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## Analogy Between Heat and Mass Transfer for Constant-Rate Period During Non-Convective Drying: Application to Water-Based Alumina Suspension for Tape Casting

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**Abstract:** Analogy between heat and mass transfer is proposed for the constant-rate period during non-convective drying process. The analogy is applied to predict a temperature-dependent heat-transfer coefficient from a temperature-dependent mass-transfer coefficient also presented for the constant-rate period during non-convective drying of water-based alumina suspension for tape casting. Predicted values of the temperature-dependent heat and mass transfer coefficients are compared with experimental data and good agreement is obtained.

**Key words:** Tape casting, ceramics, transport coefficients

### INTRODUCTION

Tape casting is a process for manufacturing large-area, thin, flat sheets of ceramics, by preparing and drying a layer of ceramic suspension on a support and cutting the material to proper shape and size (Briscoe *et al.*, 1998; Puyate, 2003). The industrial applications of tape-cast products include substrates for thin film circuitry, capacitors, solid electrolytes for sensors and solid oxide fuel cells, piezoelectric ceramics for actuators and transducers and magnesium oxide based materials for photovoltaic solar energy cells (Briscoe *et al.*, 1998). Although organic solvents (alcohol, ketones) are frequently used to prepare concentrated ceramic suspensions, the development of water-based tape casting suspensions is considered desirable in view of the inherent advantages and the extensive knowledge available in processing such systems (Hotza and Griel, 1996; Ryu *et al.*, 1993; Nagata, 1993; Briscoe *et al.*, 1998; Puyate, 1999).

Briscoe *et al.* (1998) presented a detailed experimental study on the formulation and non-convective drying kinetics of water-based alumina suspension for tape casting. They showed that the suspension dries in a two-stage process; the first stage is called the constant-rate period and the second stage is the falling-rate period. During the constant-rate period, about 90 wt.% of moisture is lost and the drying rate is constant. During the falling-rate period, the drying rate gradually decreases as the moisture content of the material falls. They also developed an empirical model which predicts the drying rate per unit surface area of the suspension as a function of temperature and relative humidity in the constant-rate period.

When a body dries with or without flow of the drying medium (gas), the humidity of the gas increases due to evaporation of the liquid. The temperature at the surface of the material falls below that of the gas and heat is transferred from the gas to the surface. At equilibrium, the rate of heat transfer from the gas to the surface just balances that required to vaporize the liquid and the surface of the material is said to be at the wet-bulb temperature. Such a dynamic equilibrium gives the drying rate in the constant-rate period as (Perry and Green, 1984; Ford, 1986):

$$\alpha = \lambda(P_{ws} - P_{wb}) = \frac{h}{E'}(T - T_{wet}) \quad (1)$$

where,  $\alpha$  is the drying rate per unit surface area of the material;  $\lambda$  is the mass-transfer coefficient;  $T$  is the ambient temperature;  $T_{wet}$  is the wet-bulb temperature;  $P_{ws}$  is the partial pressure of water vapour at the drying surface, which is taken here as the saturation vapour pressure of water at a given ambient temperature;  $P_{wb}$  is the partial pressure of water vapour in the surrounding (i.e., bulk air);  $h$  is the heat-transfer coefficient and  $E'$  is taken here as the latent heat of evaporation at a given ambient temperature expressed in  $\text{kJ kg}^{-1}$ . Thus, the drying rate in the constant-rate period can be characterized by either a mass-transfer coefficient or a heat-transfer coefficient. Equation 1 is valid for convective and non-convective drying processes and  $\lambda$  and  $h$  may be defined accordingly.

Puyate (1999, 2003) estimated a constant value of the mass-transfer coefficient,  $\lambda = 7.55 \times 10^{-9} \text{ kg m}^{-2} \text{ sec}^{-1} \text{ Pa}^{-1}$ , for the constant-rate period during non-convective air-drying of the water-based alumina suspension reported in

Briscoe *et al.* (1998) in a temperature range from 298 to 328 K. The dependence of the mass-transfer coefficient on temperature and/or humidity has not been investigated. Also, the heat-transfer coefficient in the constant-rate period during non-convective drying of the suspension has not been presented. This study partly presents a temperature-dependent mass-transfer coefficient for the constant-rate period during non-convective air-drying of water-based alumina suspension for tape casting. Reynolds analogy between heat and mass transfer (Coulson and Richardson, 1977) provides a relationship between heat and mass transfer coefficients which may be used to predict a heat-transfer coefficient for a given transport process if the mass-transfer coefficient is known and vice versa, but the analogy cannot be applied in the constant-rate period for reasons indicated in the following section. This study also proposes an analogy between heat and mass transfer in the constant-rate period for non-convective drying process, which is applied to predict a temperature-dependent heat-transfer coefficient from a temperature-dependent mass-transfer coefficient for the constant-rate period during non-convective air-drying of the water-based alumina suspension.

#### **MODEL DEVELOPMENT**

For the purpose of modeling transport of water vapour in the constant-rate period during non-convective drying of water-based material, air and water vapour are assumed to be two components, with each component exhibiting a concentration gradient between the drying surface and the surrounding. Puyate and Lawrence (1999) also assumed air and water vapour to be two components in their analysis for locating a liquid-gas interface within a concrete element where evaporation of the liquid (salt water) takes place and mass-transfer of water vapour from the interface to the surrounding was modeled as equimolecular counter-diffusion of air and water vapour with bulk flow of these components. As in the case considered by Puyate and Lawrence (1999), the concentration of water vapour in the constant-rate period is higher at the drying surface (where it is produced by evaporation) than in the surrounding, whereas the concentration of air is higher in the surrounding than at the drying surface. Therefore, air will be transferred from the surrounding to the drying surface by diffusion, while water vapour diffuses in the opposite direction. In other words, drying in the constant-rate period involves counter-diffusion of air and water vapour, resulting in a constant evaporative flux of water vapour from the drying surface. The partial pressure of air or water vapour is not constant, but the total pressure (i.e., the sum of the partial

pressures of air and water vapour) is taken to be constant everywhere and equal to the atmospheric pressure. This is true if equal number of moles of air and water vapour diffuse in opposite direction (equimolecular counter-diffusion) and is also true for non-equimolecular counter-diffusion.

During the constant-rate period, moisture movement within a material is rapid enough to maintain a saturated condition in the vapour at the drying surface and the rate of drying is controlled by the rate of moisture transfer away from the evaporating surface. Drying proceeds by diffusion of water vapour from the drying surface of the material across a stagnant air-film into the environment. When water vapour from the drying surface diffuses across the stagnant air-film, it disappears into the surrounding air of relative humidity less than 100%, so a partial pressure gradient is set up causing bulk flow of both water vapour and air away from the drying surface in addition to the transfer of water vapour by diffusion. Since air is not absorbed at the drying surface (i.e., no net transfer of air), the transfer of air by diffusion from the surrounding to the surface exactly balances its transfer by bulk flow away from the drying surface. This mass-transfer process is similar to the one modeled by Puyate and Lawrence (1999) so that equimolecular counter-diffusion of air and water vapour may be assumed in the constant-rate period.

Another case of equimolecular counter-diffusion with bulk flow of two diffusing components is treated in the standard application of Reynolds analogy to mass transfer (Coulson and Richardson, 1977) for gas absorption process, where there is net transfer of the absorbate and no net transfer of the other component. Drying and absorption are identical mass-transfer processes that involve bulk flow of the diffusing components. Although there are similarities between the heat and mass transfer processes that take place in the constant-rate period and the ones presented in Reynolds analogy (Coulson and Richardson, 1977), there are differences which make the original Reynolds analogy between heat and mass transfer inapplicable to the constant-rate period. The similarities include (i) temperature difference exists between a stationary surface and the surrounding, (ii) binary diffusion processes, with opposite fluxes of the diffusing components which is assumed to be equimolecular in Reynolds analogy and (iii) no net transfer of one component (i.e., air in the case of drying) and net transfer of the other component (i.e., water vapour in the case of drying), with bulk flow of the two components. The net transfer of water vapour in the constant-rate period may be thought to occur when the water vapour generated during the drying process is

absorbed by the surrounding air, resulting in the bulk flow of both air and water vapour away from the drying surface. The differences include: (i) the heat and mass transfer processes considered in Reynolds analogy were taking place in a flowing fluid with turbulent flow regime, but the present drying process is non-convective and (ii) most important is the fact that ‘net transfer’ of the absorbate (in absorption process) is the quantity modeled in the application of Reynolds analogy to mass transfer (Coulson and Richardson, 1977), while the evaporative flux of water vapour in the constant-rate period is the result of normal diffusion of water vapour and its bulk flow away from the drying surface which, of course, is the sum of these two fluxes of water vapour (and not the ‘net flux’, i.e., the difference between the two fluxes).

**Analogy for mass transfer in constant-rate period during non-convective drying:** The procedure used in obtaining net transfer of the absorbate in the absorption process considered in the application of Reynolds analogy to mass transfer (Coulson and Richardson, 1977) is adequate. The same procedure is applied in this paper to model the mass transfer process in the constant-rate period as equimolecular counter-diffusion of air and water vapour through the stagnant air-film at the drying surface with bulk flow of the diffusing components to obtain:

$$N_w = \frac{n}{At} \frac{C_{ws}}{C_{As}} \quad (2)$$

where,  $N_w$  is the total molar flux (not net molar flux) of water vapour from the drying surface to the surrounding resulting from normal diffusion of water vapour and its bulk flow away from the surface,  $n = n_{ws} + n_{As} = n_{wb} + n_{Ab}$  is the total number of moles of air and water vapour,  $n_{ws}$  is the number of moles of water vapour at the drying surface,  $n_{As}$  is the number of moles of air at the drying surface,  $n_{wb}$  is the number of moles of water vapour in the surrounding,  $n_{Ab}$  is the number of moles of air in the surrounding,  $C_{ws}$  is the molar concentration of water vapour at the drying surface,  $C_{As}$  is the molar concentration of air at the drying surface,  $A$  is the area of the drying surface and  $t$  is the time of transfer of water vapour. It may be observed that Eq. 2 is not dimensionally consistent due to bulk flow of water vapour and air. However, it is well known that the air-water vapour system considered in the analysis is dilute with  $C_{wb} \ll C_{Ab}$ , where  $C_{wb}$  and  $C_{Ab}$  are the molar concentrations of water vapour and air, respectively in the surrounding. As will be seen in the present analysis,  $C_{ws} \ll C_{As}$  such that  $(n/n_{As}) \approx 1$  in Eq. 2, making the effect of dimensional inconsistency in this equation insignificant.

The total pressure,  $P$  (atmospheric pressure), acting on the fluid (a mixture of air and water vapour) at the drying surface in the constant-rate period is equal to the sum of the partial pressures of air and water vapour at the surface and may be expressed as:

$$P = \frac{\text{Force}}{\text{Area}} \quad (3)$$

The force due to atmospheric pressure acting on the fluid at the drying surface is equal to the force exerted by the fluid on the surface. Since the fluid at the drying surface is stationary for non-convective drying process, the force exerted by the fluid on the surface may be expressed in terms of the weight of the fluid and Eq. 3 becomes:

$$P = \frac{m_s g}{A} = \frac{\bar{\rho}_{ms} V g}{A} = \frac{n \bar{\rho}_{ms} g}{C_T A} \quad (4)$$

where,  $m_s = \bar{\rho}_{ms} V$ , is the mass of a mixture of water vapour and air at the drying surface,  $V$  is the total volume of a mixture of water vapour and air,  $C_T = C_{ws} + C_{As} = C_{wb} + C_{Ab}$  is the total molar concentration of air and water vapour,  $g$  is the acceleration due to gravity and  $\bar{\rho}_{ms}$  is the mean mass density of a mixture of water vapour and air at the drying surface which may be calculated as:

$$\bar{\rho}_{ms} = \rho_{ws} y_{ws} + \rho_{As} y_{As} \quad (5)$$

where,  $\rho_{ws}$  is the mass density of water vapour at the drying surface,  $\rho_{As}$  is the mass density of air at the drying surface,  $y_{ws}$  is the mole fraction of water vapour at the drying surface and  $y_{As}$  is the mole fraction of air at the drying surface.

The atmospheric pressure can also be expressed as a sum of the partial pressures of water vapor and air at the drying surface in the form:

$$P = P_{ws} + P_{As} \quad (6)$$

where,  $P_{As}$  is the partial pressure of air at the drying surface. We note that  $P_{ws}$  and  $P_{As}$  can be expressed in terms of the mass densities of water vapour and air, respectively assuming Ideal Gas behaviour of these components. However, since air and water vapour are treated as two separate components (not a mixture of the two components) in Eq. 6,  $P_{ws}$  and  $P_{As}$  in this equation may be expressed in terms of the mean mass density of air and water vapour at the drying surface (not the mean mass density,  $\bar{\rho}_{ms}$  of a mixture of air and water vapour at the drying surface) to obtain:

$$P = \frac{2\bar{\rho}_{A_{ws}} Vg}{A} \tag{7}$$

with

$$P_{ws} = \frac{n_{ws}\bar{\rho}_{A_{ws}}g}{C_{ws}A} \quad P_{As} = \frac{n_{As}\bar{\rho}_{A_{ws}}g}{C_{As}A} \tag{8}$$

where,  $\bar{\rho}_{A_{ws}} = (\rho_{ws} + \rho_{As})/2$  is the mean mass density of air and water vapour at the drying surface. Equating Eq. 4 and (7) gives:

$$\rho_{ws} + \rho_{As} = \rho_{ws}Y_{ws} + \rho_{As}Y_{As} \tag{9}$$

which holds for a dilute system such as a mixture of air and water vapour at the drying surface where  $C_{ws} \ll C_{As}$ . Note, for example, that replacing  $\bar{\rho}_{A_{ws}}$  with  $\bar{\rho}_{ms}$  in Eq. 7 and equating the resulting expression to Eq. 4 yields  $2\bar{\rho}_{ms} = \bar{\rho}_{ms}$  (which is meaningless), indicating that it is invalid to use  $\bar{\rho}_{ms}$  in Eq. (6-8).

From the Ideal Gas relations  $PV = nR_gT$  and  $P_{ws}V = n_{ws}R_gT$ , we have  $(P/P_{ws}) = (n/n_{ws})$  which when combined with the expression for  $P_{ws}$  in Eq. 8 yields:

$$P = \frac{n\bar{\rho}_{A_{ws}}g}{C_{ws}A} \tag{10}$$

where,  $R_g$  is the universal gas constant (8.314 kJ kmol<sup>-1</sup> K<sup>-1</sup>). It may be necessary to indicate that P in Eq. 10 can also be expressed in terms of  $C_{As}$  but this is not used in the present analysis since transport of water vapour is being modeled. Dividing Eq. 2 by 10 and further rearrangement yields:

$$N_w = \frac{C_{ws}^2}{C_{As}} \frac{P}{\bar{\rho}_{A_{ws}}gt} = h_D(C_{ws} - 0) \tag{11}$$

to give the analogy for mass transfer in the constant-rate period for non-convective drying process as:

$$h_D \frac{C_{As}}{C_{ws}} = \frac{P}{\bar{\rho}_{A_{ws}}gt} \tag{12}$$

It may be seen that  $N_w$  in Eq. 11 is the drying rate (in molar units) per unit area in the constant-rate period for dry air with  $C_{wb} = 0$ , where  $h_D = (C_{ws}/C_{As}) P/\bar{\rho}_{A_{ws}}gt$  is the mass-transfer coefficient expressed in kmol m<sup>-2</sup> sec<sup>-1</sup> (kmol m<sup>-3</sup>)<sup>-1</sup> and is equivalent to  $\lambda$  in Eq. 1 expressed in kg m<sup>-2</sup> sec<sup>-1</sup> Pa<sup>-1</sup>.

**Analogy for heat transfer in constant-rate period during non-convective drying:** The heat necessary for evaporation in the constant-rate period is transferred from the ambient moist air (i.e., a mixture of air and water

vapour) to the drying surface. If the temperature difference between the ambient moist air and the drying surface is  $\Delta T$  and the specific heat capacity of the ambient moist air is  $C_p$ , the quantity of heat transferred from the surrounding to the drying surface is  $m_b C_p \Delta T$ , where,  $m_b = \bar{\rho}_{mb} V$  is the mass of ambient moist air and  $\bar{\rho}_{mb}$  is the mean mass density of a mixture of water vapour and air in the surrounding moist air which may be calculated as:

$$\bar{\rho}_{mb} = \rho_{wb}Y_{wb} + \rho_{Ab}Y_{Ab} \tag{13}$$

where,  $\rho_{wb}$  and  $\rho_{Ab}$  are the mass densities of water vapour and air, respectively in the ambient moist air and  $y_{wb}$  and  $y_{Ab}$  are the mole fractions of water vapour and air respectively in the ambient moist air. An equation similar to Eq. 9 can be written for the ambient moist air (where  $C_{wb} \ll C_{Ab}$ ) through Eq. 13 as:

$$\rho_{wb} + \rho_{Ab} = \rho_{wb}Y_{wb} + \rho_{Ab}Y_{Ab} \tag{14}$$

The relation

$$P = P_{ws} + P_{As} = P_{wb} + P_{Ab} \tag{15}$$

yields

$$\rho_{ws} + \rho_{As} = \rho_{wb} + \rho_{Ab} \tag{16}$$

such that Eq. 9 and 14 can be combined to give  $\bar{\rho}_{ms} = \bar{\rho}_{mb}$  and consequently  $m_s = m_b$ , where  $P_{wb}$  and  $P_{Ab}$  are the partial pressures of water vapour and air respectively in the ambient moist air. Equation 16 indicates that  $\bar{\rho}_{A_{ws}}$  can also be calculated as  $\bar{\rho}_{A_{ws}} = (\rho_{wb} + \rho_{Ab})/2$ .

The quantity of heat transferred from the ambient moist air to the drying surface per unit time is given by:

$$qA = \frac{m_b C_p \Delta T}{t} \tag{17}$$

where, q is the heat flux. Equation 4 holds in the constant-rate period during the heat-transfer process and is restated here for the sake of convenience as:

$$P = \frac{m_s g}{A} \tag{18}$$

Dividing Eq. 17 by 18 and further rearrangement gives the heat-transfer coefficient, h, between the surrounding moist air and the drying surface as:

$$h = \frac{q}{\Delta T} = \frac{PC_p}{gt} \tag{19}$$

to obtain

$$\frac{h}{\bar{\rho}_{A_{ws}} C_p} = \frac{P}{\bar{\rho}_{A_{ws}} g t} \tag{20}$$

Equation 20 is the analogy for heat transfer in the constant-rate period during non-convective drying process. In this heat-transfer analysis, no provision is made for variation in physical properties of the fluid with temperature.

**Analogy between heat and mass transfer in constant-rate period during non-convective drying:** Combining Eq. 12 and 20 gives the relationship between heat and mass transfer in the constant-rate period for non-convective drying process as:

$$\frac{h}{C_p \bar{\rho}_{A_{ws}}} = \frac{h_D C_{As}}{C_{ws}} \tag{21}$$

**Application of present analogy between heat and mass transfer to suspension drying:** For the air-water vapour system considered in the analysis,  $P_{ws}$  and  $P_{wb}$  are both small compared to atmospheric pressure so that  $P_{wb}/P_{ws}$  is approximately equal to the relative humidity expressed as a fraction. The mass-transfer part of Eq. 1 may then be expressed as:

$$\alpha = \lambda P_{ws} \left( 1 - \frac{RH}{100} \right) \tag{22}$$

where, RH is the relative humidity expressed as a percentage. During the constant-rate period, the surface of a material behaves like a free liquid surface and the rate of evaporation per unit area from such surface is given by Briscoe *et al.* (1998) and Davies and Rideal (1961):

$$\alpha' = B \exp(-E/R_g T) \tag{23}$$

where, B is a constant independent of temperature and humidity and E is the latent heat of evaporation at a given ambient temperature. Equation 23 is analogous to the expression for the rate of desorption of a species from a surface in which the effect of the partial pressure of the species in the gaseous phase is insignificant and neglected (Morrison, 1990); that is, Eq. 23 corresponds approximately to the drying rate of a material in the constant-rate period for dry air which is indicated by the prime in this equation.

Briscoe *et al.* (1998) introduced the effect of relative humidity into Eq. 23 by assuming B to be a linear function of relative humidity and obtained the drying rate per unit area in the constant-rate period during non-convective drying of water-based alumina suspension for tape casting as:

$$\alpha = (a_n + b_n RH) \exp(-E^*/R_g T) \tag{24}$$

where,  $a_n = 389.8 \text{ kg m}^{-2} \text{ sec}^{-1}$  and  $b_n = -3.94 \text{ kg m}^{-2} \text{ sec}^{-1} \text{ RH}^{-1}$  are constants and  $E^* = 41.2 \text{ kJ mol}^{-1}$  is the average latent heat of vaporization of the suspension in a temperature range from 298 to 328 K. Putting  $RH = 0$  into Eq. 22 and equating the resulting expression to Eq. 23 indicates that  $P_{ws}$  depends upon temperature in an Arrhenius form which when substituted back into Eq. 22 shows that Eq. 22 and 24 are the same. However, inspection of Eq. 24 reveals that it does not satisfy the zero-drying rate condition at 100% relative humidity with negligible error of about 1.0% at this relative humidity. Thus Eq. 22 and 24 may be equated to obtain an approximate model for the mass-transfer coefficient in the constant-rate period during non-convective drying of the suspension as:

$$\lambda(T) \approx \frac{a_n}{P_{ws}} \exp(-E^*/R_g T) \tag{25}$$

or in the equivalent form

$$h_D(T) \approx \frac{a_n}{M_w \bar{C}_{ws}} \exp(-E^*/R_g T) \tag{26}$$

where,  $\lambda(T)$  and  $h_D(T)$  indicate that the mass-transfer coefficients are functions of temperature in their respective units,  $M_w$  is the molar mass of water vapour and the bar at the top of the molar concentration variable means average value of the variable in the temperature range  $298 \text{ K} \leq T \leq 328 \text{ K}$  (range of temperature in the experiments of Briscoe *et al.* (1998) used in the analysis). Replacing  $h_D$  in Eq. 21 with the expression for  $h_D(T)$  gives the corresponding heat-transfer coefficient for the constant-rate period during non-convective drying of the suspension as:

$$h(T) = \frac{a_n C_p \bar{\rho}_{A_{ws}} \bar{C}_{As}}{M_w \bar{C}_{ws}^2} \exp(-E^*/R_g T) \tag{27}$$

The total mass density,  $\rho_T$ , of a mixture is a function of temperature. Quite often, the dependence of  $\rho_T$  on temperature is weak and the total mass density can be considered constant for a range of temperature (Stephanopoulos, 1984). Thus,  $\rho_T$  is taken to be constant

in the present analysis where the temperature range for drying of tape-cast films is narrow (Briscoe *et al.*, 1998). The total molar concentration ( $C_T$ ) of a mixture and  $\rho_T$  are related in the form  $C_T = \rho_T/M_m$ , where,  $M_m$  is the molar mass of the mixture. Since  $\rho_T$  is constant,  $C_T$  would also be constant which requires averaging  $C_T$  for the specified temperature range. This explains the averaged molar concentration variables in Eq. 26 and 27 noting that  $\bar{C}_T = \bar{C}_{ws} + \bar{C}_{As}$ , but  $P = P_{As} + P_{ws} = \bar{P}_{As} + \bar{P}_{ws}$ .

**CALCULATION OF PARAMETERS OF AIR AND WATER VAPOUR**

The total molar concentration,  $C_T$  is calculated assuming ideal gas behaviour as:

$$C_T = P/R_g T \tag{28}$$

where,  $P$  is the total pressure (atmospheric). Water vapour is assumed to be saturated at the drying surface of the suspension. The humidity ( $H_0$ ) of saturated water vapour at 100% relative humidity and a given ambient temperature is estimated from a humidity-temperature chart for air-water vapour system at atmospheric pressure. The partial pressure of saturated water vapour at the drying surface,  $P_{ws}$ , is then calculated from  $H_0$  using the relation (Coulson and Richardson, 1977):

$$H_0 = \frac{P_{ws} M_w}{(P - P_{ws}) M_A} \tag{29}$$

where,  $M_A$  is the molar mass of air. Accordingly,  $C_{ws}$  is calculated from  $P_{ws}$  as:

$$C_{ws} = P_{ws}/R_g T \tag{30}$$

The experiments of Briscoe *et al.* (1998) were carried out at different temperatures (298 K, 313 K, 328 K), with the relative humidity at each temperature varied in succession as 40, 65 and 90%. The tape-cast films dried by Briscoe *et al.* (1998) are very thin (0.6, 0.9 and 1.25 mm thicknesses) so that low-temperature heating within the range used is required to avoid cracking, warping, or fracture of the material at high temperature. Table 1 shows calculated values of  $\bar{C}_{ws}$ ,  $\bar{C}_{As}$  and  $\bar{C}_T$  using Eq. (28-30) at the experimental temperatures.

A value of  $C_p$  for ambient moist air is calculated as (Coulson and Richardson, 1977; Eastop and McConkey, 2003):

$$C_p = 1.0 + 1.9H \tag{31}$$

with

Table 1: Calculated parameters of air and water vapour at the drying surface

Temperature (K)	$H_0$ (kg kg <sup>-1</sup> )	$P_{ws}$ (Pa)	$C_{ws} (\times 10^3)$ (kmol m <sup>-3</sup> )	$C_T (\times 10^3)$ (kmol m <sup>-3</sup> )	$C_{As} (\times 10^3)$ (kmol m <sup>-3</sup> )
298	0.019	3012.79	1.22	40.90	39.68
313	0.045	6856.58	2.63	38.94	36.31
328	0.112	15503.28	5.69	37.16	31.47
Mean			$\bar{C}_{ws} = 3.18$	$\bar{C}_T = 39.0$	$\bar{C}_{As} = 35.82$

Table 2: Calculated values of  $C_p$  for ambient moist air

RH (K)	$C_p$ (kJ kg <sup>-1</sup> K <sup>-1</sup> )			
	40%RH	65%RH	90%RH	Mean
298	1.014	1.023	1.032	1.023
313	1.033	1.054	1.076	1.054
328	1.077	1.130	1.190	1.130
Mean	1.041	1.069	1.099	$C_p^* = 1.07$

Table 3: Calculated mass densities of air and water vapour at the drying surface

Temperature (K)	$H_0$ (kg kg <sup>-1</sup> )	$P_{ws}$ (Pa)	$\rho_{ws}$ (kg m <sup>-3</sup> )	$\rho_{As}$ (kg m <sup>-3</sup> )
298	0.019	3012.79	0.022	1.15
313	0.045	6856.58	0.047	1.05
328	0.112	15503.28	0.102	0.91
Mean			$\bar{\rho}_{ws} = 0.057$	$\bar{\rho}_{As} = 1.04$

$$H = \frac{P_{ws} M_w}{(P - P_{ws}) M_A} \quad P_{ws} = P_{ws} (RH/100) \tag{32}$$

where,  $H$  is the humidity of air. Calculated values of  $C_p$  using Eq. 31 and 32 at the experimental conditions of Briscoe *et al.* (1998) are presented in Table 2.

An overall mean value of  $C_p$  denoted by  $C_p^*$ , is calculated as the mean of the mean values of  $C_p$  for all the rows or columns in Table 2.

The mass densities of water vapour and air at the drying surface, denoted by  $\rho_{ws}$  and  $\rho_{As}$ , respectively, are calculated as (Coulson and Richardson, 1977):

$$\rho_{ws} = M_w P_{ws} / R_g T \quad \rho_{As} = M_A (P - P_{ws}) / R_g T \tag{33}$$

Table 3 shows calculated mass densities of air and water vapour using Eq. 33 at the experimental temperatures, where  $\bar{\rho}_{ws}$  and  $\bar{\rho}_{As}$  are the mean values of  $\rho_{ws}$  and  $\rho_{As}$ , respectively for the different temperatures. An overall mean mass density of air and water vapour at the drying surface, denoted by  $\hat{\rho}_{Aws}$ , is then calculated as  $\hat{\rho}_{Aws} = (\bar{\rho}_{ws} + \bar{\rho}_{As}) / 2$ .

**RESULTS AND DISCUSSION**

Equation 1 was used to calculate experimental values of the heat and mass transfer coefficients shown in Table 4 and 5, respectively from experimental drying rate

Table 4: Experimental values of heat-transfer coefficient for the suspension

Temperature (K)	$h (\times 10^2 \text{ kJ m}^{-2} \text{ sec}^{-1} \text{ K}^{-1})$			
	40%RH	65%RH	90%RH	$\bar{h}$
298	3.08	3.56	3.67	3.44
313	5.77	5.74*	5.095*	5.54*
328	10.027	12.56	13.25	12.036

\*Incorrect value

Table 5: Experimental values of mass-transfer coefficient for the suspension

Temperature (K)	$\lambda (\times 10^9 \text{ kg m}^{-2} \text{ sec}^{-1} \text{ Pa}^{-1})$			
	40%RH	65%RH	90%RH	$\bar{\lambda}$
298	7.07	7.13	7.99	7.40
313	6.95	6.27*	4.93*	6.05*
328	6.39	6.51	7.47	6.79

\*Incorrect value

Table 6: Calculated values of  $P_{wb}$  at experimental conditions

Temperature (K)	$P_{wb}$ (Pa)		
	40%RH	65%RH	90%RH
298	1205.12	1958.31	2711.51
313	2742.63	4456.78	6170.92
328	6201.31	10077.13	13952.95

data (Briscoe *et al.*, 1998) on non-convective air-drying of the water-based suspension in the constant-rate period, where  $E' \approx 2289 \text{ kJ kg}^{-1}$  is the average latent heat of evaporation of the water-based alumina suspension for the specified temperature range and values of  $P_{wb}$  calculated from Eq. 32 at the experimental conditions are presented in Table 6.

The value of  $T_{wet}$  corresponding to a given ambient temperature and relative humidity was estimated from a humidity-temperature chart (Coulson and Richardson, 1977; Himmelblau, 2003) for air-water vapour system at atmospheric pressure. Table 7 shows estimated values of  $T_{wet}$  at the experimental conditions of Briscoe *et al.* (1998).

The diameter of the cylindrical sample holder used in the weight-loss experiments (Briscoe *et al.*, 1998) on non-convective air-drying of the suspension was 13 mm; this gives the surface area of the suspension as  $A = 1.33 \times 10^{-4} \text{ m}^2$ .

It may be seen by studying the trend of values of  $h$  and  $\lambda$  in Table 4 and 5, respectively that these transport coefficients increase with increase in relative humidity at a given ambient temperature,  $h$  increases with increase in temperature at a given relative humidity while  $\lambda$  decreases with increase in temperature at a given relative humidity. These conditions are not fully satisfied by the values marked asterisk (\*) in Table 4 and 5, indicating that the experimental drying rates presented by Briscoe *et al.* (1998) corresponding to the asterisked values are incorrect. It may also be seen from Table 4 that the variation of  $h$  with relative humidity at a given ambient temperature is weaker than its variation with temperature at a given relative humidity. Thus, the mean value of  $h$

Table 7: Corresponding values of  $T$  and  $T_{wet}$  at experimental conditions

Temperature (K)	$T_{wet}$ (K)		
	40%RH	65%RH	90%RH
298	288.50	293.16	296.50
313	301.67	307.00	311.48
328	314.44	321.56	326.00

Table 8: Experimental and predicted values of heat and mass transfer coefficients for the suspension

Temperature (K)	$\bar{h}(\text{exp.}) \times 10^3$	$\hat{h}(\text{pred}) \times 10^3$	$\bar{\lambda}(\text{exp.}) \times 10^9$	$\lambda(\text{pred.}) \times 10^9$
	$(\text{kJ m}^{-2} \text{ sec}^{-1} \text{ K}^{-1})$	$(\text{kJ m}^{-2} \text{ sec}^{-1} \text{ K}^{-1})$	$(\text{kg m}^{-2} \text{ sec}^{-1} \text{ Pa}^{-1})$	$(\text{kg m}^{-2} \text{ sec}^{-1} \text{ Pa}^{-1})$
298	3.44	2.71	7.40	7.76
313	5.54*	6.01	6.05*	7.57
328	12.036	12.39	6.79	6.90

\*Incorrect value

(denoted by  $\bar{h}$ ) for a range of relative humidity at a given ambient temperature is used as calculated in Table 4. Also, the mean value of  $\lambda$  (denoted by  $\bar{\lambda}$ ) for a range of relative humidity at a given ambient temperature is used as calculated in Table 5.

Replacing  $C_p$  and  $\bar{P}_{Aws}$  in Eq. 27 with  $C_p^*$  and  $\hat{P}_{Aws}$ , respectively, gives:

$$\hat{h}(T) = \frac{a_n C_p^* \hat{P}_{Aws} \bar{C}_{As}}{M_w C_{ws}^2} \exp(-E' / R_g T) \tag{34}$$

where,  $\hat{h}(T)$  is the temperature-dependent heat-transfer coefficient based on  $C_p^*$  and  $\hat{P}_{Aws}$ . Table 8 shows the comparison between experimental and predicted values of the heat and mass transfer coefficients in the constant-rate period during non-convective air-drying of the suspension, where  $\bar{h}(\text{exp.})$  and  $\bar{\lambda}(\text{exp.})$  denote experimental mean values of the heat and mass transfer coefficients, respectively for a range of relative humidity at a given ambient temperature,  $\lambda(\text{pred.})$  is predicted value of the mass-transfer coefficient from Eq. 25 and  $\hat{h}(\text{pred.})$  is predicted value of the heat-transfer coefficient from Eq. 34 (based on the proposed analogy between heat and mass transfer in the constant-rate period).

It may be seen from Table 8 that good agreement is obtained between the experimental and predicted data even though the asterisked values are lower than expected. It is also evident from Table 8 that a constant value of the heat-transfer coefficient is inadequate in the constant-rate period, while a constant value of the mass-transfer coefficient such as the one presented by Puyate (2003) may be used to characterize the drying rate in the constant-rate period of the suspension.

### CONCLUSION

Analogy between heat and mass transfer is proposed for the constant-rate period during non-convective drying



process. A model is also presented for temperature-dependent mass-transfer coefficient in the constant-rate period during non-convective drying of water-based alumina suspension for tape casting. The proposed analogy between heat and mass transfer is applied to predict a temperature-dependent heat-transfer coefficient from the temperature-dependent mass-transfer coefficient in the constant-rate period during non-convective drying of the suspension. It is shown that the models presented for the temperature-dependent heat and mass transfer coefficients in the constant-rate drying period of the suspension are adequate, indicating that the proposed analogy between heat and mass transfer in the constant-rate period for non-convective drying process is also adequate. It would be worthwhile to test the proposed analogy using experimental data from different workers, but such data with the required parameters are not available at the moment. The present analysis also indicates that the mean mass density of a dilute two-component mixture like the air-water vapour system considered in the analysis equals the total mass density of the mixture. The development of analogy between heat and mass transfer for constant-rate period during convective drying process remains the focus of continuing work.

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