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Application of Homotopy Perturbation Method to Solve Linear and Non-Linear Systems of Ordinary Differential Equations and Differential Equation of Order Three

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Abstract: In this study, Homotopy Perturbation Method (HPM) is implemented to solve system of differential equations. The HPM deforms a difficult problem into a simple problem which can be easily solved. The results are compared with the results obtained by exact solutions and Adomian's decomposition method. The results reveal that the HPM is very effective, convenient and quite accurate to systems of nonlinear equations. Some examples are presented to show the ability of the method for linear and non-linear systems of differential equations.

Key words: Systems of differential equations, homotopy perturbation method, ordinary differential equations, third order differential equation, nonlinear

INTRODUCTION

A system of ordinary differential equations of the first order can be considered as:

$$\begin{cases} y_1' = f_1(x, y_1, \ldots, y_n) \\ y_2' = f_2(x, y_1, \ldots, y_n) \\ \quad . \\ \quad . \\ \quad . \\ y_n' = f_n(x, y_1, \ldots, y_n) \end{cases} \qquad (1)$$

where each equation represents the first derivative of one of the unknown functions as a mapping depending on the independent variable x and n unknown functions $f_1,\ldots,f_n$.

Many problem in the field of engineering yield to a system of differential equations and task of solving these systems plays important role. For instance, Gerstmayr and Irschik (2003) studied the vibrations of elasto-plastic beams with rigid-body degrees-of-freedom and the formulation leads to a non-linear system of differential algebraic equations. As an another example, Durán and Monteagudo (2004) has made Modelling of soot and SOF emissions from a typical European turbocharged diesel engine and his study yield to the system of ordinary differential equations. Hu and Chen (2008) investigated the bifurcation and chaos in atomic force microscope. They simplified the partial-differential equation that governs the motions of the microcantilever to a set of ordinary differential equations. Yuan (2008) constructed a class of transonic shock in a divergent nozzle which is a part of an angular sector (for two-dimensional case) or a cone (for three-dimensional case) which does not contain the vertex. His idea involved is to solve discontinuous solutions of a class of two-point boundary value problems for systems of ordinary differential equations.

On the other hand, since every ordinary differential equation of order n can be written as a system consisting of n ordinary differential equation of order one, we restrict our study to a system of differential equations of the first order.

Biazar *et al.* (2004) used the Adomian's Decomposition Method (ADM) to solve this problem.

The motivation of this study is to extend the Homotopy Perturbation Method (HPM) proposed by He (1999, 2000, 2004a-c, 2005a-c, 2006a-c) to solve the epidemic model. The HPM is useful to obtain exact and approximate solutions of linear and nonlinear differential equations.

No need to linearization or discretization, large computational work and round-off errors is avoided. It has been used to solve effectively, easily and accurately a large class of nonlinear problems with approximations. These approximations converge rapidly to accurate solutions (Rafei and Ganji, 2006; Ganji and Rafei, 2006; Gorji *et al.*, 2007; Siddiqui *et al.*, 2006; Abbasbandy, 2006a-c).

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ANALYSIS OF HE'S HOMOTOPY PERTURBATION METHOD

To illustrate the basic ideas of this method, we consider the following nonlinear differential Equation:

$$A(u) - f(r) = 0, r \in \Omega \quad (2)$$

Considering the boundary conditions of:

$$B(u, \partial u/\partial n) = 0, \quad r \in \Gamma \quad (3)$$

where, A is a general differential operator, B a boundary operator, f(r) a known analytical function and Γ is the boundary of the domain Ω.

The operator A can be, generally divided into two parts of L and N, where L is the linear part, while N is the nonlinear one Eq. 2 can, therefore, be rewritten as:

$$L(u) + N(u) - f(r) = 0. \quad (4)$$

By the homotopy technique, we construct a homotopy as v (r, p): Ω × [0, 1] → R which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \ p \in [0,1], \ r \in \Omega \quad (5)$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad (6)$$

where, $p \in [0,1]$ is an embedding parameter and u_0 is an initial approximation of Eq. 2 which satisfy the boundary conditions. Obviously, considering Eq. 5 and 6, we will have:

$$H(v,0) = L(v) - L(u_0) = 0 \quad (7)$$

$$H(v,1) = A(v) - f(r) = 0 \quad (8)$$

The changing process of p from zero to unity is just that of v(r,p) from $u_0(r)$ to u(r). In topology, this is called deformation and L(v)-L(u_0) and A(v)-f(r) are called homotopy.

According to HPM, we can first use the embedding parameter p as a small parameter and assume that the solution of Eq. 5 and 6 can be written as a power series in p:

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \quad (9)$$

Setting p = 1 results in the approximate solution of Eq. 2:

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \dots. \quad (10)$$

The combination of the perturbation method and the homotopy method is called the homotopy perturbation method, which lessens the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantages of the traditional perturbation techniques.

The series (10) is convergent for most cases. However, the convergence rate depends on the nonlinear operator A(v). The following opinions are suggested by He (2006b):

- The second derivative of N(v) with respect to v must be small because the parameter p may be relatively large, i.e., p → 1.
- The norm of $L^{-1}\, \partial N/\partial v$ must be smaller than one so that the series converges He (2006a-c).

APPLICATION OF THE HPM

Numerical examples: In this part we present three examples. The first and the second examples are considered to illustrate the method for linear and non-linear systems of ordinary differential equations of order one. While in third example we solve a differential equation of order three by transforming it into a system of differential equations of the first order.

Example 1: Consider the following system of differential equations, with initial values $y_1(0) = 1$, $y_2(0) = 0$ and $y_3 = 2$.

$$\begin{cases} y_1' = y_3 - \cos(x), \\ y_2' = y_3 - e^x, \\ y_3' = y_1 - y_2, \end{cases} \quad (11)$$

From Eq. 9, if the two terms approximations are sufficient, we will obtain:

$$\begin{aligned} v_1 &= v_{1,0} + p\, v_{1,1} + p^2 v_{1,2} + \cdots, \\ v_2 &= v_{2,0} + p\, v_{2,1} + p^2 v_{2,2} + \cdots, \\ v_3 &= v_{3,0} + p\, v_{3,1} + p^2 v_{3,2} + \cdots, \end{aligned} \quad (12)$$

where, $v_{i,j}$, i, j = 1,2,3,... are functions yet to be determined. Therefore from Eq. 10:

$$\begin{aligned} y_1(x) &= \lim_{p \to 1} v_1(x) = \sum_{k=0}^{k=2} v_{1,k}(x), \\ y_2(x) &= \lim_{p \to 1} v_2(x) = \sum_{k=0}^{k=2} v_{2,k}(x), \\ y_3(x) &= \lim_{p \to 1} v_3(x) = \sum_{k=0}^{k=2} v_{3,k}(x), \end{aligned} \quad (13)$$

According to the HPM, we can construct a homotopy of system (11) as follows:

$$\begin{aligned}&(1-p)(\dot{v}_1 - v_3 - \dot{u}_{1,0}) + p(\dot{v}_1 - v_3 + \cos(x)) = 0,\\&(1-p)(\dot{v}_2 - v_3 - \dot{u}_{2,0}) + p(\dot{v}_2 - v_3 + e^x) = 0,\\&(1-p)(\dot{v}_3 - \dot{u}_{3,0}) + p(\dot{v}_3 - v_1 + v_2) = 0,\end{aligned} \qquad (14)$$

where dot denotes differentiation with respect to x and the initial approximations are as follows:

$$\begin{aligned}&v_{1,0}(0) = y_1(0) = 1,\\&v_{2,0}(0) = y_2(0) = 0,\\&v_{3,0}(0) = y_3(0) = 2,\end{aligned} \qquad (15)$$

Substituting Eq. 12 and 15 into Eq. 14 and rearranging based on powers of p-terms, we have:

$$\begin{aligned}&(\dot{v}_{1,0} - v_{3,0}) + (\dot{v}_{1,1} + \cos(x) - v_{3,1})p + (\dot{v}_{1,2} - v_{3,2})p^2 + \ldots = 0,\\&(\dot{v}_{2,0} - v_{3,0}) + (\dot{v}_{2,1} + e^x - v_{3,1})p + (\dot{v}_{2,2} - v_{3,2})p^2 + \ldots = 0,\\&(\dot{v}_{3,0}) + (\dot{v}_{3,1} + v_{2,0} - v_{1,0})p + (v_{3,2} + v_{2,1} - v_{1,1})p^2 + \ldots = 0,\end{aligned} \qquad (16)$$

In order to obtain the unknowns $v_{i,j}$, i, j = 1,2,3,... we must construct and solve the following system which includes nine equations with nine unknowns:

$$\begin{aligned}&\dot{v}_{1,0} - v_{3,0} = 0\\&\dot{v}_{1,1} + \cos(x) - v_{3,1} = 0\\&\dot{v}_{1,2} - v_{3,2} = 0\\&\dot{v}_{2,0} - v_{3,0} = 0\\&\dot{v}_{2,1} + e^x - v_{3,1} = 0\\&\dot{v}_{2,2} - v_{3,2} = 0\\&\dot{v}_{3,0} = 0\\&\dot{v}_{3,1} + v_{2,0} - v_{1,0} = 0\\&v_{3,2} + v_{2,1} - v_{1,1} = 0\end{aligned} \qquad (17)$$

Therefore,

$$\begin{aligned}&v_{1,0}(x) = 2x + 1,\\&v_{1,1}(x) = -\sin(x) + \frac{1}{2}x^2,\\&v_{1,2}(x) = \sin(x) + e^x - \frac{1}{2}x^2 - 2x - 1,\\&v_{2,0}(x) = 2x,\\&v_{2,1}(x) = -e^x + \frac{1}{2}x^2 + 1,\\&v_{2,2}(x) = \sin(x) + e^x - \frac{1}{2}x^2 - 2x - 1,\\&v_{3,0}(x) = 2,\\&v_{3,1}(x) = x,\\&v_{3,2}(x) = \cos(x) + e^x - x - 2,\end{aligned} \qquad (18)$$

Therefore from Eq. 13:

$$y_1(x) = e^x, \qquad (19)$$

$$y_2(x) = \sin(x), \qquad (20)$$

$$y_3(x) = \cos(x) + e^x, \qquad (21)$$

Which is exactly the same as obtained by Adomian's decomposition method 1 and exact solution of the system 11.

Example 2: In this example we solve the following nonlinear system of differential equations, with exact solutions $y_1 = e^{2x}$, $y_2 = e^x$ and $y_3 = xe^x$.

$$\begin{cases}y_1' = 2y_2^2,\\y_2' = e^{-x}y_1,\\y_3' = y_2 + y_3,\end{cases} \qquad (22)$$

From Eq. 9, if the two terms approximations are sufficient, we will obtain:

$$\begin{aligned}&v_1 = v_{1,0} + pv_{1,1} + p^2v_{1,2} + p^3v_{1,3} + \cdots,\\&v_2 = v_{2,0} + pv_{2,1} + p^2v_{2,2} + p^3v_{2,3} + \cdots,\\&v_3 = v_{3,0} + pv_{3,1} + p^2v_{3,2} + p^3v_{3,3} + \cdots,\end{aligned} \qquad (23)$$

where, $v_{i,j}$, i, j = 1,2,3,..., are functions yet to be determined. Therefore from Eq. 10:

$$\begin{aligned}&y_1(x) = \lim_{p\to 1} v_1(x) = \sum_{k=0}^{k=3} v_{1,k}(x),\\&y_2(x) = \lim_{p\to 1} v_2(x) = \sum_{k=0}^{k=3} v_{2,k}(x),\\&y_3(x) = \lim_{p\to 1} v_3(x) = \sum_{k=0}^{k=3} v_{3,k}(x),\end{aligned} \qquad (24)$$

According to the HPM, we can construct a homotopy of system (22) as follows:

$$\begin{aligned}&(1-p)(\dot{v}_1 - \dot{u}_{1,0}) + p(\dot{v}_1 - 2v_2^2) = 0,\\&(1-p)(\dot{v}_2 - \dot{u}_{2,0}) + p(\dot{v}_2 - e^{-x}v_1) = 0,\\&(1-p)(\dot{v}_3 - \dot{u}_{3,0}) + p(\dot{v}_3 - v_2 - v_3) = 0,\end{aligned} \qquad (25)$$

where, dot denotes differentiation with respect to x and the initial approximations are as follows:

$$\begin{aligned}&v_{1,0}(0) = y_1(0) = 1,\\&v_{2,0}(0) = y_2(0) = 1,\\&v_{3,0}(0) = y_3(0) = 0,\end{aligned} \qquad (26)$$

Substituting Eq. 23 and 26 into Eq. 25 and rearranging based on powers of p-terms, we have:

$$\begin{aligned}
&(\dot{v}_{1,0}) + (\dot{v}_{1,1} - 2v_{2,0}^2)p + (\dot{v}_{1,2} - 4v_{2,0}v_{2,1})p^2 + \\
&(\dot{v}_{1,3} - 4v_{2,0}v_{2,2} - 2v_{2,1}^2)p^3 + \ldots = 0, \\
&(\dot{v}_{2,0}) + (\dot{v}_{2,1} - v_{1,1}e^{-x})p + (\dot{v}_{2,2} - v_{1,2}e^{-x})p^2 + \\
&(\dot{v}_{2,3} - v_{1,3}e^{-x})p^3 + \ldots = 0, \\
&(\dot{v}_{3,0}) + (\dot{v}_{3,1} - v_{2,0} - v_{3,0})p + (\dot{v}_{3,2} - v_{2,1} - v_{3,1})p^2 + \\
&(\dot{v}_{3,3} - v_{2,2} - v_{3,2})p^3 + \ldots = 0,
\end{aligned} \tag{27}$$

In order to obtain the unknowns $v_{i,j}$, i, j = 1,2,3,... we must construct and solve the following system which includes nine equations with nine unknowns:

$$\begin{aligned}
&\dot{v}_{1,0} = 0 \\
&\dot{v}_{1,1} - 2v_{2,0}^2 = 0 \\
&\dot{v}_{1,2} - 4v_{2,0}v_{2,1} = 0 \\
&\dot{v}_{1,3} - 4v_{2,0}v_{2,2} - 2v_{2,1}^2 = 0 \\
&\dot{v}_{2,0} = 0 \\
&\dot{v}_{2,1} - v_{1,1}e^{-x} = 0 \\
&\dot{v}_{2,2} - v_{1,2}e^{-x} = 0 \\
&\dot{v}_{2,3} - v_{1,3}e^{-x} = 0 \\
&\dot{v}_{3,0} = 0 \\
&\dot{v}_{3,1} - v_{2,0} - v_{3,0} = 0 \\
&\dot{v}_{3,2} - v_{2,1} - v_{3,1} = 0 \\
&\dot{v}_{3,3} - v_{2,2} - v_{3,2} = 0
\end{aligned} \tag{28}$$

Therefore,

$$\begin{aligned}
v_{1,0}(x) &= 1, \\
v_{1,1}(x) &= -e^{(-2x)} + 8e^{(-x)} + 8x - 7, \\
v_{1,2}(x) &= \frac{1}{3}e^{(-4x)} - \frac{56}{9}e^{(-3x)} - 16e^{(-2x)}x + 6e^{(-2x)} \\
&\quad + \frac{272}{3}e^{(-x)} + 64e^{(-x)}x + \frac{112}{3}x - \frac{817}{9}, \\
v_{1,3}(x) &= -9e^{-4x} + \frac{1448}{15}x + \frac{328}{5}e^{-x} - \frac{896}{9}xe^{-3x} \\
&\quad - \frac{1096}{9}e^{-3x} - \frac{11}{135}e^{-6x} + \frac{544}{225}e^{-5x} - 64e^{-2x}x^2 \\
&\quad - \frac{80}{3}e^{-2x}x + \frac{3377}{9}e^{-2x} + 8xe^{-4x} + 448e^{-x}x - \frac{210857}{675}, \\
v_{2,0}(x) &= -e^{(-x)} + 2,
\end{aligned}$$

$$\begin{aligned}
v_{2,1}(x) &= \frac{1}{3}e^{(-3x)} - 4e^{(-2x)} - 8e^{(-x)}x - e^{(-x)} + \frac{14}{3}, \\
v_{2,2}(x) &= -\frac{1}{15}e^{(-5x)} + \frac{14}{9}e^{(-4x)} + \frac{16}{3}xe^{(-3x)} - \frac{2}{9}e^{(-3x)} \\
&\quad - \frac{184}{3}e^{(-2x)} - 32e^{(-2x)}x - \frac{112}{3}e^{(-x)}x + \frac{481}{9}e^{(-x)} + \frac{298}{45},
\end{aligned}$$

$$\begin{aligned}
v_{2,3}(x) &= \frac{37}{25}e^{-5x} - \frac{1448}{15}e^{-x}x + \frac{145697}{675}e^{-x} - \frac{724}{5}e^{-2x} \\
&+ \frac{224}{9}xe^{4x} + \frac{110}{3}e^{-4x} + \frac{11}{945}e^{-7x} - \frac{272}{675}e^{6x} + \frac{64}{3}x^2e^{-3x} \\
&+ \frac{208}{9}xe^{-3x} - \frac{3169}{27}e^{-3x} - \frac{8}{5}xe^{-5x} - 224e^{-2x}x + \frac{4498}{525},
\end{aligned}$$

$$\begin{aligned}
v_{3,0}(x) &= 0, \\
v_{3,1}(x) &= e^{(-x)} + 2x - 1, \\
v_{3,2}(x) &= -\frac{1}{9}e^{(-3x)} + 2e^{(-2x)} + 8e^{(-x)}x + 8e^{(-x)} + x^2 + \frac{11}{3}x - \frac{89}{9}, \\
v_{3,3}(x) &= \frac{1}{75}e^{-5x} - \frac{7}{18}e^{-4x} - \frac{16}{9}xe^{-3x} - \frac{13}{27}e^{-3x} + \frac{113}{3}e^{-2x} + 16e^{-2x}x \\
&\quad + \frac{88}{3}e^{-x}x - \frac{289}{9}e^{-x} + \frac{1}{3}x^3 + \frac{11}{6}x^2 - \frac{49}{15}x - \frac{6343}{1350},
\end{aligned} \tag{29}$$

Therefore from Eq. 24:

$$y_1(x) = -3 + 6x + 4e^{-x}, \tag{30}$$

$$y_2(x) = 4 - e^{-x} - 2(1+x)e^{-x}, \tag{31}$$

$$y_3(x) = -1 + 2x + e^{-x} + \frac{1}{2}x^2, \tag{32}$$

Example 3: Consider the following non-linear ordinary differential equation of order 3, with the initial conditions $y(0) = 0$, $y'(0) = 1$ and $y''(0) = 2$ and the exact solution $y(x) = xe^x$.

$$y''' = \frac{1}{x}y + y', \tag{33}$$

Considering three functions, $y_1(x) = y(x)$, $y_2(x) = y'(x)$ and $y_3(x) = y''(x)$ and we can convert (33) into the following non-linear system of three differential equation of order one.

$$\begin{aligned}
y_1' &= y_2, \\
y_2' &= y_3, \\
y_3' &= \frac{1}{x}y_1 + y_3.
\end{aligned} \tag{34}$$

From Eq. 9, if the two terms approximations are sufficient, we will obtain:

$$\begin{aligned}
v_1 &= v_{1,0} + pv_{1,1} + p^2v_{1,2} + p^3v_{1,3} + p^4v_{1,4} + \cdots, \\
v_2 &= v_{2,0} + pv_{2,1} + p^2v_{2,2} + p^3v_{2,3} + p^4v_{2,4} + \cdots, \\
v_3 &= v_{3,0} + pv_{3,1} + p^2v_{3,2} + p^3v_{3,3} + p^4v_{3,4} + \cdots,
\end{aligned} \tag{35}$$

where, $v_{i,j}$, i,j = 1,2,3,4,..., are functions yet to be determined.

Therefore from Eq. 10:

$$\begin{aligned}
y_1(x) &= \lim_{p\to 1} v_1(x) = \sum_{k=0}^{k=4} v_{1,k}(x), \\
y_2(x) &= \lim_{p\to 1} v_2(x) = \sum_{k=0}^{k=4} v_{2,k}(x), \\
y_3(x) &= \lim_{p\to 1} v_3(x) = \sum_{k=0}^{k=4} v_{3,k}(x),
\end{aligned} \tag{36}$$

As in the previous examples if we apply the HPM, we can construct a homotopy of system (34) as follows:

$$\begin{aligned}&(1-p)(\dot{v}_1 - v_2 - \dot{u}_{1,0}) + p(\dot{v}_1 - v_2) = 0,\\&(1-p)(\dot{v}_2 - v_3 - \dot{u}_{2,0}) + p(\dot{v}_2 - v_3) = 0,\\&(1-p)(\dot{v}_3 - \dot{u}_{3,0}) + p(\dot{v}_3 - \frac{1}{x}v_1 - v_3) = 0,\end{aligned} \tag{37}$$

Substituting initial conditions and (35) into Eq. 37 and rearranging based on powers of p-terms, we have:

$$\begin{aligned}&(\dot{v}_{2,0} - v_{3,0}) + (\dot{v}_{2,1} - v_{3,1})p + (\dot{v}_{2,2} - v_{3,2})p^2 +\\&(\dot{v}_{2,3} - v_{3,3})p^3 + (\dot{v}_{2,4} - v_{3,4})p^4 + \ldots = 0,\\&(\dot{v}_{3,0}) + (\dot{v}_{3,1} - v_{3,0} - \frac{1}{x}v_{1,0})p + (\dot{v}_{3,2} - \frac{1}{x}v_{1,1} - v_{3,1})p^2 +\\&(\dot{v}_{3,3} - \frac{1}{x}v_{1,2} - v_{3,2})p^3 + (\dot{v}_{3,4} - \frac{1}{x}v_{1,3} - v_{3,3})p^4 + \ldots = 0,\end{aligned} \tag{38}$$

In order to obtain the unknowns $v_{i,j}$, i,j = 1,2,3,4... we must construct and solve the following system which includes fifteen equations with fifteen unknowns:

$$\begin{aligned}&v_{1,0}(x) = x^2 + x,\\&v_{1,1}(x) = \frac{1}{24}x^4 + \frac{1}{2}x^3,\\&v_{1,2}(x) = \frac{1}{2880}x^6 + \frac{1}{60}x^5 + \frac{1}{8}x^4,\\&v_{1,3}(x) = \frac{1}{967680}x^8 + \frac{13}{100800}x^7 + \frac{11}{2880}x^6 + \frac{1}{40}x^5,\\&v_{1,4}(x) = \frac{1}{696729600}x^{10} + \frac{113}{304819200}x^9 + \frac{19}{691200}x^8\\&\qquad + \frac{67}{100800}x^7 + \frac{1}{240}x^6,\\&v_{2,0}(x) = 2x + 1,\\&v_{2,1}(x) = \frac{1}{6}x^3 + \frac{3}{2}x^2,\\&v_{2,2}(x) = \frac{1}{480}x^5 + \frac{1}{12}x^4 + \frac{1}{2}x^3,\\&v_{2,3}(x) = \frac{1}{120960}x^7 + \frac{13}{14400}x^6 + \frac{11}{480}x^5 + \frac{1}{8}x^4,\\&v_{2,4}(x) = \frac{1}{69672960}x^9 + \frac{113}{33868800}x^8 + \frac{19}{86400}x^7\\&\qquad + \frac{67}{14400}x^6 + \frac{1}{40}x^5,\\&v_{3,0}(x) = 2,\\&v_{3,1}(x) = \frac{1}{2}x^2 + 3x,\\&v_{3,2}(x) = \frac{1}{96}x^4 + \frac{1}{3}x^3 + \frac{3}{2}x^2,\\&v_{3,3}(x) = \frac{1}{17280}x^6 + \frac{13}{2400}x^5 + \frac{11}{96}x^4 + \frac{1}{2}x^3,\\&v_{3,4}(x) = \frac{1}{7741440}x^8 + \frac{113}{4233600}x^7 + \frac{133}{86400}x^6\\&\qquad + \frac{67}{2400}x^5 + \frac{1}{8}x^4,\end{aligned} \tag{39}$$

Therefore from Eq. 36:

$$\begin{aligned}y_1(x) = {}& x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{24}x^5 + \frac{1}{120}x^6 + \frac{1}{1260}x^7\\&+ \frac{23}{806400}x^8 + \frac{113}{304819200}x^9 + \frac{1}{696729600}x^{10},\end{aligned} \tag{40}$$

$$\begin{aligned}y_2(x) = {}& 1 + 2x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{5}{24}x^4 + \frac{1}{20}x^5 + \frac{1}{180}x^6\\&+ \frac{23}{100800}x^7 + \frac{113}{33868800}x^8 + \frac{1}{696729600}x^9,\end{aligned} \tag{41}$$

$$\begin{aligned}y_3(x) = {}& 2 + 3x + 2x^2 + \frac{5}{6}x^3 + \frac{1}{4}x^4 + \frac{1}{30}x^5 + \frac{23}{14400}x^6\\&+ \frac{113}{4233600}x^7 + \frac{1}{7741440}x^8,\end{aligned} \tag{42}$$

The solution, $y_1(x)$ in a closed form is found to be:

$$y(x) = y_1(x) = x\,e^x, \tag{43}$$

Which is exactly the same as obtained by Adomian's decomposition method 1 and exact solution of the system 32.

CONCLUSIONS

In this study, the homotopy perturbation method has been successfully used to solve system of ordinary differential equations. The solution obtained by means of the homotopy perturbation method is an infinite power series for appropriate initial condition, which can be, in turn, expressed in a closed form. The results obtained here were compared with the exact solutions and the results reported by using other method. All the examples show that the results of the present method are in excellent agreement with those obtained by the analytical method and exact solution. The HPM has got many merits and much more advantages than the Adomian's decomposition method. This method is to overcome the difficulties arising in calculation of Adomian polynomials. Also the HPM does not require small parameters in the equation, so that the limitations of the traditional perturbation methods can be eliminated and also the calculations in the HPM are simple and straightforward. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

For computations we used Maple 9.5.

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