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Peristaltic Flow Through a Porous Medium in a Non-Uniform Channel

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Abstract: In this study, the problem of peristaltic flow through a porous medium is studied for the case of non-uniform channel. The problem is formulated and analyzed using a perturbation series of a wave number as a parameter. We obtained an explicit form for the velocity components and the pressure gradient to the second order. Moreover, the pressure rise and the average pressure rise are computed and are explained graphically. The results show that both pressure rise and average pressure rise decrease as permeability parameter k increases.

Key words: Peristalsis, porous medium, non-uniform channel

INTRODUCTION

Peristalsis is a mechanism of pumping fluids in a tube by means of a contractile ring around the tube which pushes the fluid forward. By peristaltic pumping we mean transporting the fluid by a wave of contraction or expansion from a region of lower pressure to higher pressure.

The problem of peristaltic flow in uniform and non uniform channel has been studied by many authors for both Newtonian and non Newtonian fluids. Some of these studies have been done by Abd El Naby *et al.* (2004), Elshehawey and Husseny (2002), Mekheimer (2005), Mishra and Ramachandra (2003), Misra and Pandey (2001) and Elshehawey and Sobh (2001).

Recently, some studies have been done to understand the influence of an inserted endoscope on peristaltic motion of some types of fluids. Some of these studies were made by Abd El Naby and El Misery (2002) and El Misery *et al.* (2003).

The effect of the porous medium on peristaltic transport of a Generalized Newtonian fluid in a uniform planar channel has been studied by Elshehawey *et al.* (2000). Also, Sobh (2004) studied the peristaltic transport of a magneto-Newtonian fluid through a porous medium in uniform channel. It has been shown that the pressure rise increases as the permeability decreases.

Since peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems and most biological organs are generally observed to be non-uniform, we purpose to study the effect of porous medium on peristaltic motion of a Newtonian fluid in a non-uniform channel.

Because of the complexity of the governing equations, we shall consider the case of creeping flow. The problem is formulated and a perturbation solution with wave number as a parameter is obtained to second order. The velocity field and pressure gradient are obtained in explicit forms. Also, the pressure rise per unit wavelength is computed numerically and is plotted with the variation of the time. Moreover, the average pressure rise is graphed versus flow rate for various values of permeability parameter k.

FORMULATION AND ANALYSIS

Consider the two dimensional flow of an incompressible Newtonian fluid in an infinite channel of non-uniform thickness with a sinusoidal wave traveling down its wall. The geometry of the wall surface is defined as:

$$\bar{H}(\bar{X},\bar{t}) = g_1(\bar{X}) + b\sin\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}), \tag{1}$$

With

$$g_1(\bar{X}) = a + K_1\bar{X} \tag{2}$$

where, $g_1(\bar{X})$ is the half width of the channel at any axial distance from the inlet, a is the half width of the channel at the inlet, $K_1 \ll 1$ is constant whose magnitude depends on the length of the channel and exit and inlet dimensions, b the is the amplitude of the wave, λ is the wavelength, c is propagation velocity of the wave, t is the time.

In the moving coordinates $(\bar{x},\bar{y})$ which travel in the $\bar{x}$ direction with the same speed as the wave, the flow is

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steady but in the fixed coordinates, the flow in the channel can be treated as unsteady. The coordinate frames are related through (Shapiro *et al*., 1969).

$$\bar{x} = \bar{X} - c\bar{t}, \qquad \bar{y} = \bar{Y}, \tag{3}$$

$$\bar{u} = \bar{U} - c, \qquad \bar{v} = \bar{V}, \tag{4}$$

where, $(\bar{U},\bar{V})$ and $(\bar{u},\bar{v})$ are the velocity components in the fixed and the moving frames, respectively.

Using the following non-dimensional parameters

$$X = \frac{\bar{X}}{\lambda}, Y = \frac{\bar{Y}}{a}, t = \frac{c\bar{t}}{\lambda}, P = \frac{a^2\bar{P}}{c\lambda\mu}, U = \frac{\bar{U}}{c}, V = \frac{\lambda\bar{V}}{ac}, Re = \frac{\rho ca}{\mu}, k = \frac{\bar{k}}{a^2}, \delta = \frac{a}{\lambda},$$

and

$$H = \frac{\bar{H}}{a} = 1 + \frac{\lambda K_1 X}{a} + \varphi \sin 2\pi(X - t), \tag{5}$$

where, δ is the wave number, Re is the Reynolds number, and φ is the amplitude ratio, $\varphi = b/a<1$, the non-dimensional equations of motion are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{6}$$

$$Re\,\delta\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial p}{\partial X} + \delta^2\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{k}, \tag{7}$$

$$Re\,\delta^3\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial p}{\partial Y} + \delta^2\left(\delta^2\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \delta^2\frac{V}{k}, \tag{8}$$

Considering the creeping flow (Re = 0), taking into account that δ is small and eliminating the pressure from Eq. 7 and 8, we obtain the following system of differential equations:

$$0 = -\frac{\partial p}{\partial X} + \delta^2\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{k}, \tag{9}$$

$$0 = \frac{\partial^3 U}{\partial Y^3} + \delta^2\left(\frac{\partial^3 U}{\partial Y\partial X^2} - \frac{\partial^3 V}{\partial X\partial Y^2}\right) - \delta^4\frac{\partial^3 V}{\partial X^3} - \frac{1}{k}\frac{\partial U}{\partial Y} + \frac{\delta^2}{k}\frac{\partial V}{\partial X}, \tag{10}$$

with the continuity Eq. 6.

The nom-dimensional boundary conditions are:

$$\frac{\partial U}{\partial Y} = 0, \quad V = 0 \quad \text{for} \quad Y = 0, \tag{11a}$$

$$U = 0, \quad V = \frac{\partial H}{\partial X} \quad \text{for} \quad Y = H. \tag{11b}$$

RATE OF VOLUME FLOW

The instantaneous volume flow rate in the stationary frame is given by:

$$Q(\bar{X},\bar{t}) = \int_0^{\bar{H}} \bar{U}(\bar{X},\bar{Y},\bar{t})d\bar{Y}, \tag{12}$$

where, $\bar{H}$ is a function of $\bar{X}$ and $\bar{t}$.

On substituting Eq. 3 and 4 into Eq. 12 and then integrating, one finds:

$$Q = \bar{q} + c\bar{H}, \tag{13}$$

Where:

$$\bar{q} = \int_0^{\bar{H}} \bar{u}(\bar{x},\bar{y})d\bar{y}, \tag{14}$$

is the rate of volume flow in the moving frame (wave frame) and independent of time. Here $\bar{H}$ is a function of $\bar{X}$ only.

The time-mean flow over a period $T = \lambda/c$ at a fixed position $\bar{X}$ is defined as:

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, d\bar{t}. \tag{15}$$

Substituting from Eq. 13 into Eq. 15 and using Eq. 1 and integrating, we obtain

$$\bar{Q} = \bar{q} + ac\sin\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}), \tag{16}$$

Defining the dimensionless time-mean flows Θ and F as follows:

$$\Theta = \frac{\bar{Q}}{ac} \text{ and } F = \frac{\bar{q}}{ac}, \tag{17}$$

Equation 16 can be rewritten as:

$$F = \Theta + \varphi\sin 2\pi(X - t), \tag{18}$$

Where:

$$F = \int_0^{H(x)} U\, dY. \tag{19}$$

METHOD OF SOLUTION

We expand the following quantities in a power series of the small parameter δ as follows:

$$U = U_0 + \delta U_1 + \delta^2 U_2 + O(\delta^3)$$
$$V = V_0 + \delta V_1 + \delta^2 V_2 + O(\delta^3)$$

$$\frac{\partial p}{\partial X} = \frac{\partial p_0}{\partial X} + \delta\left(\frac{\partial p_1}{\partial X}\right) + \delta^2\left(\frac{\partial p_2}{\partial X}\right) + O(\delta^3)$$

$$F = F_0 + \delta F_1 + \delta^2 F_2 + O(\delta^3)\,. \tag{20}$$

The use of expansions (20) with Eq. 6, 9, 10 and 11 gives the following systems:

System of order zero

$$\frac{\partial U_0}{\partial X} + \frac{\partial V_0}{\partial Y} = 0, \tag{21}$$

$$\frac{\partial p_0}{\partial X} = \frac{\partial^2 U_0}{\partial Y^2} - \frac{U_0}{k}, \tag{22}$$

$$\frac{\partial^3 U_0}{\partial Y^3} - \frac{1}{k}\frac{\partial U_0}{\partial Y} = 0, \tag{23}$$

With the dimensionless boundary conditions

$$\frac{\partial U_0}{\partial Y} = 0, \quad V_0 = 0 \ \text{ for } \ Y = 0, \tag{24a}$$

$$U_0 = 0, \quad V_0 = \frac{\partial H}{\partial X} \ \text{ for } \ Y = H. \tag{24b}$$

The solution of this system for U_0 subject to the boundary conditions is:

$$U_0 = k\left(\frac{\partial p_0}{\partial X}\right)\left[\frac{\cosh\left(\frac{Y}{\sqrt{k}}\right)}{\cosh\left(\frac{H}{\sqrt{k}}\right)} - 1\right]. \tag{25}$$

The instantaneous volume flow rate F_0 in the moving coordinates is given by:

$$F_0 = \int_0^H U_0\, dY = k\left(\frac{\partial p_0}{\partial X}\right)\left[\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right) - H\right],$$

which implies that

$$\frac{\partial p_0}{\partial X} = \frac{F_0}{k^{3/2}\tanh\left(\frac{H}{\sqrt{k}}\right) - kH}. \tag{26}$$

Using Eq. 21, 25 and 26, we obtain the alternative form of U_0 and V_0 as:

$$U_0 = c_0 + c_1 \cosh\left(\frac{Y}{\sqrt{k}}\right), \tag{27a}$$

$$V_0 = -c_0' Y - c_1' \sqrt{k}\sinh\left(\frac{Y}{\sqrt{k}}\right), \tag{28b}$$

Where:

$$c_0 = -a_0 F_0 \cosh\left(\frac{H}{\sqrt{k}}\right), \quad c_1 = a_0 F_0,$$

and

$$a_0 = \left[\sqrt{k}\sinh\left(\frac{H}{\sqrt{k}}\right) - H\cosh\left(\frac{H}{\sqrt{k}}\right)\right]^{-1}. \tag{29}$$

System of order one: Equating the coefficients of δ on both sides in Eq. 6, 9, 10 and 11, we get

$$\frac{\partial U_1}{\partial X} + \frac{\partial V_1}{\partial Y} = 0, \tag{30}$$

$$\frac{\partial p_1}{\partial X} = \frac{\partial^2 U_1}{\partial Y^2} - \frac{U_1}{k}, \tag{31}$$

$$\frac{\partial^3 U_1}{\partial Y^3} - \frac{1}{k}\frac{\partial U_1}{\partial Y} = 0, \tag{32}$$

with the boundary conditions

$$\frac{\partial U_1}{\partial Y} = 0, \qquad V_1 = 0 \ \text{ for } \ Y = 0, \tag{33a}$$

$$U_1 = 0, \qquad V_1 = 0 \ \text{ for } \ Y = H. \tag{33b}$$

Solving this system for U_1, after using Eq. 27, we obtain

$$U_1 = k\left(\frac{\partial p_1}{\partial X}\right)\left[\frac{\cosh\left(\frac{Y}{\sqrt{k}}\right)}{\cosh\left(\frac{H}{\sqrt{k}}\right)} - 1\right]. \tag{34}$$

Again, the alternative form for U_1, in which $\partial p_1/\partial X$ is replaced by an equivalent expression in term of F_1, is given by:

$$U_1 = c_2 + c_3 \cosh\left(\frac{Y}{\sqrt{k}}\right), \tag{35}$$

Where:

$$c_2 = -a_0 F_1 \cosh\left(\frac{H}{\sqrt{k}}\right), \quad c_3 = a_0 F_1 \tag{36}$$

$$F_1 = \int_0^H U_1\, dY = k\left(\frac{\partial p_1}{\partial X}\right)\left[\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right) - H\right],$$

and

$$\frac{\partial p_1}{\partial X} = \frac{F_1}{k^{3/2}\tanh\left(\frac{H}{\sqrt{k}}\right) - kH}. \tag{37}$$

Using the continuity equation, one finds

$$V_1 = c_2' y - c_3'\sqrt{k}\sinh\left(\frac{Y}{\sqrt{k}}\right), \tag{38}$$

System of order two: Equating the coefficients of δ^2 on both sides in Eq. 6, 9, 10 and 11, we get

$$\frac{\partial U_2}{\partial X} + \frac{\partial V_2}{\partial Y} = 0, \tag{39}$$

$$\frac{\partial p_2}{\partial X} = \frac{\partial^2 U_2}{\partial Y^2} + \frac{\partial^2 U_0}{\partial X^2} - \frac{U_2}{k}, \tag{40}$$

$$\frac{\partial^3 U_2}{\partial Y^3} - \frac{1}{k}\frac{\partial U_2}{\partial Y} = \frac{\partial^3 V_0}{\partial X \partial Y^2} - \frac{\partial^3 U_0}{\partial Y \partial X^2} - \frac{1}{k}\frac{\partial V_0}{\partial X}, \tag{41}$$

with the boundary conditions

$$\frac{\partial U_2}{\partial Y} = 0, \qquad V_2 = 0 \quad \text{for} \quad Y = 0, \tag{42a}$$

$$U_2 = 0, \qquad V_2 = 0 \quad \text{for} \quad Y = H. \tag{43b}$$

Solving this system for U_2, after using Eq. 27, we obtain

$$U_2 = k\left(\frac{\partial p_2}{\partial X} - c_0''\right)\left[\frac{\cosh\left(\frac{Y}{\sqrt{k}}\right)}{\cosh\left(\frac{H}{\sqrt{k}}\right)} - 1\right] + c_0''\frac{(H^2+2k)\cosh\left(\frac{Y}{\sqrt{k}}\right)}{2\cosh\left(\frac{H}{\sqrt{k}}\right)}$$

$$+ c_1''\frac{\sqrt{k}H}{2}\tanh\left(\frac{H}{\sqrt{k}}\right)\cosh\left(\frac{Y}{\sqrt{k}}\right) - c_1''\frac{\sqrt{k}Y}{2}\sinh\left(\frac{Y}{\sqrt{k}}\right) - \frac{c_0''}{2}(Y^2+2k). \tag{44}$$

The instantaneous volume flow rate F_2 is given by:

$$F_2 = \int_0^H U_2 dY = \left[\left(\frac{\partial p_2}{\partial X}\right) - c_0''\right]\left[k^{3/2}\tanh\left(\frac{H}{\sqrt{k}}\right) - kH\right] + c_0''\frac{(H^2+2k)}{2}\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)$$

$$+ c_1''\frac{kH}{2}\tanh\left(\frac{H}{\sqrt{k}}\right)\sinh\left(\frac{Y}{\sqrt{k}}\right) - c_1''\frac{kH}{2}\cosh\left(\frac{H}{\sqrt{k}}\right) + \frac{c_1''k^{3/2}}{2}\sinh\left(\frac{H}{\sqrt{k}}\right).$$

Solving the above equation for $\partial p_2/\partial X$, we get

$$\frac{\partial p_2}{\partial X} = b_0 F_2 - \frac{c_1'' b_0 k}{2}H\tanh\left(\frac{H}{\sqrt{k}}\right)\sinh\left(\frac{H}{\sqrt{k}}\right) - \frac{b_0 c_0''}{2}(H^2+2k)\sqrt{k}\tanh\left(\frac{H}{\sqrt{k}}\right)$$

$$+ \frac{b_0 c_0''}{6}H(H^2+6k) + \frac{b_0 c_1''}{2}kH\cosh\left(\frac{H}{\sqrt{k}}\right) - \frac{c_1'' b_0 k^{3/2}}{2}\sinh\left(\frac{H}{\sqrt{k}}\right) + c_0'', \tag{45}$$

Where:

$$b_0 = \left[k^{3/2}\tanh\left(\frac{H}{\sqrt{k}}\right) - kH\right]^{-1} \tag{46}$$

Substituting Eq. 45 into Eq. 44 we obtain the alternative form of U_2 as:

$$U_2 = c_4 + c_5 Y^2 + c_6\cosh\left(\frac{Y}{\sqrt{k}}\right) + c_7 Y\sinh\left(\frac{Y}{\sqrt{k}}\right), \tag{47}$$

Where:

$$c_4 = -kb_0F_2 + \frac{c_1''b_0k^2}{2}H\tanh\left(\frac{H}{\sqrt{k}}\right)\sinh\left(\frac{H}{\sqrt{k}}\right) + \frac{b_0c_0''k^{3/2}}{2}(H^2+2k)\tanh\left(\frac{H}{\sqrt{k}}\right) - \frac{b_0c_0''k}{6}H(H^2+6k)$$

$$- \frac{b_0c_1''}{2}k^2H\cosh\left(\frac{H}{\sqrt{k}}\right) + \frac{c_1''b_0k^{5/2}}{2}\sinh\left(\frac{H}{\sqrt{k}}\right) - c_0''k, \tag{48}$$

$$c_5 = -\frac{c_0''}{2},\quad c_6 = -\frac{(c_0''H^2 - 2c_4)}{2\cosh\left(\frac{H}{\sqrt{k}}\right)} + \frac{c_1''\sqrt{k}H}{2}\tanh\left(\frac{H}{\sqrt{k}}\right),$$

$$c_7 = -\frac{c_1''\sqrt{k}}{2}$$

Now, the axial velocity component U and the pressure gradient can be expressed, to second order where $F_0 = F - \delta F_1 - \delta^2 F_2$, as:

$$U = c_0 + c_1\cosh\left(\frac{Y}{\sqrt{k}}\right) + \delta\left[c_2 + c_3\cosh\left(\frac{Y}{\sqrt{k}}\right)\right] + \delta^2\left[c_4 + c_5Y^2 + c_6\cosh\left(\frac{Y}{\sqrt{k}}\right) + c_7Y\sinh\left(\frac{Y}{\sqrt{k}}\right)\right] \tag{49}$$

$$\frac{dp}{dX} = b_0F + \delta^2F\left[c_1'' + \frac{b_0}{6}\left\{c_1''H^3 - 3\sqrt{k}\,c_1''H^2\tanh\left(\frac{H}{\sqrt{k}}\right) + 3kc_0''H\,\mathrm{sech}\left(\frac{H}{\sqrt{k}}\right) + 6c_1''kH - 3k^{3/2}c_0''\sinh\left(\frac{H}{\sqrt{k}}\right) + 6k^{3/2}c_1''\tanh\left(\frac{H}{\sqrt{k}}\right)\right\}\right] \tag{50}$$

The pressure rise per wave length $\Delta p_\lambda(t)$ is given by:

$$\Delta p_\lambda(t) = \int_0^1 \frac{dp}{dX}dX \tag{51}$$

RESULTS AND DISCUSSION

It is clear that we have obtained analytical form, to second order of δ, for the velocity field and the pressure gradient, Eq. 49 and 50. As k tends to infinity, we obtain

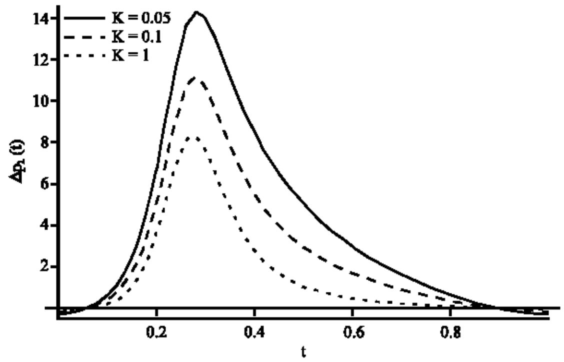

Fig. 1: The pressure rise versus the time at $\delta = 0.02$, $\varphi = 0.8$ and $\Theta = 0$

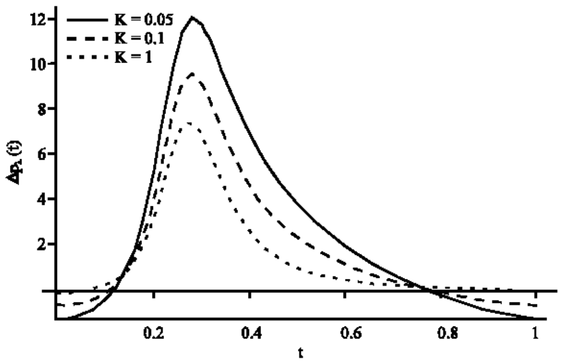

Fig. 2: The pressure rise versus the time at $\delta = 0.02$, $\varphi = 0.8$ and $\Theta = 0.1$

the velocity field and the pressure gradient of peristaltic flow of Newtonian fluid in non-uniform channel.

We shall now compute the dimensionless pressure rise $\Delta p_\lambda(t)$ over the channel length for different values of the dimensionless flow rate Θ and permeability parameter k.

The average pressure rise $\overline{\Delta p_\lambda}(t)$ is then evaluated by averaging $\Delta p_\lambda(t)$ over one period of the wave. As the integral in Eq. 51 is not integrable in the closed form, it is evaluated numerically using the MATHEMATICA package. Following Srivastava and Srivastava (1984), we take $K_1 = 3a/\lambda$ and then plot Eq. 51.

Figure 1-3 represent the variation of dimensionless pressure rise with the dimensionless time for $\delta = 0.02$, $\varphi = 0.8$, (k = 0.05, 0.1, 1) and $\Theta = 0$, 0.1 and 0.2, respectively. We have observed that the pressure rise decreases with increasing flow rate and becomes maximal at zero flow rate. Also, it is clear from these figures that the pressure rise decreases with increasing permeability parameter k. Also, the behavior of the pressure rise is the same for various values of flow rate Θ.

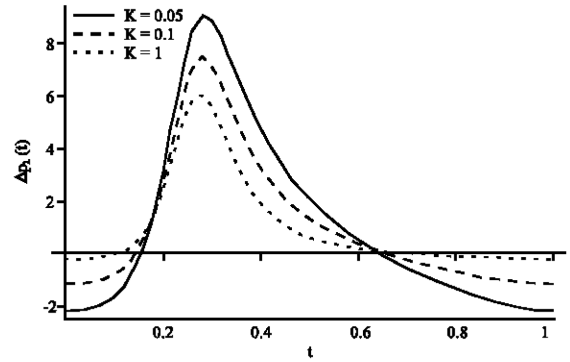

Fig. 3: The pressure rise versus the time at $\delta = 0.02$, $\varphi = 0.8$ and $\Theta = 0.2$

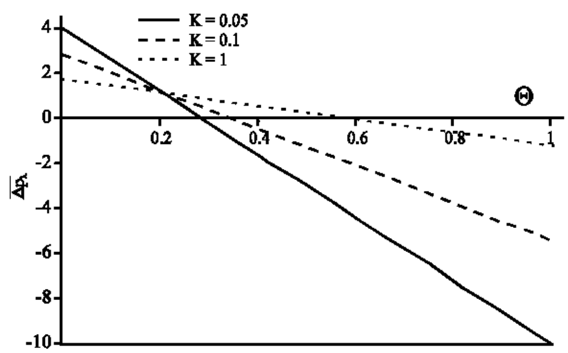

Fig. 4: The average pressure rise versus flow rate at $\delta = 0.02$ and $\varphi = 0.8$

The average pressure rise versus the flow rate is plotted in Fig. 4 for $\delta = 0.02$, $\varphi = 0.8$, (k = 0.05, 0.1, 1). As shown, the average pressure rise decreases as the permeability parameter increases. This is because of the resistance caused by the porous medium.

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