ , then  $T_1^* \geq W/D, T_2^* \geq W/D, T_2^* < M$  and  $T_3^* < M$ . So, we have

$$TVC_1'(\frac{W}{D}) \leq 0, TVC_2'(\frac{W}{D}) \leq 0 \text{ and } TVC_2'(M) = TVC_3'(M) > 0.$$

Equation 14a-c, 15a-c and 16a-c imply that

- (i)  $TVC_3(T)$  is increasing on  $[M, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, T_2^*]$  and increasing on  $[T_2^*, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i), (ii) and (iii), we have  $TVC(T^*) = TVC_2(T_2^*)$ . Hence  $T^* = T_2^*$ .

- If  $\Delta_1 \leq 0, \Delta_2 \leq 0$  and  $\Delta_3 \leq 0$ , then  $T_1^* \geq W/D, T_2^* \geq W/D, T_2^* \geq M$  and  $T_3^* \geq M$ . So, we have

$$TVC_1'(\frac{W}{D}) \leq 0, TVC_2'(\frac{W}{D}) \leq 0 \text{ and } TVC_2'(M) = TVC_3'(M) \leq 0.$$

Equation 14a-c, 15a-c and 16a-c imply that

- (i)  $TVC_3(T)$  is decreasing on  $[M, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (ii)  $TVC_2(T)$  is decreasing on  $[W/D, M]$ .
- (iii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i), (ii) and (iii), we have  $TVC(T^*) = TVC_3(T_3^*)$ . Hence  $T^* = T_3^*$ .

**Proof of Theorem 2:**

- If  $\Delta_1 > 0$  and  $\Delta_2 \geq 0$ , then  $T_1^* < W/D$  and  $T_3^* \leq W/D$ . So, we have  $TVC_1'(\frac{W}{D}) > 0$  and  $TVC_3'(\frac{W}{D}) \geq 0$ . Equation

14a-c and 16a-c imply that

- (i)  $TVC_3(T)$  is increasing on  $[W/D, \infty)$ .
- (ii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i) and (ii), we have  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_3(W/D)\}$ . Hence  $T^*$  is  $T_1^*$  or  $W/D$  associated with the least cost.

- If  $\Delta_1 \leq 0$  and  $\Delta_2 < 0$ , then  $T_1^* \geq W/D$  and  $T_3^* > W/D$ . So, we have  $TVC_1'(\frac{W}{D}) \leq 0$  and  $TVC_3'(\frac{W}{D}) < 0$ . Equation 14a-c

and 16a-c imply that

- (i)  $TVC_3(T)$  is decreasing on  $[W/D, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .
- (ii)  $TVC_1(T)$  is decreasing on  $(0, W/D)$ .

Combining (i) and (ii), we have  $TVC(T^*) = TVC_3(T_3^*)$ . Hence  $T^* = T_3^*$ .

- If  $\Delta_1 > 0$  and  $\Delta_2 < 0$ , then  $T_1^* < W/D$  and  $T_3^* > W/D$ . So, we have  $TVC_1'(\frac{W}{D}) > 0$  and  $TVC_3'(\frac{W}{D}) < 0$ . Equation 14a-c

and 16a-c imply that

- (i)  $TVC_3(T)$  is decreasing on  $[W/D, T_3^*]$  and increasing on  $[T_3^*, \infty)$ .

(ii)  $TVC_1(T)$  is decreasing on  $(0, T_1^*]$  and increasing on  $[T_1^*, W/D)$ .

Combining (i) and (ii), we have  $TVC(T^*) = \min \{TVC_1(T_1^*), TVC_3(T_3^*)\}$ . Hence  $T^*$  is  $T_1^*$  or  $T_3^*$  associated with the least cost.

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