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Floquet Scattering Through a Heterostructure in a Time-Periodic Potential

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Abstract: We studied the resonances of a one-dimensional potential barrier driven by an external periodic force considering the Position Dependent Effective Mass (PDEM) formalism. The Floquet S matrix will be constructed then the transmission probabilities of our system are calculated. We found that the change in the effective mass of the square well does not vary the energy level where resonance occurs.

Key words: Oscillating barrier, Fano transmission resonance, scattering matrix, bound states

INTRODUCTION

The study of time-periodic quantum models is usually carried out within the context of Floquet theory (Kazanskii *et al*., 1976; Holthaus and Hone, 1993; Wagner, 1997; Azbel and Tsukernik, 1998). Floquet Eigen states are Eigen states of the one-period time evolution operator and are the natural states for describing time-periodic systems (Büttiker and Landauer, 1982; Stovneng and Hauge, 1989). Some of the Floquet states will be metastable and localized in phase space. These states are known as resonances. The resonances are metastable states for which the electrons are trapped by the localized potential (Timberlake and Reichl, 1999). The driving field induces a series of nonlinear resonances of odd order in the square well system, with higher-order resonances occurring at lower energies (Lin and Reichl, 1986; Fuka *et al*., 1995). This quantum system is periodic in time and can be described in terms of Floquet eigenstates, which are eigenstates of the one-period time evolution operator.

We choose the model of sinusoidally driven square well because of its simplicity and its connection with experimental work in solid-state physics. The driven square well serves as a highly simplified model for experiments involving electrons confined in $GaAs/Al_xGa_{1-x}As$ wells and subjected to intense far-infrared radiation (Galdrikian *et al*., 1995; Birnir *et al*., 1993). This model is also advantageous because it has been the subject of many theoretical studies, both classical and quantum. We consider the quantum barrier consisting of two static regions I and III with electron effective mass μ_I^* and a central driven quantum barrier region II with electron effective mass μ_{II}^*. We let several values of μ_I^* and μ_{II}^* in each region. The values of the effective masses used are: 0.067 m_e for GaAs, 0.092 m_e for AlGaAs and 0.15 m_e for AlAs.

The proposed Hamiltonian is periodic in time with period $2\pi/\omega$. Hence, a Floquet approach is used. Any solution of the time-dependent Schrödinger equation are expanded as a linear combination of time-periodic states-called Floquet states of the system-with coefficients oscillating in time as $\exp(-iE_i t)$ where E_i is called the quasi-energy of the Floquet state.

A Floquet Scattering matrix has been constructed by Li and Reichl for periodically driven mesoscopic systems (Li and Reichl, 1999). The Floquet S-matrix connects the outgoing propagating modes to the incoming propagating modes and is a unitary matrix which conserves probability.

One of the interesting features of localized time-periodic potentials is the presence of resonances or quasi-bound states, which could be thought of as electrons dynamically trapped by the oscillating potential (Frensley, 1990; Timberlake and Reichl, 1999; Lin and Reichl, 1986). It is well known that the quasi-bound states of an open system are related to the true bound states of the corresponding closed system (Li and Reichl, 1999; Frensley, 1990). In this study, we present a technique which is as the direct solution of an eigenvalue problem, to compute the positions and life times of quasi-bound states and the energies of transmission ones and zeros.

We are going to solve the time-periodic Schrödinger equation for the proposed system using the Floquet theorem and then construct the Floquet S matrix. Also we will study the behavior of transmission probabilities for the proposed one-dimensional (1D) modulated square potential and study numerically the transmission resonances for both different values of the effective masses of the well region and its neighbors cases.

The Floquet S matrix: We consider electrons transmitting through a modulated potential which extends from

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-L/2 to L/2 subjected to the harmonic driving force $V_1 \cos(\omega t)$. By the Floquet theorem the solution of the time-dependent Schrödinger equation could be converted into a time-independent Eigen value problem (Li and Reichl, 1999; Bagwell and Lake, 1992). The wave function outside the barrier consists of many Floquet sidebands with energy spacing hw. The wave function in these regions is a superposition of an infinite number of these sidebands and can be written in regions I and II, respectively as:

$$\psi^{I}(x,t) = \sum_{n=-\infty}^{\infty} (A_n^i e^{ik_n x} + A_n^o e^{-ik_n x}) e^{-iE_n t/\hbar} \quad (1)$$

$$\psi^{III}(x,t) = \sum_{n=-\infty}^{\infty} (B_n^i e^{-k_n x} + B_n^o e^{+ik_n x}) e^{-iE_n t/\hbar} \quad (2)$$

the expression for the Floquet state inside the oscillating region $\psi^{II}(x,t)$:

$$\psi^{II}(x,t) = e^{-iE_F t/\hbar} \sum_{n=-\infty}^{\infty} \phi(x) e^{-in\omega t}, \quad (3)$$

Where:

$$\phi(x) = \sum_{m=-\infty}^{\infty} (a_m e^{iq_m x} + b_m e^{-iq_m x}) J_{n-m}(\frac{V1}{\hbar\omega}) \quad (4)$$

A_n^i and B_n^i are the probability amplitudes of the incoming waves from the left and right, respectively, while A_n^o and A_n^o are those of the outgoing waves and

$$k_n = \sqrt{\frac{2\mu_I^* E_n}{\hbar^2}}, \quad q_m = \sqrt{\frac{2\mu_{II}^*}{\hbar^2}(E_F + m\hbar\omega - V_0)},$$

Matching the wave function $\psi(x,t)$ and its derivative at the boundaries $x = \pm L/2$, finally we get the following matrix equation:

$$2\gamma k_n (A_n^i \pm B_n^i) e^{-ik_n L/2} = \sum_{m=-\infty}^{\infty} \begin{Bmatrix} (\gamma k_n + q_m) e^{-iq_m L/2} \\ \pm(\gamma k_n - q_m) e^{iq_m L/2} \end{Bmatrix} J_{n-m}(\frac{V_1}{\hbar\omega}) C^{\pm} \quad (5)$$

where,

$$C^{\pm} = a_m \pm b_m \quad \text{and} \quad \gamma = \frac{\mu_{II}^*}{\mu_I^*}.$$

following the notations and approach by Al-Sahhar (2006), the last equation can be written by the matrix format as:

$$M_e^{\pm} . C^{\pm} = M_r . (A_n^i \pm B_n^i), \quad (6)$$

The probability amplitudes of the outgoing waves written in the matrix form are:

$$\begin{aligned} A^o &= M_C^{+} .a + M_C^{-} .b - M_i . A^i \\ A^o &= M_{AA} . A^i + M_{AB} . B^i \end{aligned} \quad (7)$$

And

$$\begin{aligned} B^o &= M_C^{-} .a + M_C^{+} .b - M_i . B^i \\ B^o &= M_{BA} . A^i + M_{BB} . B^i \end{aligned} \quad (8)$$

Combining Eq. 7 and 8, we obtain we obtain the matrix S

$$\begin{pmatrix} A^o \\ B^o \end{pmatrix} = \begin{pmatrix} M_{AA} & M_{AB} \\ M_{BA} & M_{BB} \end{pmatrix} \begin{pmatrix} A^i \\ B^i \end{pmatrix} \quad (9)$$

Each element S_{nm} of the matrix S gives the probability amplitude that the electron is scattered from Floquet sideband m to sideband n [n,m $\in(-\infty, \infty)$]. If we only keep the propagating modes [n,m $\in(0, \infty)$], then we obtain the scattering matrix $\bar{S}$,

$$\begin{pmatrix} \bar{A}^o \\ \bar{B}^o \end{pmatrix} = \bar{S} \begin{pmatrix} \bar{A}^i \\ \bar{B}^i \end{pmatrix}, \quad (10)$$

Transmission resonances: The transmission coefficient T is given by:

$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{k_n}{k_m} |t_{nm}|^2$$

where, t_{nm} and r_{nm} are for propagating modes incident from the left. Thus, we consider an infinite number of incoming and outgoing waves with the Floquet energy spacing of $\hbar\omega$ between adjacent channels and compute the transmission coefficients. Keeping the frequency of the time periodic field constant. We plot the transmission coefficient T as a function of electron incident energy E (Fig. 1), through (Fig. 2) for some heterostructures with different values of electron effective mass.

In case that the effective mass of the square well $\mu_{II}^* = 0.067m_e$, (Fig. 1a) the system has only one Fano transmission resonance which for small amplitude of the driving field is associated with the n = -1 localized Floquet evanescent mode which has its origin in the bound state of the undriven system. A resonance occurs at E ≈ 0.825 meV, corresponding to abound state energy -0.175 meV, which is in a good agreement to the calculated value up to two significant digits. The binding energy of the ground state in the field-free case for the deep quantum well (V_0 = -20 meV, L = 10 Å) E_b = -0.174 meV.

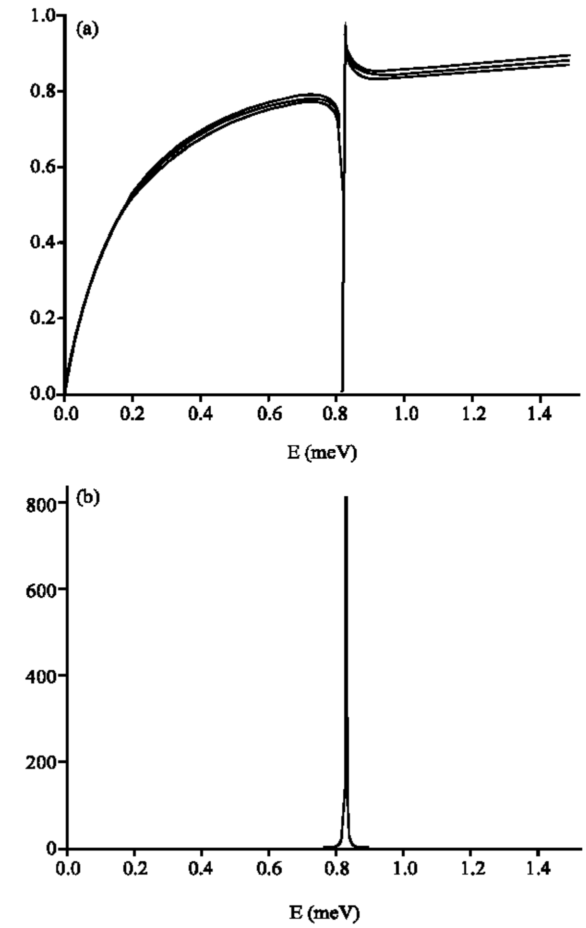

Fig. 1: (a) The transmission coefficient T as a function of incident energy for system parameters $\hbar\omega$ = 1 meV, V_0 = -20 meV, V_1 = 5 meV, L = 10Å and the effective masses of the well region 0.041, 0.092 and 0.15 m_e. A resonance occurs at E ≈ 0.825 meV and (b) The transmission amplitudes of sideband $|t_{-1,0}|^2$ for the same parameters as in (a) shows the accumulation of electrons in the bound state

The Fano-resonances, which are indicated by a dip or a transmission peak/dip in the coefficient T, correspond to quasibound states of the system. We now focus on the transmission coefficient T and discuss how it probes the quasibound states of the system.

At the energy level where the resonance occurs, electrons in the incident channel ($E = \hbar\omega - |E_0| \approx 0.826$ meV) can emit photons and drop to the bound state. Also electrons in the bound state can absorb photons and jump to the incident channel (or other Floquet channels). A transmission resonance takes place when the energy difference between the incident channel and the bound state is equal to the energy of one or more photons.

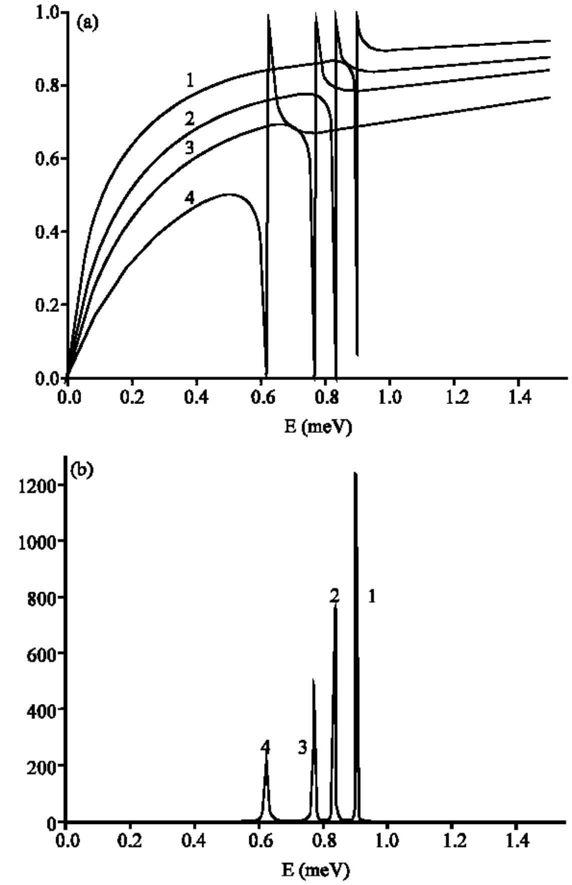

Fig. 2: (a) The transmission coefficient T as a function of incident energy for system parameters $\hbar\omega$ = 1 meV, V_0 = -20 meV, V_1 = 5 meV, L = 10 Å and the effective masses in regions I and III, (where $\mu_I^* = \mu_{III}^*$ and μ_{II}^* = 0.067 m_e) (1) 0.041 m_e (2) 0.067 m_e (3) 0.092 m_e and (4) 0.15 m_e and (b) The transmission amplitudes of sideband $|t_{-1,0}|^2$ for the same parameters as in (a) shows the accumulation of electrons in the bound state

When the effective mass of the square potential is varied to take the other values 0.041, 0.092 and 0.15 m_e, T has the same Fano transmission resonance (Fig. 1a). The sharp rise (Fig. 1b) is due to the fact that the density of states is very large at this bound state. The transmission coefficient T as a function of electron incident energy E, for different values of the effective masses in the regions I and III, the effective mass of the square well μ_{II}^* = 0.067m_e keeps unvaried (Fig. 2a). For this case the system displays one Fano transmission resonance for each configuration of the effective masses in regions I, II and III (E_r = 0.8916, 0.8258, 0.7632 and 0.6150 meV). The bound states are determined by $E_b = -(\hbar\omega - E_r)$, which are found to be in good agreement with the calculated values (Table 1).

Table 1: Comparison of the calculated bound state energy and that derived from S matrix for $\hbar\omega$ = 1 meV, V_0 = -20 meV, V_1 = 5 meV, L = 10 Å

	E_r meV	$E_b = -(\hbar\omega - E_r)$ meV	Calculated E_b meV	(Calculated E_b) $-E_b$ meV
0.041 m_e	0.8916	-0.1084	-0.1069	0.0015
0.067 m_e	0.8258	-0.1742	-0.1738	0.0004
0.092 m_e	0.7632	-0.2368	-0.2376	0.0008
0.15 m_e	0.6150	-0.3850	-0.3837	0.0013

Note in Fig. 2a that the difference between the transmission coefficients T_n for various effective masses becomes more prominent (high up) with decreasing μ_I^* and μ_{III}^*. The reason is that as μ_I^* and μ_{III}^* are decreased more Floquet channels interfere with the incident electron wave and significantly contribute to the total transmission coefficient.

CONCLUSION

The change in the effective mass of the square well does not vary the energy level where resonance occurs and hence the bound state energy of the well can be calculated. In calculating the bound state energy one has to use that of the neighbors of the quantum well. A comparison of the transmission coefficients reveals that as the effective mass is decreased the higher order resonances, for $E > \hbar\omega$, become stronger.

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