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A Search Algorithm for Determination of Economic Order Quantity in a Two-Level Supply Chain System with Transportation Cost

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Abstract: This study considers a two-level supply chain system consisting of one warehouse and a number of identical retailers. In this system, we incorporate transportation costs into inventory replenishment decisions. The transportation cost contains a fixed cost and a variable cost. We assume that the demand rate at each retailer is known and the demand is confined to a single item. First, we derive the total cost which is the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. Then, we propose a search algorithm to find the economic order quantities for the warehouse and retailers which minimize the total cost.

Key words: Economic order quantity, inventory control, transportation, supply chain management

INTRODUCTION

The goal of most research efforts related to the supply chain management is to present mechanisms to reduce operational costs. The most important operational costs in a supply chain are the inventory cost and the transportation cost.

In the replenishment process, other than the inventory cost, the transportation cost is a major cost factor that affects the optimal shipment size.

Some articles in supply chain consider the transportation cost as a part of the ordering cost and assume it is independent of the shipment size (Hill, 1997; Goyal and Nebebe, 2000; Hoque and Goyal, 2000). Schuster and Bassok (1997), Qu *et al.* (1999), Nozick and Turnquist (2001) and Aghezzaf *et al.* (2006) studied the integrated inventory-transportation systems. In their works, the transportation cost is affected by the routing decisions and does not depend on the shipment size.

In many practical cases, the transportation cost is affected by the shipment size and vice versa. So, it is important to determine the economic order quantity which minimizes the overall logistics costs.

Bregman *et al.* (1990) proposed a heuristic method for the control of inventory in a multi-level environment with transportation cost under deterministic and dynamic demand and finite horizon. Ganeshan (1999) introduced a three-level supply chain consisting of a number of identical retailers, one central warehouse and a number of identical suppliers. In his model, the objective function consists of the ordering, the holding and the transportation costs. He considered the transportation cost as a function of the order quantity but ignored the capacity of the vehicle. Swenseth and Godfrey (2002) studied the effect of the transportation cost discounts on ordering decision when over declaring a shipment is possible. Huang *et al.* (2005) considered a two-level supply chain system with transportation capacity constraint. They applied the Zero Inventory Ordering (ZIO) policy in which the replenishment is made at equally spaced time intervals and orders are placed only when inventory levels are zero. Ertogral *et al.* (2007) considered a vendor-buyer supply chain model and incorporated the transportation cost. In their study, the transportation is made by one type of vehicle whose cost is a function of the shipment size; this function has an all-unit-discount structure. Our model differs from the one proposed by Ertogral *et al.* (2007) in the sense that we assume there are three types of vehicles which are defined as small, medium and large. Each type has its own fixed cost, variable cost and the capacity size.

THE MODEL

In this study, we consider a two-level supply chain consisting of one warehouse and a number of identical retailers (Fig. 1). We assume that the demand rate at each retailer is known and the demand is confined to a single item. Shortage is allowed neither at the retailers nor at the warehouse. The transportation time for an order to arrive at a retailer from the warehouse is assumed to be constant. The warehouse orders to an external supplier.

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Table 1: Transportation scheme

Vehicle type	Capacity	Fixed cost	Variable cost
S	q_1	F_1	v_1
M	q_2	F_2	v_2
L	q_3	F_3	v_3

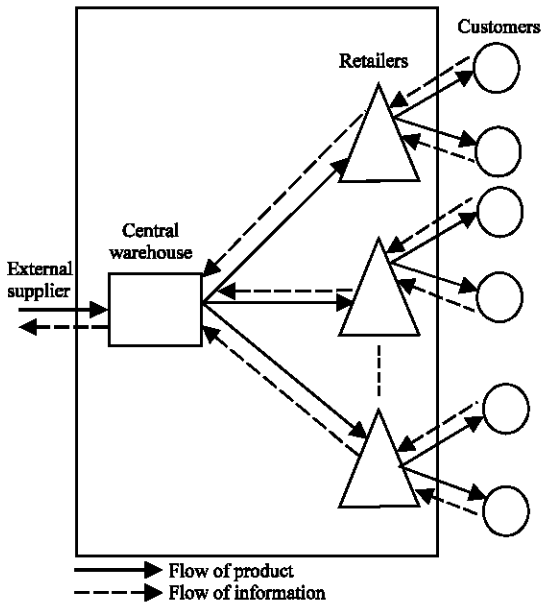

Fig. 1: A two-level supply chain

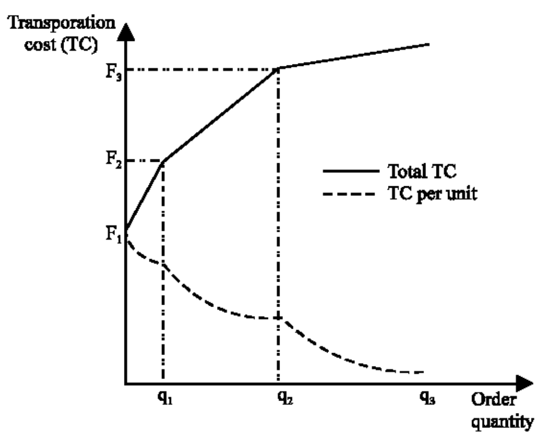

Fig. 2: Variations of transportation cost

The lead time for an order to arrive at the warehouse is assumed to be constant. We assume that the retailers are identical i.e., the parameters related to the retailers such as the demand rate, the rate of holding cost, the ordering cost and the transportation time are same for all the retailers. The objective is to find the economic order quantities for the warehouse and retailers which minimize the total cost. The total cost is the sum of the holding and ordering costs at the warehouse and retailers as well as the transportation cost from the warehouse to retailers.

In this model, we suppose that there are three types of vehicles and delivery of each order from warehouse to a retailer is made by a single vehicle without splitting. It is a common transportation scheme in most practical cases. We define these types as small (S), medium (M) and large (L). Each type has its own fixed cost, variable cost and the capacity size (Table 1).

It is assumed that $F_1<F_2<F_3$, $v_1>v_2>v_3$, $q_1<q_2<q_3$, $F_2 = F_1+q_1(v_1-v_2)$ and $F_3 = F_2+q_2(v_2-v_3)$. These equations are supposed to avoid any over declaration. Hence, the transportation cost varies according to the order quantity as shown in Fig. 2.

FORMULATION OF THE TOTAL COST

In this section, we intend to derive the total cost. The total cost is the sum of the holding and ordering costs at the warehouse and retailers as well as the transportation cost from the warehouse to retailers.

The notations used in the formulation are as follows:

D_r = Demand rate at a retailer
A_r = Ordering cost for a retailer
A_w = Ordering cost for the warehouse
h_r = Rate of holding cost at a retailer
h_w = Rate of holding cost at the warehouse
Q_r = Order quantity at a retailer
Q_w = Order quantity at the warehouse
m = Number of retailers

We have assumed that the demand rate at the retailers and the transportation time to the retailers are constant and shortage is not allowed at the retailers. Hence, the inventory level at the retailers is a simple EOQ model.

It is assumed that there is no lot-splitting at the warehouse. Furthermore, shortage is not allowed at the warehouse so the order quantity of the warehouse includes an integer multiple (n) of the order quantity of each retailer. Since there are m identical retailers therefore the order quantity of the warehouse is $Q_w = mnQ_r$. For optimal solution the arrival of an order to the warehouse corresponds to the delivery of an order to each retailer. Thus, the maximum inventory level at the warehouse is Q_w-mQ_r.

The total cost is the sum of the holding and ordering costs at the retailers and the warehouse plus the transportation cost from the warehouse to retailers. Thus, the total cost can be written as:

$$C_{Ti}(Q_w,Q_r)=\frac{D_wA_w}{Q_w}+\frac{h_w(Q_w-mQ_r)}{2}+m\left(\frac{D_rA_r}{Q_r}+\frac{h_rQ_r}{2}+\frac{D_rF_i}{Q_r}+D_rv_i\right),\quad i=1,2,3 \tag{1}$$

To minimize the above cost we must consider the following constraints:

$$q_{(i-1)} < Q_r \leq q_i \tag{1.1}$$

$$Q_w = mnQ_r \tag{1.2}$$

$$n \text{ is a positive int and } q_0 = 0 \tag{1.3}$$

Index i in (1) denotes the vehicle types; 1, 2 and 3, respectively for S, M and L. D_w is the demand rate at the warehouse which is sum of the demand rates at the retailers, $D_w = mD_r$.

Substituting mnQ_r for Q_w and mD_r for D_w in (1) then our mathematical problem can be defined as:

$$\begin{aligned}\text{Min } C_{Ti}(n, Q_r) &= \frac{D_rA_w}{nQ_r} + \frac{h_w m Q_r (n-1)}{2} \\ &+ m\left(\frac{D_rA_r}{Q_r} + \frac{h_rQ_r}{2} + \frac{D_rF_i}{Q_r} + D_r v_i\right), \quad i = 1,2,3\end{aligned} \tag{2}$$

s.t.

$$q_{(i-1)} < Q_r \leq q_i \tag{2.1}$$

$$n \text{ is a positive int and } q_0 = 0 \tag{2.2}$$

SEARCH ALGORITHM TO FIND THE OPTIMAL SOLUTION

The total cost function (2) has the piece-wise convex property for a given value of n. This property is originated by the transportation scheme supposed in the model (Fig. 3).

Figure 2 shows that the transportation cost has an incremental discount structure. Hence, for a given value of n the method of obtaining the optimal value of Q_r is the same as the one described by Hadley and Whitin (1963) for incremental quantity discount model.

We develop a search algorithm to obtain the optimal value of n and Q_r. As mentioned earlier, we apply the incremental quantity discount method for a given value of n. To create our search algorithm we need a lower bound and an upper bound for n. Since n is a positive integer thus 1 can be a lower bound for n. The following proposition generates an upper bound for n.

Proposition: The upper bound of n is:

$$n_u = \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \right\rfloor, \text{ if } \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \right\rfloor = 0 \text{ then } n_u = 1$$

($\lfloor X \rfloor$ represents the largest integer less than or equal to X).

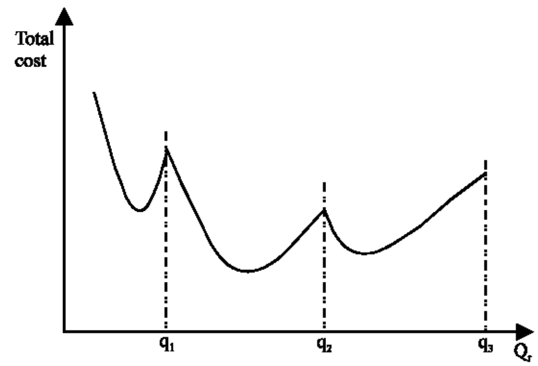

Fig. 3: Total cost for a given value of n

Proof: In the first interval of Q_r, the total cost function is:

$$\begin{aligned}C_{T1}(n, Q_r) &= \frac{D_rA_w}{nQ_r} + \frac{h_w m Q_r (n-1)}{2} \\ &+ m\left(\frac{D_rA_r}{Q_r} + \frac{h_rQ_r}{2} + \frac{D_rF_1}{Q_r} + D_r v_1\right)\end{aligned} \tag{3}$$

If we set the derivatives of C_{T1} with respect to Q_r and n equal to zero we obtain:

$$Q^*_r = \sqrt{\frac{2D_r(\frac{A_w}{n} + mA_r + mF_1)}{m(h_w(n-1) + h_r)}} \tag{4}$$

$$n^* = \frac{1}{Q_r}\sqrt{\frac{2D_rA_w}{mh_w}} \tag{5}$$

Substituting Q^*_r in (3) we have:

$$C_{T1}(n) = \sqrt{2D_r m(\tfrac{A_w}{n} + mA_r + mF_1)(h_w(n-1) + h_r)} + Dv_1 \tag{6}$$

The value of n which optimizes $C_{T1}(n)$ is obtained as:

$$n^* = \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \tag{7}$$

From Eq. 5, it is clear that n and Q_r have an inverse relation. The value of Q_r obtained from Eq. 4 is a lower bound on Q_r, because there is no gain to decrease Q_r less than Q^*_r. Hence, the n* in Eq. 7 would be an upper bound on n.

In our search algorithm we need the incremental quantity discount method. The steps of this method are as follows (Hadley and Whitin, 1963):

Step 1: Compute Q_{ri}, the value of Q_r which minimizes $C_{Ti}(n, Q_r)$ for i = 1, 2, 3. From Eq. 2 we see that:

$$Q_{ri} = \sqrt{\frac{2D_r(\frac{A_w}{n} + mA_r + mF_i)}{m(h_w(n-1) + h_r)}} \tag{8}$$

Step 2: For those Q_{ri} which are $q_{(i-1)} < Q_{ri} \le q_i$ determine $C_{Ti}(n, Q_{ri})$.

Step 3: The Q_{ri} corresponding to the minimum of those costs (in step 2) is the optimal value of Q_r.

In summary, the search algorithm to obtain the optimal values of n and Q_r is as follows:

Algorithm:

Step 1:

$$\text{Set } n_u = \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_l)}} \right\rfloor, \text{ if } \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_l)}} \right\rfloor = 0 \text{ than } n_u = 1$$

Step 2: For n = 1, 2,…,n_u find the corresponding optimal value of Q_r as follows:

Step 2.1. Compute Q_{ri} using Eq. 8.
Step 2.2. For those Q_{ri} which are $q_{(i-1)} < Q_{ri} \le q_i$ determine $C_{Ti}(n, Q_{ri})$.
Step 2.3. The Q_{ri} corresponding to the minimum of those costs (in step 2.2) is the optimal value of Q_r.

Step 3: For n = 1, 2, …, n_u and the corresponding optimal value of Q_r calculate the total cost.

Step 4: The solution which has the minimum total cost among the solutions in step 3 is the overall optimal solution.

NUMERICAL EXAMPLE

To clarify the steps of the search algorithm, we solve a numerical example. The problem data is as follows:

A_w = 300 €
A_r = 25 €
h_w = 2 € unit^{-1} year^{-1}
h_r = 10 € unit^{-1} year^{-1}
D_r = 1500 unit year^{-1}
m = 3

Table 2 gives the relevant data on the transportation.

Solution Algorithm:
Step 1:

$$n_u = \left\lfloor \sqrt{\frac{300 \times (10-2)}{3 \times 2 \times (25+10)}} \right\rfloor = 3$$

Table 2: Transportation data

Vehicle type	Capacity (unit)	Fixed cost (€)	Variable cost (€ unit^{-1})
S	100	10	0.30
M	200	20	0.20
L	275	30	0.15

Step 2:
For n = 1:
Step 2.1: Using Eq. 8 Q_{r1} = 201.2, Q_{r2} = 208.6 and Q_{r3} = 215.6.
Step 2.2: 200<Q_{r3}< = 275 and C_{T3}(1, 215.6) = 7144.2.
Step 2.3: For n = 1 the optimal value of Q_r is 215.6.
For n = 2:
Step 2.1: Using Eq. 8 Q_{r1} = 145.8, Q_{r2} = 154.1 and Q_{r3} = 162.0.
Step 2.2: 100<Q_{r2}< = 200 and C_{T2}(2, 154.1) = 6448.0.
Step 2.3: For n = 2 the optimal value of Q_r is 154.1.
For n = 3:
Step 2.1: Using Eq. 8 Q_{r1} = 121.0, Q_{r2} = 129.6 and Q_{r3} = 137.6.
Step 2.2: 100<Q_{r2}< = 200 and C_{T2}(3, 129.6) = 6341.5.
Step 2.3: For n = 3 the optimal value of Q_r is 129.6.

Step 3:
For n = 1 and Q_r = 215.6:
C_{T3}(1, 215.6) = 7144.2.
For n = 2 and Q_r = 154.1:
C_{T2}(2, 154.1) = 6448.0.
For n = 3 and Q_r = 129.6:
C_{T2}(3, 129.6) = 6341.5

Step 4:
The optimal solution is n = 3, Q_r = 129.6 and C_{T2}(3, 129.6) = 6341.5.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this study, we considered a two-level supply chain system consisting of one warehouse and a number of identical retailers. Unlike the common practice which determines the economic order quantity according to inventory costs only, in this model we incorporated transportation costs into inventory replenishment decisions. We derived the total cost which is the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. The total cost function is a piece-wise convex function. Based on this property, we proposed a search algorithm to obtain the optimal solution. We provided a numerical example to show that one can apply easily the steps of the search algorithm.

For future research one can expand this model by including the multi-item lot-sizing problem. This model can be more practical if the number of vehicles is a decision variable.

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