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On Hermite Matrix Polynomials of Two Variables

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Abstract: This study deals with the two-variable Hermite matrix polynomials, some relevant matrix functions appear interims of the two-variable Hermite matrix polynomials the relationships with Hermite matrix polynomials of one variable, Chepyshev matrix polynomials of the second kind have been obtained and expansion of the. Gegenbauer matrix polynomials as series of Hermite matrix polynomials.

Key words: Hermite matrix polynomials, recurrence relation, Rodrigue's formula, Chepyshev matrix polynomials and Gegenbauer matrix polynomials

INTRODUCTION

Laguerre, Hermite and Gegenbauer matrix polynomials were introduced and studied (Defez and Jo'dar, 1998; Msayyed *et al*., 2004; Jo'dar *et al*., 1994), for matrix in $C^{N\times N}$. Moreover, some properties of the Hermite matrix polynomials are given (Defez and Jo'dar, 1998; Defez *et al*., 2002) and a generalized form of the Hermite matrix polynomials has been introduced and studied in (Msayyed *et al*., 2003).

Jo'dar and Company (1996) introduced the class of Hermite matrix polynomials $H_n(x, A)$ which appear as finite series solutions of second order matrix differential equations $Y''-xAY+nAY = 0$, for a matrix A in $C^{N\times N}$ whose eigen values are all in right open half-plane. If A is a matrix in $C^{N\times N}$, it spectrum $\sigma(A)$ denotes the set of all eigenvalues of A. If f(z), g(z) are holomorphic functions in an open set Ω of the complex plane and if $\sigma(A)\subset\Omega$ we denote by f(A), g(A), respectively, the image by the Riesez-Dunford functional calculus of the functions f(z), g(z), respectively, acting on the matrix A and

$$f(A)\, g(A) = g(A)\, f(A)$$

If D_0 is the complex plane cut along the negative real axis and log(z) denotes the principal logarithm of z, then $z^{1/2}$ represents exp(1/2log(z)). If A is a matrix in $C^{N\times N}$ with $\sigma(A)\subset D_0$ then $A^{1/2}=\sqrt{A}$ denotes the image by $z^{1/2}$ of the matrix functional calculus acting on the matrix A.

Let A is a matrix in $C^{N\times N}$ such that,

$$\mathrm{Re}(z)>0 \quad \text{For all } z \in \sigma(A) \tag{1}$$

then n^{th} Hermite matrix polynomial $H_n(x, A)$ is defined by Jo'dar and Company (1996)

$$H_n(x,A)=n!\sum_{n=0}^{[n/2]}\frac{(-1)^k\left(x\sqrt{2A}\right)^{n-2k}}{k!(n-2k)!}\quad ,\ n\geq 0 \tag{2}$$

and the following Rodrigues formula holds

$$H_n(x,A)=\exp\left(\frac{Ax^2}{2}\right)(-1)^n\left(\frac{A}{2}\right)^{-n/2}\left[\frac{d^n}{dx^n}\exp\left(\frac{Ax^2}{2}\right)\right],\quad n\geq 0 \tag{3}$$

and satisfy the three terms recurrence relation ship.

$$\begin{aligned}&H_n(x,A)=xI\sqrt{2A}\,H_{n-1}(x,A)-2(n-1)\\&H_{n-2}(x,A) \qquad\qquad n\geq 1\\&H_{-1}(x,A)=0,\qquad H_0(x,A)=I\end{aligned} \tag{4}$$

where, I is the identity matrix in $C^{R\times R}$.
by Jo'dar and Company (1996) we also have

$$e^{xt\sqrt{2A}-t^2I}=\sum_{n\geq 0}^{\infty}\frac{1}{n!}H_n(x,A)t^n \qquad |t|<+\infty \tag{5}$$

Batahan (2006) define the two-variable Hermite matrix polynomials by

$$H_n(x,y,A)=n!\sum_{n=0}^{\left[\frac{n}{2}\right]}\frac{(-1)^k\left(x\sqrt{2A}\right)^{n-2k}y^k}{k!(n-2k)!} \tag{6}$$

and satisfy the recurrence relation ship.

$$\begin{aligned}&H_0(x,y,A)=I,\ H_1(x,y,A)=x\sqrt{2A}\\&H_n(x,y,A)=y^{n/2}H_n\left(x/\sqrt{y},A\right),\\&H_n(x,1,A)=H_n(x,A)\end{aligned} \tag{7}$$

where, $H_n(x, A)$ is defined in (2)
we shall use the relations (Defez and Jo'dar, 1998; Jo'dar and Company, 1996)

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A(k,n)=\sum_{n=0}^{\infty}\sum_{k=0}^{n}A(k,n-k) \tag{8}$$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A(k,n)=\sum_{n=0}^{\infty}\sum_{k=0}^{[n/2]}A(k,n-2k) \qquad (9)$$

where, A (k, n) is a matrix on $C^{N\times N}$ and the relation (Msayyed *et al*., 2004)

$$\frac{(-1)^k}{(n-k)!}I=\frac{(-n)_k}{n!}I=\frac{(-nI)_k}{n!}, \qquad 0\le k\le n \qquad (10)$$

Khan and Abukahmmash (1998) obtained the generating function for $H_n(x, y)$ by

$$\sum_{n=0}^{\infty}\frac{H_n(x,y)t^n}{n!}=e^{2xt-(y+1)t^2} \qquad (11)$$

for the Hermite polynomials of two variables

$$H_n(x,y)=\sum_{r=0}^{[n/2]}\frac{n!(-y)^r H_{n-2r}(x)}{r!(n-2r)!} \qquad (12)$$

where, $H_n(x)$ is the well known Hermite polynomial of one-variable and it's equivalent to the following explicit representation of $H_n(x, y)$ by

$$H_n(x,y)=\sum_{r=0}^{\left[\frac{n}{2}\right]}\sum_{s=0}^{\left[\frac{n}{2}-r\right]}\frac{(-n)_{2r+2s}(-1)^s(2x)^{n-2r-2s}(-y)^r}{r!s!} \qquad (13)$$

Kahmmash (2007) define Gegenbauer matrix polynomials of two variables by

$$C_{n,k}^{A}(x,y)=\sum_{r=0}^{[n/2]}\sum_{j=0}^{[k/2]}\frac{(-1)^{r+j}(A)_{n+k-r-j}2^{n+k}x^{n-2r}y^{k-2r}}{r!j!(n-2r)!(k-2j)!} \qquad (14)$$

where, $C_{n,k}^{A}(x,y)$ is a polynomial in two variables x and y of degree n in x and k in y thus $C_{n,k}^{A}(x,y)$ is a polynomial in two variables x and y of degree n+k.

The aim of this study is to establish the two variable extension of the Hermite matrix polynomials and the generating function for these matrix polynomials and Rodrigues formula, expansion series of Hermite matrix polynomials. Chepyshev matrix polynomial of the second kind and expansion of the Gegenbauer matrix polynomials as series of Hermite matrix polynomials.

HERMITE MATRIX POLYNOMIALS OF TWO VARIABLES

Let A be a matrix in $C^{N\times N}$ satisfying the condition (1). we define two variable Hermite matrix polynomials by

$$H_n(x,y,t)=\exp\left(xt\sqrt{2A}-(y+1)t^2I\right)=\sum_{n\ge 0}\frac{H_n(x,y,A)t^n}{n!}, \qquad |t^n|<\infty \qquad (15)$$

we can write,

$$\exp\left(xt\sqrt{2A}-(y+1)t^2I\right)=\sum_{n,k=0}^{\infty}\frac{(-1)^k(x)^n\left(\sqrt{2A}\right)^n(y+1)^k t^{n+2k}}{n!k!} \qquad (16)$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{(-1)^k(x)^{n-2k}\left(\sqrt{2A}\right)^{n-2k}(y+1)^k t^n}{n!\,k!}$$

$$H_n(x,y,A)=\sum_{k=0}^{\left[\frac{n}{2}\right]}\frac{n!(-1)^k(x)^{n-2k}\left(\sqrt{2A}\right)^{n-2k}(y+1)^k}{(n-2k)!\,k!} \qquad (17)$$

for y = 0, (17) reduces to Hermite matrix polynomial $H_n(x, A)$ of one variable (Jo'dar and Company, 1996)

RECURRENCE RELATIONS

Now, since

$$\exp\left(xt\sqrt{2A}-(y+1)t^2I\right)=\sum_{n\ge 0}\frac{H_n(x,y,A)t^n}{n!} \qquad (18)$$

differentiating (18) partially w.r.t. y, we get

$$-t^2Ie^{2xt-(y+1)t^2I}=\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{\partial}{\partial y}H_n(x,y,A) \qquad (19)$$

$$-\sum_{n=0}^{\infty}\frac{1}{n!}H_n(x,y,A)t^{n+2}=\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{\partial}{\partial y}H_n(x,y,A) \qquad (20)$$

which with a shift of index on left yields,

$$\frac{\partial}{\partial y}H_0(x,y,A)=0,\ \frac{\partial}{\partial y}H_1(x,y,A)=0 \text{ and for } n\ge 2$$
$$-\frac{H_{n-2}(x,y,A)}{(n-2)!}=\frac{1}{n!}\frac{\partial}{\partial y}H_n(x,y,A) \qquad (21)$$

iteration of (21), gives

$$\frac{\partial^r}{\partial y^r}H_n(x,y,A)=\frac{(-1)^r n!H_{n-2r}(x,y,A)}{(n-2r)!} \qquad (22)$$

differentiating (22) partially w.r.t. x, we get.

$$\sqrt{2A}\,t\,e^{2xt-(y+1)t^2I}=\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{\partial}{\partial x}H_n(x,y,A)$$
$$\sum_{n=0}^{\infty}\frac{\sqrt{2A}H_{n-1}(x,y,A)t^{n+1}}{(n-1)!}=\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{\partial}{\partial x}H_n(x,y,A) \quad (23)$$

which with a shift of index on left yields,

$$\frac{\partial}{\partial x}H_0(x,y,A)=0 \text{ and for } n\geq 1$$

$$\frac{\sqrt{2A}H_{n-1}(x,y,A)}{(n-1)!}=\frac{1}{n!}\frac{\partial}{\partial x}H_n(x,y,A)$$

or

$$\frac{\partial}{\partial x}H_n(x,y,A)=n\sqrt{2A}H_{n-1}(x,y,A) \quad (24)$$

iteration of (24), gives

$$\frac{\partial^k}{\partial x^k}H_n(x,y,A)=\frac{(\sqrt{2A})^k n!}{(n-k)!}H_{n-k}(x,y,A) \quad (25)$$

from (20), we get

$$\frac{\partial}{\partial y}H_n(x,y,A)=-n(n-1)H_{n-2}(x,y,A) \quad (26)$$

let k = 2, in (25), we get

$$\frac{\partial^2}{\partial x^2}H_n(x,y,A)=\frac{(\sqrt{2A})^2 n!}{(n-2)!}H_{n-2}(x,y,A)$$

(26), yields

$$\frac{\partial^2}{\partial x^2}H_n(x,y,A)+2A\frac{\partial}{\partial y}H_n(x,y,A)=0 \quad (27)$$

Equation 22, 25 and 27 are similar to the results given by Batahan (2006) differentiating (18) partially w.r.t. wt, we get.

$$2\sqrt{2A}\,x-2(y+1)tI\,e^{(2xt\sqrt{2A}-(y+1)t^2I)}=\sum_{n=1}^{\infty}\frac{H_n(x,y,A)t^{n-1}}{(n-1)!} \quad (28)$$

multiplying (23), (19) and (28), by $\left(x-\left(tI/\sqrt{2A}\right)\right)$, 2y and -t, respectively and adding, we get

$$\left(x-\frac{tI}{\sqrt{2A}}\right)\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{\partial}{\partial x}H_n(x,y,A)-y\sum_{n=0}^{\infty}\frac{t^n}{n!}\frac{\partial}{\partial y}H_n(x,y,A)$$
$$-t\sum_{n=1}^{\infty}\frac{t^{n-1}}{(n-1)!}H_n(x,y,A)=0$$

or

$$\sum_{n=0}^{\infty}\frac{xt^n}{n!}\frac{\partial}{\partial x}H_n(x,y,A)-\frac{I}{\sqrt{2A}}\sum_{n=1}^{\infty}\frac{t^n}{(n-1)!}\frac{\partial}{\partial y}H_n(x,y,A)$$
$$-\sum_{n=1}^{\infty}\frac{2yt^n}{n!}\frac{\partial}{\partial y}H_n(x,y,A)=\sum_{n=0}^{\infty}\frac{t^{n+1}}{n!}\frac{\partial}{\partial x}H_n(x,y,A)$$

equating the coefficients of t^n, we get

$$\frac{x}{n!}\frac{\partial}{\partial x}H_n(x,y,A)-\frac{I}{\sqrt{2A}(n-1)!}H_n(x,y,A)$$
$$-\frac{y}{n!}\frac{\partial}{\partial y}H_n(x,y,A)=\frac{1}{(n-1)!}\frac{\partial}{\partial x}H_{n-1}(x,y,A)$$

or

$$x\frac{\partial}{\partial x}H_n(x,y,A)-\frac{nI}{\sqrt{2A}}H_n(x,y,A)$$
$$-y\frac{\partial}{\partial y}H_n(x,y,A)=n\frac{\partial}{\partial x}H_{n-1}(x,y) \quad (29)$$

combination of (24), (26) and (29) yields

$$n\sqrt{2A}x\,H_{n-1}(x,y,A)-\frac{nI}{\sqrt{2A}}H_n(x,y,A)+2n(n-1)yH_{n-2}(x,y,A)$$
$$=n\frac{\partial}{\partial x}H_{n-1}(x,y) \quad (30)$$

from (29) and (30) we obtain the pure recurrence relation

$$n\sqrt{2A}x\,H_{n-1}(x,y,A)-\frac{nI}{\sqrt{2A}}H_n(x,y,A)$$
$$=n(n-1)\left(2y+\sqrt{2A}\right)H_{n-2}(x,y,A) \quad (31)$$

from (24) and (29) the partial differential Equation given by

$$x\frac{\partial}{\partial x}H_n(x,y,A)-\frac{nI}{\sqrt{2A}}H_n(x,y,A)$$
$$-y\frac{\partial}{\partial y}H_n(x,y,A)=n\frac{\partial}{\partial x}H_{n-1}(x,y)$$

or

$$\frac{1}{\sqrt{2A}}\frac{\partial^2}{\partial x^2}H_n(x,y,A)-x\frac{\partial}{\partial x}H_n(x,y,A)$$
$$-\frac{nI}{\sqrt{2A}}\frac{\partial}{\partial y}H_n(x,y,A)-2y\frac{\partial}{\partial y}H_n(x,y,A)=0$$
$$\frac{\partial^2}{\partial x^2}H_n(x,y,A)-\sqrt{2A}x\frac{\partial}{\partial x}H_n(x,y,A)$$
$$-nI\,H_n(x,y,A)-2\sqrt{2A}y\frac{\partial}{\partial y}H_n(x,y,A)=0 \quad (32)$$

RELATIONSHIPS BETWEEN H_n (x, y, A) AND H_n (x, A)

Since,

$$e^{\left(xt\sqrt{2A}-t^2 I\right)}=\sum_{n=0}^{\infty}\frac{H_n(x,A)t^n}{n!} \quad (33)$$

where, H_n (x, A) is well known Hermite matrix polynomial of one variable (Jo'dar and Company, 1996) replacing x by $x/\sqrt{y+1}$ and t by $\sqrt{y+1}t$ in (33), we get

$$e^{\left(2xt\sqrt{2A}-\sqrt{y+1}t^2 I\right)}=\sum_{n=0}^{\infty}\frac{H_n\left(\frac{x}{\sqrt{y+1}},A\right)\left(\sqrt{y+1}t\right)^n}{n!} \quad (34)$$

in view of (15), we get

$$\sum_{n=0}^{\infty}\frac{H_n(x,y,A)t^n}{n!}=\sum_{n=0}^{\infty}\frac{H_n\left(\frac{x}{\sqrt{y+1}},A\right)(y+1)^{\frac{n}{2}}t^n}{n!}$$

equating coefficients of t", we get

$$H_n(x,y,A)=(y+1)^{\frac{n}{2}}H_n\left(\frac{x}{\sqrt{y+1}},A\right) \quad (35)$$

now

$$H_n(-x,y,A)=(-1)^n H_n\left(\frac{x}{\sqrt{y+1}},A\right) \quad (36)$$

for y = 0, (35) reduces

$$H_n(x,0,A)=(-1)^n H_n(x,A) \quad (37)$$

and for x = 0, we get

$$H_n(0,y,A)=(y+1)^{\frac{n}{2}}H_n(0,A) \quad (38)$$

for x = y = 0, we get

$$H_n(0,0,A)=H_n(0,A) \quad (39)$$

THE RODRIGUES FORMULA

Examination of the defining relation

$$e^{\left(xt\sqrt{2A}-(y+1)t^2 I\right)}=\sum_{n=0}^{\infty}\frac{H_n(x,y,A)t^n}{n!} \quad (40)$$

in the light of maclaurin's theorem gives us at once.

$$H_n(x,y,A)=\left[\frac{d^n}{dt^n}e^{xt\sqrt{2A}-(y+1)t^2 I}\right]_{t=0} \quad (41)$$

the function $e^{-\frac{x^2(2A)}{2(y+1)}}$ is independent of t. So we may write

$$e^{-\frac{Ax^2}{2(y+1)}}H_n(x,y,A)=\left[\frac{d^n}{dt^n}e^{\left(\frac{x\sqrt{2A}}{2\sqrt{y+1}}-\sqrt{y+1}tI\right)^2}\right]_{t=0}$$

now put $\frac{x\sqrt{2A}}{2\sqrt{y+1}}-\sqrt{y+1}tI=\omega$ then

$$\left(\frac{x\sqrt{2A}}{2\sqrt{y+1}}-\sqrt{y+1}tI\right)^2=(\omega)^2$$

then

$$\left(\frac{2A}{4}\right)\left[\frac{x}{\sqrt{y+1}}1-(A/2)^{-1/2}\sqrt{y+1}tI\right]^2=\omega^2$$

or

$$\left(\frac{A}{2}\right)\left[\frac{x}{\sqrt{y+1}}I-\frac{1}{\sqrt{2A}}\sqrt{y+1}tI\right]^2=\omega^2$$

$$e^{-\frac{Ax^2}{2(y+1)}}H_n(x,y,A)=(-1)^n(y+1)^{\frac{n}{2}}\left(\frac{A}{2}\right)^{-\frac{n}{2}}\left[\frac{d^n}{dt^n}e^{-\omega^2}\right]_{\omega=\frac{x}{\sqrt{y+1}}}$$

or

$$H_n(x,y,A)=(-1)^n\,(y+1)^{n/2}\left(\frac{A}{2}\right)^{-\frac{n}{2}}e^{\frac{Ax^2}{2(y+1)}}\frac{d^n}{d\omega^n}e^{-\frac{Ax^2}{2(y+1)}} \quad (42)$$

a formula of the same nature as Rodrigue's formula for Hermite matrix polynomial of one variablenote, setting y = 0 in (42) it gives the Rodrigue's formula (Jo'dar and Company, 1996) of H_n = (x, A),

$$H_n(x,A)=e^{\frac{Ax^2}{2}}(-1)^n\left(\frac{A}{2}\right)^{-\frac{n}{2}}\left[\frac{d^n}{dx^n}e^{-\frac{Ax^2}{2}}\right],\quad n\geq 0$$

EXPANSION OF TWO-VARIABLES HERMITE MATRIX POLYNOMIALS

Since

$$e^{xt\sqrt{2A}-(y+1)t^2 I}=\sum_{n=0}^{\infty}\frac{H_n(x,y,A)t^n}{n!} \quad (43)$$

it follows that

$$\sum_{n=0}^{\infty}\frac{\left(\sqrt{2A}xt\right)^2}{n!}=\sum_{n=0}^{\infty}\frac{(y+1)^n t^{2n}}{n!}\sum_{n=0}^{\infty}\frac{H_n(x,y,A)t^n}{n!}$$
$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{H_n(x,y,A)(y+1)^k t^{n+2k}}{n!k!}$$
$$=\sum_{n=0}^{\infty}\sum_{k=0}^{[n/2]}\frac{H_{n-2k}(x,y,A)(y+1)^k t^n}{(n-2k)!k!}$$

equating coefficients of t^n, we get

$$\left(\sqrt{2A}x\right)^n I=\sum_{k=0}^{[n/2]}\frac{n!H_{n-2k}(x,y,A)(y+1)^k}{(n-2k)!k!} \qquad (44)$$

for y = 0, (44), gives the expansion of one variable Hermite matrix polynomials (Defez and Jo'dar, 1998).

THE CHEPYSHEV MATRIX POLYNOMIALS

The two-variable Hermite matrix polynomials will be exploited here to define a matrix Version of Chepyshev polynomials. The Chepyshev polynomials of the second kind (Davis, 1975) are defined by

$$U_n(z)=\sum_{k=0}^{[n/2]}\frac{(-1)^k(n-k)!(2z)^{n-2k}}{(n-2k)!k!} \qquad (45)$$

suppose that A is a matrix in $C^{N\times N}$ satisfying the condition (1), by (16) it follows that

$$\frac{1}{n!}\int_0^{\infty}e^{-t}t^nH_n\left(x,\frac{1-t}{t},A\right)dt$$
$$=\int_0^{\infty}e^{-t}t^n\sum_{k=0}^{[n/2]}\frac{(-1)^k\left(\sqrt{2A}\right)^{n-2k}(x)^{n-2k}t^{-k}}{(n-2k)!k!}$$

since the summation in the right-hand side of the above equality is finite, then the series and the integral can be permuted. also, in view of

$$n!=\int_0^{\infty}e^{-t}t^n\,dt$$

we can write

$$\frac{1}{n!}\int_0^{\infty}e^{-t}t^nH_n\left(x,\frac{1-t}{t},A\right)dt=\sum_{k=0}^{[n/2]}\frac{(-1)^k(n-k)!\left(\sqrt{2A}\right)^{n-2k}(x)^{n-2k}}{(n-2k)!k!} \qquad (46)$$

hence, the chepyshev matrix polynomials of the second kind can be defined by

$$U_n(x,A)=\sum_{k=0}^{[n/2]}\frac{(-1)^k(n-k)!\left(\sqrt{2A}x\right)^{n-2k}}{(n-2k)!k!}$$

or by using the two-variable Hermite matrix polynomials in the form

$$U_n(x,A)=\frac{1}{n!}\int_0^{\infty}e^{-t}t^nH_n\left(x,\frac{1-t}{t},A\right)dt$$

in similar way, we define the generalized Chepyshev matrix polynomials of the second kind as follows

$$U_n(x,y,A)=\sum_{k=0}^{[n/2]}\frac{(-1)^k\left(\sqrt{2A}x\right)^{n-2k}(y+1)^k}{(n-2k)!k!}$$

and

$$U_n(x,y,A)=\frac{1}{n!}\int_0^{\infty}e^{-t}t^nH_n\left(x,(y+1)\frac{1-t}{t},A\right)dt$$

it's evident that

$$U_n(x,y,A)=(y+1)^{n/2}U_n\left(x/\sqrt{y+1},A\right) \qquad (47)$$

EXPAND THE GEGENBAUER MATRIX POLYNOMIALS OF TWO VARIABLES IN SERIES OF H_n (x, y, A)

Let us now employ (14), (9) and (44) and taking into account that each matrix commutes with it self.
from (14), one gets

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}C_{n,k}^{A}(x,y)t^{n+k}$$
$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{[n/2]}\sum_{j=0}^{[k/2]}\frac{(-1)^{r+j}(A)_{n+k-r-j}2^{n+k}x^{n-2r}y^{k-2r}t^{n+k}}{r!j!(n-2r)!(k-2j)!} \qquad (48)$$

which on applying (9) becomes.

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{r+j}(A)_{n+k+r+j}2^{n+k}x^ny^kt^{n+k+2r+2j}}{r!j!n!k!}$$

From (44), we get

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}C_{n,k}^{A}(x,y)t^{n+k}\left(\sqrt{2A}\right)^n2^{-(n+k)}$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\sum_{s=0}^{[n/2]}\frac{(-1)^{r+j}(A)_{n+k+r+j}n!y^{k}(y+1)^{s}t^{n+k+2r+2j}H_{n-2s}(x,y,A)}{r!j!n!k!(n-2s)!}$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\sum_{s=0}^{\infty}\frac{(-1)^{r+j}(A)_{n+k+r+j+2s}y^{k}(y+1)^{s}t^{n+k+2r+2j+2s}H_{n}(x,y,A)}{n!k!r!j!s!}$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\sum_{s=0}^{k}\frac{(-1)^{r+j}(A)_{n+k+r+j+s}y^{k-s}(y+1)^{s}t^{n+k+2r+2j+s}H_{n}(x,y,A)}{n!(k-s)!r!j!s!}$$

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\sum_{s=0}^{k}\frac{(-1)^{r+j}\left[A+(n+k+r+j)I\right]_{S}y^{k-s}(y+1)^{s}t^{n+k+2r+2j+s}}{n!k!r!j!s!(k-s)!}$$

by (10) it follows that

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{r+j}y^{k}}{n!k!r!j!}\sum_{s=0}^{k}\frac{\left[A+(n+k+r+j)I\right]_{S}(-kI)_{S}}{(-1)^{s}s!}$$
$$\cdot\left(\frac{y+1}{y}\right)(A)_{n+k+r+j}H_{n}(x,y,A)t^{n+k+2r+2j}$$

we may write as

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{\infty}\sum_{j=0}^{\infty}\frac{(-1)^{r+j}y^{k}}{n!k!r!j!}\,{}_{2}F_{0}\left[-kI,A+(n+k+r+j)I;-;\left(-\frac{y+1}{y}t\right)\right]$$
$$\cdot\,(A)_{n+k+r+j}H_{n}(x,y,A)t^{n+k+2r+2j}$$

again from (9), one gets

$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{r=0}^{[n/2]}\sum_{j=0}^{[k/2]}\frac{(-1)^{r+j}y^{k-2j}}{n!k!r!j!}\,{}_{2}F_{0}\left[-kI,A+(n+k+r+j)I;-;\left(-\frac{y+1}{y}t\right)\right]$$
$$\cdot\,(A)_{n+k+r+j}H_{n}(x,y,A)t^{n+k}$$

equating the coefficient of t^{n+k} we obtain an expansion of the two-variable Gegenbauer matrix Polynomials as series of two-variable Hermite matrix polynomials in the form

$$C_{n,k}^{A}(x,y)=\sum_{r=0}^{[n/2]}\sum_{j=0}^{[k/2]}\frac{(-1)^{r+j}y^{k-2j}}{!r!j!(n-2r)!(k-2j)!}$$
$$\cdot{}_{2}F_{0}\left[-kI,A+(n+k+r+j)I;-;\left(-\frac{y+1}{y}t\right)\right](A)_{n+k-r-j}H_{n}(x,y,A)\quad(49)$$

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