Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 8 (7): 1242-1248, 2008
ISSN 1812-5654

Dynamic Material Removal Rate and Tool Replacement Optimization with Calculus of Variations

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Abstract: This study mathematically presents an optimum material removal control model, where the Material Removal Rate (MRR) is comprehensively introduced, to accomplish the dynamic machining control and tool life determination of a cutting tool under an expected machining quantity. To resolve the incessant cutting-rate control problem, Calculus of Variations is implemented for the optimum solution. Additionally, the decision criteria for selecting the dynamic solution are suggested and the sensitivity analyses for key variables in the optimal solution are fully discussed. The versatility of this study is furthermore exemplified through a numerical illustration from the real-world industry with BORLAND C++ BUILDER. It is shown that the theoretical and simulated results are in good agreement. This study absolutely explores the very promising solution to dynamically organize the MRR in minimizing the machining cost of a cutting tool for the contemporary machining industry.

Key words: Material removal rate, calculus of variations, dynamic optimization, expected machining quantity

INTRODUCTION

Our earlier work (Lan *et al*., 2002; Yeh and Lan, 2002) have indicated that the material removal rate is an important control factor of machining operation and the control of machining rate is also critical for production planners. Many researches on the dynamic Material Removal Rate (MRR) control were established in adaptive controllers for optimization of machining operations, a few of which are presented in this study. Fuh *et al*. (1996) have designed a Variable Structure System (VSS) controller on commercial Computer Numerical Controlled (CNC) turning machines. It utilized overriding the spindle speed to dynamically manage the MRR. These PC-based controllers have also been realized to on-line override the programmed feed rates by Rober and Shin (1995) on the CNC milling machines as well as by Kim and Kim (1996) on the machining centers. Therefore, by overriding the feedrate and/or spindle speed on various CNC machines, the practicality of dynamically controlling the MRR for most CNC machining operations is doubtless.

It has been stated by Choudhury and Appa Rao (1999) that the tool life is also a critical parameter of the cutting process. Many researches have been performed on the determination of tool life. Lee *et al*. (1992) proposed a method of optimal control to ensure maximum tool life. Novak and Wilklund (1996) proposed an appropriate implementation to predict tool life and Iakovou *et al*. (1996) developed an optimal tool replacement policy to minimize the machining cost per part. Meng *et al*. (2000) also provided a modified Taylor tool life equation to minimize tool cost. However, their maximum tool life or minimum tool cost will not guarantee the minimal cost of the overall machining operation because they are hardly related to the machines.

Earlier studies (Lan *et al*., 2002; Yeh and Lan, 2002; Jung and Ahluwalia, 1995) have mentioned that the cost to machine each part is a function of the machining time; and the machining cost is divided into two different categories, the operational cost and holding cost. While Kamien and Schwartz (1991) described the marginal operation cost of production is a linear function of production rate, the operational cost of a machine is also proposed in direct proportion to the square of the material removal rate in previous researches (Lan *et al*., 2002; Yeh and Lan, 2002) as well as this study. It is that the more machining rate results more operational cost such as machine maintenance and depreciation costs.

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Galante *et al*. (1998) mentioned that the dependency of a reliability model on the cutting conditions is the aim to optimize the manufacturing system. Kim *et al*. (1996) have discussed several time series modeling on the control of machining process decision-making; nevertheless, none is characterized to complete the minimum machining cost. In this study, the modelling of MRR control and the dynamic optimization for a cutting tool with tool life determination are treated. As Wang and Luh (1996) claimed that the advanced Computer Numerical Controlled (CNC) machines are broadly used to perform from job shops to Flexible Manufacturing Systems (FMS), the attention in the minimum-cost machining control grows up in the systems. The dynamic model proposed in this study definitely provides the functional solution to the technique and contributes the economic approach to control the machining operation with profound insight.

MATHEMATICAL MODELLING

In this study, the cutting process is regarded to be a continuous single-tool turning operation without breakdown as in our earlier works (Lan *et al*., 2002; Yeh and Lan, 2002). The ductile materials are considered for the machining operation in this study. The upper limit of MRR is generated from the maximum acceptable conditions (speed, feed rate and depth of cut) suggested in the machining handbook. Therefore, no chattering or scrapping of parts will occur during machining.

The machining costs are divided into two different categories, the operational cost and holding cost (Lan *et al*., 2002; Yeh and Lan, 2002). The operational cost of the machine is directly proportional to the square of the MRR (Lan *et al*., 2002; Yeh and Lan, 2002). As many of the real manufacturing cases, all chips from cutting and the finished products are usually held and stored at the machine shop until a tool change. As the tool cost for a given amount of work was introduced into many of the machining optimization processes (Amin *et al*., 1998; Cauchick-Miguel and Coppini, 1996), the expected machining quantity for a single cutting tool is considered known in this study.

In this study, $\int_0^T bM'^2(t)dt$ signifies the operational cost during time interval [0, T] and $\int_0^T cM(t)dt$. denotes the overall chip holding cost during time interval [0, T]. Additionally, $\int_0^T c_1 dt$ presents the labor cost during time interval [0, T]. Thus, the dynamic model and its constraints are thus elaborated as below.

$$\begin{cases} \underset{M,T}{\text{minimize}} & \left\{ \int_0^T \left[bM'^2(t) + cM(t) + c_1 \right] dt \right\} \\ \text{subject to} & M(0) = 0 \\ & M(T) = aQ \\ & T \text{ is free} \\ & 0 \le M'(t) \le B \\ & \forall\, t \in [0,T] \end{cases}$$

OPTIMAL SOLUTION

Let M* (t) and T* be the optimal solution of the dynamic model and suppose that time interval $(0,\tilde{t})$ is the maximal subinterval of [0, T] to satisfy Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992). There are two feasible cases to be discussed as follows.

Situation 1: M*' (t) will not touch B before tool life T. $(\tilde{t} = T)$

The optimal solution for Situation 1 is shown as follows:

$$T^* = \frac{2\sqrt{b}}{c}\left(\sqrt{acQ + c_1} - \sqrt{c_1}\right) \tag{1}$$

$$M^{*\prime}(t) = \frac{c}{2b}t + \sqrt{\frac{c_1}{b}} \qquad 0 \le t \le T \tag{2}$$

$$M^*(t) = \frac{c}{4b}t^2 + \sqrt{\frac{c_1}{b}}\,t \qquad 0 \le t \le T \tag{3}$$

The detailed processes are described in Appendix A.

Before solving the optimum solution for the other case, one property is proposed and discussed as follows:

Property: If the line y = M*' (t) touches the line, y = B these two lines should overlap to be y = B from the touch point $\tilde{t}$ to the end point T.

Proof: From Eq. 2, y = M*' is a strictly increasing linear function of t. And it holds for the subinterval of $[0,\tilde{t})$ subject to $0 \le M^{*\prime}(t) \le B$. Since it cannot contradict Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992) to be a decreasing linear function of t, the property is verified.

Situation 2: M*' (t) will touch B before tool life T. $(\tilde{t} \in [0,T))$.

We assume that the material removal rate M*' (t) will reach the upper limit B at $t = \tilde{t}$ where $\tilde{t} \in [0,T)$. The optimal solution for situation 2 is shown as follows:

$$\tilde{t}^* = \frac{2b}{c}\left(B - \sqrt{\frac{c_1}{b}}\right) \quad (4)$$

$$T^* = \frac{1}{B}\left[aQ - \frac{(bB^2 - c_1)}{c}\right] + \frac{2b}{c}\left(B - \sqrt{\frac{c_1}{b}}\right) \quad (5)$$

$$M^{*'}(t) = \begin{cases} \frac{c}{2b}t + \sqrt{\frac{c_1}{b}} & \text{if } t \in [0, \tilde{t}] \\ B & \text{if } t \in (\tilde{t}, T] \end{cases} \quad (6)$$

$$M^*(t) = \begin{cases} \frac{c}{4b}t^2 + \sqrt{\frac{c_1}{b}}\,t & \text{if } t \in [0, \tilde{t}] \\ \frac{\left(bB^2 - c_1\right)}{c} + B(t - \tilde{t}) & \text{if } t \in (\tilde{t}, T] \end{cases} \quad (7)$$

The detailed processes for the solutions above are described in Appendix B.

Decision criteria: From Eq. B5, two possible decision criteria are derived and classified as follows:

- When, $acQ \le bB^2 - c_1$ it means $\tilde{t} \ge T$. This contradicts the assumption of Situation 2. It is that the optimal control of the material removal rate will not reach the upper speed limit within the tool life. The optimal solution is situation 1.
- When $acQ > bB^2 - c_1$, it means $\tilde{t} < T$. This is that the optimal control of the material removal rate will reach the upper speed limit within the tool life. The optimal solution is Situation 2.

The procedure in achieving the optimal solution of the dynamic model provides a continuous function indicating the optimal path to be followed by the variables through time or space. It is argued (Lan *et al.*, 2002; Kamien and Schwartz, 1991; Chiang, 1992) that using the properties of the Calculus of Variations for dynamic optimization, the completeness and the optimality of the solution are guaranteed.

SENSITIVITY ANALYSIS

Sensitivity Analysis for Situation 1: From Eq. 1, it is claimed that the optimum tool life T* is directly proportional to the material removal per unit product a, the marginal operation constant b, or the expected machining quantity per unit tool Q. It is also shown that increasing the overall holding cost of unit chip per unit time, c, will shorten the optimum tool life T*.

By Eq. 2 and 3, the cumulative volume of material machined M* (t) and material removal rate $M^{*'}$ (t) are both increasing with the overall holding cost of unit chip per unit time c, or the labor cost per unit time c_1; but decreasing with the marginal operation constant b.

The overall sensitivity analysis for Situation 1 is shown in Table 1.

Table 1: The sensitivity analysis for situation 1. ($\tilde{t} = T$)

Decision variables	Parameters a	b	c	Q	c_1	Reference
T*	+	+	-	+	#	Eq. 1
$M^{*'}$ (t)	#	-	+	#	+	Eq. 2
M* (t)	#	-	+	#	+	Eq. 3

+: Decision variable is directly proportional to the parameter, -: Decision variable is inversely proportional to the parameter, #: Decision variable depends on the changes of other relevant parameters

Table 2: The sensitivity analysis for situation 2. ($\tilde{t} \in [0, T)$)

Decision variables	Parameters a	b	c	B	Q	c_1	Reference
$\tilde{t}$	#	#	-	+	#	-	Eq. 4
T*	+	#	#	#	+	#	Eq. 5

+: Decision variable is directly proportional to the parameter, -: Decision variable is inversely proportional to the parameter, #: Decision variable depends on the changes of other relevant parameters

Sensitivity analysis for situation 2: From Eq. 4, it is derived that the time to reach upper limit $\tilde{t}$ is inversely proportional to the overall holding cost of unit chip per unit time c, or the labor cost per unit time c_1. It is also derived that increasing the upper limit of MRR, B, will delay the time to reach the upper limit $\tilde{t}$.

By Eq. 5, the optimum tool life T* is increasing with the material removal per unit product a or the expected machining quantity per unit tool Q.

The overall sensitivity analysis for situation 2 is shown in Table 2.

NUMERICAL EXAMPLE

To demonstrate the extensive versatility of the dynamic model, a numerical example from the real-world industry of the previous study (Lan *et al.*, 2002) is introduced. The machining process of a single-tool turning operation for specific fixture plates from AirTAC Corporation in Taiwan, R.O.C. is referenced for the simulation. The operation is assigned to a MIYANO LX-21 CNC lathe with ISCR tool holder and WNMG 06T304-NF insert. All data compiled are transformed into SI units as well as US dollars and listed as follows:

Q = 40 parts, a = 17355 mm^3, b = 1.7×10^{-8} (dallars-min)/mm^6, c = 6.625×10^{-8} dallars (min-mm^3), c_1 = 0.135 dollars min^{-1}, B = 16470 mm^3 min^{-1} and $\bar{T}$ = 70 min.

To compare present dynamic model and the AirTAC's traditional machining model on the aspect of machining cost per unit tool under the expected machining quantity, a computer program written in BORLAND C++ BUILDER is then developed. The concept of the flow chart is described as follows:

Q, a, b, c, c_1, B and $\bar{T}$ should be given before the following algorithm.

Step 1: Compute $\bar{B} = \frac{aQ}{\bar{T}}$ and the production cost for traditional machining model.

$$\bar{O}_b = \int_0^{\bar{T}} \left[b\bar{B}^2 + c\bar{B}t + c_1\right] dt$$

Go to Step 2.

Step 2: If, $acQ > bB^2 - c_1$, go to Step 4.
Otherwise, go to Step 3.

Step 3: Compute $T^* = \frac{2\sqrt{b}}{c}\left(\sqrt{acQ + c_1} - \sqrt{c_1}\right)$ and the production cost for the dynamic model.

$$O_b = \int_0^{T} \left[bM'^2(t) + cM(t) + c_1\right] dt$$

Go to Step 5.

Step 4: Compute $\underline{T} = \frac{2b}{c}\left(B - \sqrt{\frac{c_1}{b}}\right)$ and $T^* = \frac{1}{B}\left[aQ - \frac{(bB^2 - c_1)}{c}\right] + \frac{2b}{c}\left(B - \sqrt{\frac{c_1}{b}}\right)$, then compute the production cost for the dynamic model.

$$O_b = \int_0^{\underline{T}} \left[bM'^2(t) + cM(t) + c_1\right] dt + \int_{\underline{T}}^{T} \left[bB^2 + cBt + c_1\right] dt$$

Go to Step 5.

Step 5: Write T and O_b for the dynamic model, $\bar{T}$ and $\bar{O}_b$ for traditional model.

From the simulated result shown in Fig. 1, it is observable that the production cost per unit tool of the dynamic model is \$56.21 dollars less costly than the traditional machining model, which is considered cost competitive through years of experiences in AirTAC Corporation. However, the tool life determined for the dynamic model is 158.32 min longer than that in AirTAC's machining model. This denotes that when the expected machining quantity Q has satisfactory due-date allowance on the machining operation, the dynamic model can significantly minimize the machining cost for the individual tool. It is shown that the theoretical and simulated results are in good agreement.

With this study, the manufacturing planning, machining cost estimating and even the contract negotiation can be then further approached.

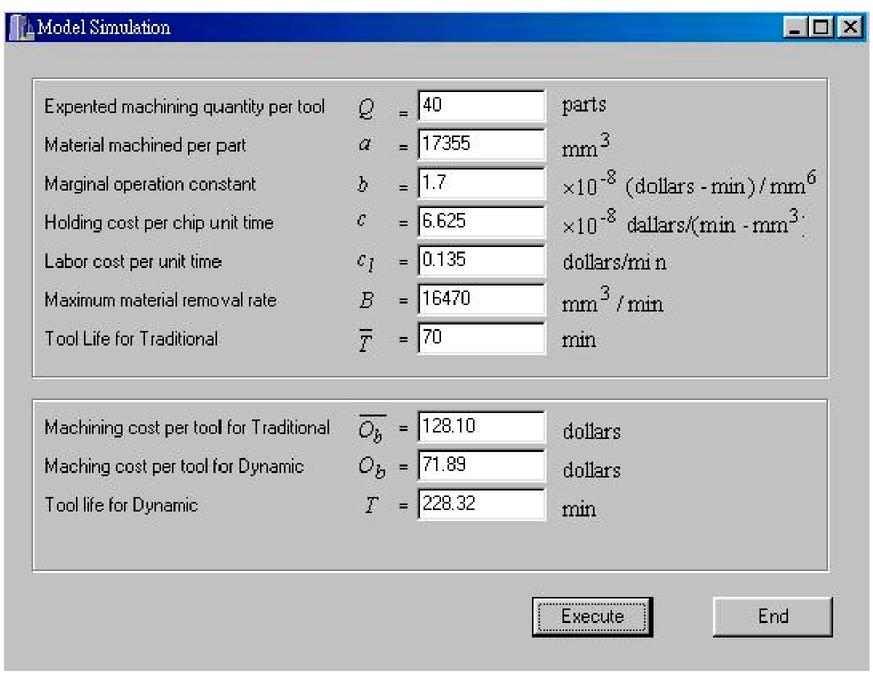

Fig. 1: Cost simulation for dynamic and traditional machining models

CONCLUSIONS

The material removal rate is an important control issue of machining operation and the control of machining rate is also critical for production schemers in modern CNC machining industry. The operational cost, holding costs, average material machined per unit part, expected machining quantity per tool, unconstrained tool life and upper speed limit of MRR are integrated concurrently to investigate the optimal control of MRR. This is deemed complicated; however, the problem becomes stabilized through the proposed dynamic model.

There are the two characters of the optimal MRR control derived from this study.

- The optimal control of material removal rate $M^{*\prime}(t)$ is a strictly increasing linear function of t before reaching the upper MRR limit.
- By the property described before, if the material removal rate $M^{*\prime}(t)$ touches the upper speed limit B, the optimal MRR will settle to be upper speed limit B for the rest of the tool life T.

Additionally, the decision criteria for selecting the optimal control of material removal rate, the sensitivity analyses on different key variables of the optimal solution, as well as the completeness and optimality of the solution are fully discussed. Moreover, a numerical study from the real-world industrial is exemplified to present advantages of the dynamic model in cost minimization. It is shown that when the expected machining quantity Q has satisfactory time allowance on the machining operation, the dynamic model can significantly minimize the machining cost for the specific tool. With this study, the manufacturing planning, machining cost estimating and even the contract negotiation can be then further approached.

Minimizing the machining cost of a cutting tool has become essential for computer-based machining industry. In this study, the modelling and optimization of the dynamic control of MRR are addressed. The proposed dynamic model not only contributes a better and practical solution to this field, but also generates a reliable and applicable concept of machine tool control to the techniques. An experimental project on a typical Computer Numerical Control (CNC) machine with PC-based Digital Signal Processor (DSP) to on-line realize and simulate the optimal material removal rate control is being conducted following this study. Future researches with the modelling of dynamic optimization on various machining processes are encouraged in this study.

Appendix A: The optimal solution for Situation 1.

Let $F = bM'^2(t)+cM(t)+c_1$ and suppose that the material removal rate $M'(t)$ will never reach the upper speed limit B before tool life T. From Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992), it is derived that

$$c = \frac{d}{dt}[2bM'(t)]$$

There exists a constant k_1 to satisfy

$$M'(t) = \frac{c}{2b}t + k_1 \quad 0 \le t \le T \tag{A1}$$

Integrating Eq. A1 with t, such that

$$M(t) = \frac{c}{4b}t^2 + k_1 t + k_2 \quad 0 \le t \le T \tag{A2}$$

Introducing the boundary condition, M (0) = 0, into Eq. A2; then

$$k_2 = 0 \tag{A3}$$

With the transversality condition for free horizon T (Kamien and Schwartz, 1991; Chiang, 1992), it is obtained that

$$bM'^2(T) = cM(T) + c_1 \tag{A4}$$

Providing Eq. A1, A2 at t = T and Eq. A3 into Eq. A4, it yields

$$b\left(\frac{c}{2b}T + k_1\right)^2 = c\left(\frac{c}{4b}T^2 + k_1 T\right) + c_1 \tag{A5}$$

Expanding Eq. A5 and collecting terms, it gives

$$k_1 = \sqrt{\frac{c_1}{b}} \tag{A6}$$

Using the boundary condition, M (T) = aQ and Eq. A4; it is derived that

$$M'(T) = \sqrt{\frac{acQ + c_1}{b}} \tag{A7}$$

Introducing Eq. A3 and A6 into Eq. A1 at t = T and then compare with Eq. A7, we have

$$\frac{c}{2b}T + \sqrt{\frac{c_1}{b}} = \sqrt{\frac{acQ + c_1}{b}} \quad \text{(A8)}$$

Therefore, the optimum tool life, T*, is then derived. Introducing Eq. A3 and A6 into Eq. A1 and A2, the optimum solution, M*'(t) and M*(t), is obtained.

Appendix B: The optimal solution for Situation 2.

Before $M^{*'}(t)$ touches the upper limit B, Eq. 2 and 3 are satisfied either. Besides, when it reaches the upper limit B, the PROPERTY is then applied.

Using the transversality condition for free end point $\tilde{t}$ (Kamien and Schwartz, 1991; Chiang, 1992), we have

$$-bM'^2(\tilde{t}) + cM(\tilde{t}) + c_1 = 0 \quad \text{(B1)}$$

Introducing $M'(\tilde{t}) = B$ into Eq. B1, it is then obtained

$$M(\tilde{t}) = \frac{bB^2 - c_1}{c} \quad \text{(B2)}$$

Since M*'(t) is a linear increasing function of t before reaching the upper limit B, thus

$$M(\tilde{t}) = \frac{\tilde{t}}{2}\left(B + \sqrt{\frac{c_1}{b}}\right) \quad \text{(B3)}$$

With Eq. B2 and B3, the optimal touch point $\tilde{t}^*$ is found From the PROPERTY and the boundary condition M (T) = aQ, it is noted that

$$aQ = M(\tilde{t}) + B(T - \tilde{t}) \quad \text{(B4)}$$

Providing Eq. B2 into Eq. B4, we have

$$T - \tilde{t} = \frac{1}{B}\left[aQ - \frac{(bB^2 - c_1)}{c}\right] \quad \text{(B5)}$$

Applying Eq. B4 into Eq. B5 and the optimum production period T* is derived.

With the Property and Eq. B2, the optimal solution, $M^{*'}$ and M* (t), is obtained.

NOTATIONS

This study is developed on the basis of the following notations.

a = Average volume of material machined per unit part
B = Upper speed limit of MRR
$\bar{B}$ = Fixed MRR for traditional machining model
bM' (t) = Marginal operation cost at the material removal rate M' (t); where b is a constant
$bM'^2(t)$ = Operational cost at time t
c = Overall holding cost of unit chip per unit time.
c_1 = Labor cost per unit time
M (t) = Cumulated volume of material machined during time interval [0, t]
M' (t) = MRR at time t
O_b = Objective (machining cost) for the dynamic model
$\bar{O}_b$ = Objective (machining cost) for traditional model
Q = Expected machining quantity per unit tool
$\bar{T}$ = Tool life for traditional machining model
T = Tool life for the dynamic model

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