

# Journal of Applied Sciences

ISSN 1812-5654





# Geomorphological Runoff Routing Modeling Based on Linear Reservoirs Cascade

<sup>1</sup>V. Nourani, <sup>2</sup>V.P. Singh, <sup>1</sup>M.T. Alami and <sup>1</sup>H. Delafroz <sup>1</sup>Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran <sup>2</sup>Faculty of Biological and Agricultural Engineering, Texas A and M University, Texas, USA

Abstract: In this research, a geomorphological hydrologic model is developed and analyzed which is based on distributed flow routing and linear reservoirs cascade. In this model, the sub-basins output hydrographs have been calculated by Nash's black box model, considering the geomorphological properties of the sub-basins; then the obtained hydrographs have been routed through the main channel using the non-linear kinematics wave model. The two most important characteristics of the model are: (a) it explicitly includes the watershed morphology in the formulation and (b) it depends on only one uncertain parameter which must be calibrated. The result of the model has been compared with Nash's black box model and another gemorphological model i.e., SCS, for Ammameh watershed in central Iran. Combination of a non-linear distributed routing model (i.e., kinematics wave) and a linear lumped rainfall-runoff model (i.e., Nash's model) causes the proposed model to be a proper runoff routing model.

**Key words:** Rainfall-runoff modeling, semi-distributed model, geomorphological model, linear reservoir, GIS, Ammameh watershed

# INTRODUCTION

The process of rainfall-runoff in any watershed is a very complex process that depends on many properties of watershed such as area, overland slope, land use, etc. For a true determination of this process, plenty of hydrological models have been developed from the past until now and Nourani *et al.* (2007) have made a list of these models that shown in Fig. 1.

Due to the complexity of rainfall-runoff process and absence of data to describe in detail the character of heterogeneous and of spatially distributed inputs, simulation of the rainfall-runoff process is generally based on the conceptual models. The linear reservoir model presented by Zoch in 1934 is the oldest and simplest model with high level of application in relation to simulation of rainfall-runoff process which is the base of most other conceptual models (Chow et al., 1988). Certainly the cascade of linear reservoirs model with equal storage coefficients is the first conceptual model which uses the real meaning of linear reservoir with a mathematical base to present an explicit mathematical formulation for Instantaneous Unit Hydrograph (IUH) of the watershed (Nash, 1957). Then, Dooge (1959) developed a more complete model for calculation of IUH considering the effect of flow transition and adding the

meaning of linear channel to the Nash's model. But since the equation of IUH was not easily solvable for complex applicable problems, different simplified models out of the above-mentioned model were propounded.

For instance, Wang and Chen (1996) and Jeng and Coon (2003) have presented approaches on the basis of cascade of linear reservoirs model. Also some computer models on the basis of linear reservoirs concept have been introduced in recent years which have been briefly explained by Singh and Woolhiser (2002).

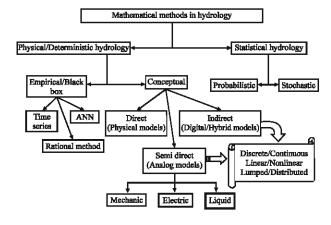


Fig. 1: Hydrological models (Nourani et al., 2007)

But these models have a great number of parameters for estimation and are usually able to calculate the hydrograph only at the watershed outlet, so hydrologists have decided to create and develop semi-distributed models (Nourani and Mano, 2007). Such models have been introduced on the basis of geomorphological routing concept and representing Geomorphological Unit Hydrograph (GUH).

It was at the end of 70th decade that utilizing this type of routing started for those watersheds with lack of complete observation data and it was tried to determine most of parameters according to the watershed morphology. Therefore reserving routing model is presented by Boyd (1978) and Boyd et al. (1979) on the basis of incorporating geomorphological properties of the watershed. Rodriguez-Iturbe and Valdes (1979) and Gupta et al. (1980) presented a geomorphological instantaneous unit hydrographs based on the theory of exponential distribution of required time that a drop of water measures in a specific way in the watershed. Rosso (1984) proposed a model which expresses the Nash's IUH parameters as functions of the Horton's indexes. Karnieli et al. (1994) and Hsieh and Wang (1999) presented different models for geomorphological routing by a similar method to the Boyd (1978). López et al. (2005) and Agirre et al. (2005) developed geomorphological instantaneous unit hydrographs based on cascade of reservoirs that have one uncertain parameter. Nourani and Mano (2007) used TOPMODEL and kinematics wave approaches to present a model that all parameters of this model were linked to geomorphologic properties except one uncertain parameter.

Furthermore, by developing GIS tools in hydrological science and rainfall-runoff modeling, it is possible to determine all hydrological and morphological parameters of the watershed precisely and easily by DEM (Digital Elevation Model) maps (Jenson and Domingue, 1988; Maidment *et al.*, 1996; Olivera and Maidment, 1999; Maidment, 2002).

In this study, first, theories of the Nash and SCS models as two classic and most applicable models are considered and then an application of a new model is shown. This is based on the linear reservoirs cascade concept that represents the structure of the watershed by its sub-watersheds, which are defined as the terrain portion draining to a channel of the drainage network and kinematics wave. At the end the result of this model for Ammameh watershed, a small watershed in central Iran, would be compared with results of Nash and SCS models.

All of the model geomorphological parameters are determined by GIS tools and only one uncertain parameter would be determined and calibrated using rainfall - runoff data sets.

#### THE NASH MODEL

The formulation of Nash's IUH was obtained under the assumption that, watershed behavior can be associated with a cascade of n equal linear reservoirs each having lag time of k, where unit rainfall instantaneously is imposed on the upper reservoir. With the above assumption, a Gamma distribution with parameters n and k is derived for IUH (Nash, 1957):

$$h(t) = \frac{e^{-\frac{t}{k}}}{k\Gamma(n)} \left[ \frac{t}{k} \right]^{n-1} \tag{1}$$

where, h(t) is IUH of Nash's model,  $\Gamma(n)$  is a Gamma function and n, k could be determined by moments method as (Singh, 1988):

$$\begin{split} &M_1(Q)\!-M_1(I)\!=\!nk \\ &M_2(Q)\!-\!2M_1(I)M_1(Q)\!+M_2(I)\!=\!nk^2(n\!-\!1) \end{split} \label{eq:mass}$$

In which  $M_1$  and M are the first and second moments of the functions and I, Q are inflow and outflow hydrographs, respectively.

Nash used his model in 1962 for some British catchments. He has established experimental relations between watershed properties and n, k parameters and presented Nash's synthetic model (Singh, 1988).

#### THE SCS MODEL

SCS method is one of the methods that can be used for computing UH in watersheds with insufficient data. First, lag time, t<sub>i</sub>, could be determined with regard to rainfall continuity then watershed physical properties such as area, main river length, average slope, CN (Curve Number) are used in order to make synthetic unit hydrograph for the watershed (Chow *et al.*, 1988).

#### THE SLRC MODEL

SLRC (Semi-distributed version of Linear Reservoirs Cascade) model, proposed in this research, is a semi-distributed version of linear reservoirs cascade. In this model, the watershed is divided into subdivisions with regard to topography, then each sub-watershed is substituted by one linear reservoirs cascade, thus the watershed is represented by reservoirs that distributed according to the watershed morphology. Finally a nonlinear equation (i.e., kinematics wave) is used for flow routing through the watershed main channel. Meantime, main precipitation is divided proportionally between sub-watersheds on the basis of areas.

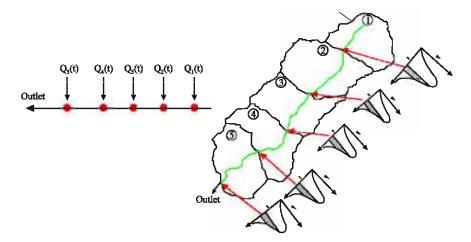


Fig. 2: Operation of SLRC model

In this model, outflow of each sub-watershed is determined with regard to geomorphology properties and using Nash's synthetic model equations (Singh, 1988):

$$k_1 = 0.79 L_1^{-0.1} A_1^{0.3} S_{0.1}^{-0.3}$$
 $n_1 = L_1^{0.1}$ 
(3)

where,  $S_0$  is average land slope, A is area (km<sup>2</sup>), L is the longest flow path in the drainage network (m) for ith sub-watershed.

Then the determined runoff applies to the main river momentarily as lateral flow. Thereafter flow is routed in the main channel using the kinematics wave equation. It can be mentioned that, kinematics wave equation may be considered as a reliable routing model for such watersheds with high land slope. The circumstance of the model operation is illustrated by Fig. 2.

Dispensing the term of pressure and acceleration in Saint-Venant equation, in shallow water and using the Manning's formula, kinematics wave equation can be obtained as (Chow *et al.*, 1988):

$$\frac{\partial Q}{\partial x} + \alpha \beta Q^{\beta - 1} \left( \frac{\partial Q}{\partial t} \right) = q \tag{4}$$

with coefficients as follow:

$$\beta = 0.6$$
,  $\alpha = \left(n_{\rm m} B^{2/3} / \sqrt{s_0'}\right)^{0.6}$  (5)

where, q is lateral flow,  $S'_0$  is channel slope,  $n_m$  is Manning coefficient and B is channel width. By solving this non-linear partial differential equation, flow discharge in any time (t) and distance (x) can be obtained through the watershed main channel.

However, in SLRC model Eq. 4 is used as the adjusted form of:

$$\frac{\partial Q}{\partial x} + \overline{\alpha} \alpha \beta Q^{\beta - 1} \left( \frac{\partial Q}{\partial t} \right) = q$$
 (6)

where,  $\bar{\alpha}$ , is a correction coefficient due to the assumptions and condensing applied to the model and is the only determinable parameter of the model.

In SLRC, Digital Elevation Model (DEM) might be used for determining channel slope. Manning coefficient could be obtained with regard to the land use and plant coverage of the watershed. Whereas, using large scale maps for extracting some hydrological properties such as channel width is not suitable and also using small scale maps for hydrological modeling is not economic, so an equation was proposed by Bandaragoda *et al.* (2004) in order to estimate channel cross section width as:

$$B = aA_{\text{upstream}}^{b} \tag{7}$$

where,  $A_{upstream}$  is upstream drainage area (km²), a = 0.0011, b = 0.518 and B is channel width (m). Correctness of Eq. 7 has been reported by other researches (e.g., Nourani and Mano, 2007) but unlike b, a may change from a watershed to the other greatly.  $A_{upstream}$  is calculated at each location with distributing area linearly along the main river.  $\bar{\alpha}$ , a watershed parameter with no dimension, is the only parameter which should be calibrated using rainfall-runoff data. As a matter of fact,  $\bar{\alpha}$  is used for correcting due to the applied assumptions and existence of uncertainty in the estimated parameters such as  $n_m$  and a.

Implicit finite difference method (Chow et al., 1988) is used in SLRC model for solving Eq. 6 and all used geomorphological parameters are extracted by GIS tools.

In order to determine the uncertain parameter of the model ( $\overline{\alpha}$ ), direct search method (Yue and Hashino, 2000) may be used among other optimization schemes.

# **EFFICIENCY CRITERIA**

For a more complete analysis of the suitability of the models, Nash and Sutcliffe index (E) (1970), correlation coefficient between observed and calculated data (R) and ratio of absolute error of peak flow (RAE<sub>P</sub> (%)) are used in the current study. These indicators are defined as follows:

$$E = 1 - \frac{\sum_{i=1}^{No} (Q_{i,obs} - Q_{i,sim})^2}{\sum_{i=1}^{No} (Q_{i,obs} - \overline{Q}_{obs})^2}$$
(8)

$$R = \frac{\sum_{i=1}^{No} ((Q_{i,obs} - \overline{Q}_{obs})(Q_{i,sim} - \overline{Q}_{sim}))}{\sqrt{\sum_{i=1}^{No} (Q_{i,obs} - \overline{Q}_{obs})^2 \sum_{i=1}^{No} (Q_{i,sim} - \overline{Q}_{sim})^2}}$$
(9)

$$RAE_{P}(\%) = \frac{\left|Q_{Pobs} - Q_{Psim}\right|}{Q_{Pobs}} \times 100 \tag{10}$$

where,  $Q_{i, obs}$  is observed discharge at t = i,  $Q_{i, sim}$  is simulated discharge at t = i, No is the number of observed data and  $Q_{P obs}$ ,  $Q_{P sim}$  are observed and simulated peak discharges, respectively.

#### WATERSHED DESCRIPTION

The Ammameh Watershed, one of the sub-watersheds of Jajrood in upstream of Latian Dam, is located in south area of Central Alborz, near Tehran (Capital of Iran) with an area of 37.2 km² between the

heights of 1900 and 3868 m. From topology point of view, it is a mountain area. Figure 3a shows aerial photograph of Ammameh watershed.

Figure 3b shows DEM of Ammameh watershed which illustrates elevation condition of watershed and also DEM is a tool for eliciting geomorphological properties of the watershed. This map obtained from topography map of the watershed (scale: 1/25000) using GIS.

Regarding vegetation coverage, about 200 hectares (5% of watershed area) include gardens and grass and the remained has vegetable coverage of bushes. Vegetation coverage of watershed extracted from aerial photographs is shown in Fig. 3c. Some of the events in this watershed have been registered from 1990 with time intervals of 30 min. In order to consider the rainfall-runoff by the models, watershed is divided into 5 sub-watersheds using GIS tools (Fig. 3d).

# RESULTS AND DISCUSSION

Due to the lake of registered data, 8 events were used for comparing simulated and observed direct hydrographs; 6 events for calibrating and 2 events for verifying. In order to find the observed direct hydrograph of any event, first, base flow specified by fixed gradient method and then observed direct hydrograph calculated. Next, penetration of each event calculated by the use of continuous theorem and fixed penetration rate method, finally this penetration deducted from observed hyetograph in order to find the excess hyetograph of the event.

Specifications of each event are shown in Table 1. In the Table 1 first column is the number of event, the second is the date of event, third is the height of precipitation, fourth is the equivalent height of direct runoff, fifth is the loss rate and the last is the transform rate of rainfall into runoff in the watershed.

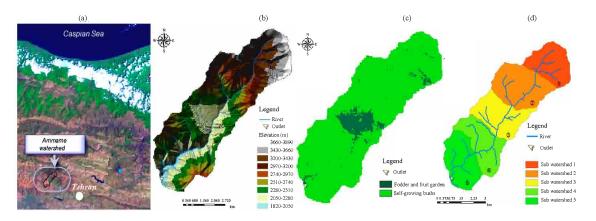


Fig. 3: (a) Situation map, (b) DEM, (c) Vegetation coverage and (d) Sub-watersheds

Table 1: Rainfall-runoff events data

No.Event	Date	h <sub>ramfall</sub> (mm)	h <sub>runoff</sub> (mm)	$\Psi$ (mm h <sup>-1</sup> )	$h_{\text{runoff}}/h_{\text{rainfall}}$	
1	10/08/1993	8.05	0.20	6.40	0.025	
2	03/27/1992	5.45	2.78	1.57	0.511	
3	04/07/1992	6.35	2.63	1.51	0.415	
4	05/09/1992	11.50	0.20	0.83	0.017	
5	04/07/1997	7.65	0.28	0.07	0.037	
6	04/17/1997	6.25	2.82	5.17	0.451	
7	07/13/1996	10.50	1.85	1.20	0.176	
8	07/18/2004	4.95	2.74	1.11	0.553	

Table 2: Geomorphological parameters of SLRC model

		Longest_Fl	Slp_Mean	Long_River				
No.	Area (m²)	(m)	(%)	(m)	S' <sub>0</sub> (%)	k (h)	n	B (m)
1	6349057	3047.08	49.90	2141	21.41	0.76	2.59	3.34
2	9259785	4325.05	66.61	3329	33.29	0.75	2.68	5.32
3	7378772	4119.52	37.97	2559	25.59	0.84	2.67	6.51
4	7106051	5460.24	53.52	2140	21.40	0.73	2.74	7.48
5	7294662	4664.72	60.12	3074	30.74	0.72	2.70	8.37

Table 3: Calibration results of Nash, SCS and SLRC models

	Nash mo	SCS mod	SCS model				SLRC						
Event	k (h)	n	E	R	$RAE_P$	t <sub>L</sub> (min)	E	R	$RAE_P$	$\overline{\alpha}$	E	R	$RAE_P$
1	0.829	1.785	0.80	0.93	16.43	102	0.53	0.81	22.10	0.43	0.78	0.81	8.80
2	1.001	4.789	0.87	0.93	36.10	135	0.32	0.90	61.74	1.50	0.85	0.95	34.09
3	1.421	2.494	0.89	0.95	22.08	148	0.53	0.97	38.49	0.74	0.93	0.99	1.56
4	1.421	2.520	0.89	0.94	22.26	151	0.55	0.98	40.61	1.38	0.94	0.98	1.04
5	1.027	4.497	0.87	0.93	35.50	125	0.34	0.91	69.95	0.81	0.87	0.96	31.47
6	1.315	2.386	0.90	0.95	16.07	133	0.54	0.98	38.47	0.52	0.92	0.99	5.87
Average	1.169	3.079	0.87	0.94	24.74	132	0.47	0.93	45.23	0.88	0.88	0.95	13.81

Table 4: Verification results of Nash, SCS and SLRC models

	Nash mo	odel			SCS model				SLRC					
Event	k (h)	n	E	R	$RAE_P$	t <sub>L</sub> (min)	E	R	$RAE_{P}$	$\overline{\alpha}$	E	R	$RAE_P$	
7	1.169	3.079	0.85	0.95	42.13	132	0.41	0.98	53.98	0.88	0.85	0.93	21.14	
8	1.169	3.079	0.82	0.93	50.47	132	0.47	0.96	55.68	0.88	0.93	0.98	23.22	
Average	1.169	3.079	0.84	0.94	46.30	132	0.44	0.97	54.83	0.88	0.89	0.96	22.18	

Geomorphological parameters extracted by GIS and necessary parameters for calculating determinable parameter of SLRC model are shown in Table 2.

We have respectively the number, area, the length of the greatest drainage of sub-watershed, average slope of sub-watershed, length of the river, river slope of the sub-watershed, n (number of reservoirs) and k (lag time or storage coefficient) as the introduced amounts in Eq. 3. Last column is channel width at the sub-watershed outlet, determined by Eq. 7. Average Manning coefficients are  $n_{\rm m}=0.024$  for sub-watersheds 1, 2, 4, 5 and  $n_{\rm m}=0.03$  for sub-watershed 3, which obtained considering vegetation coverage extracted from aerial photography using ERDAS IMAGINE software.

The watershed considered as a single piece with no sub-watershed for modeling by SCS method. Curve Number (CN) was chosen 79 according to the vegetation coverage. Lag time parameter was chosen as the SCS variable parameter and for calculating optimum value of the parameter, direct search method (Yue and Hashino, 2000) was used and results are given in Table 3.

Calibration results of Nash's model with determinable parameters calculated by moments method (Eq. 2) and SCS and SLRC models with parameters calculated by direct search method are presented by Table 3. The graphs of calibration step and observed direct hydrographs are shown in Fig. 4.

Then the models were verified by two events data sets that results are presented by Table 4. Average values of parameters, obtained from calibration process, were used in verification step. Verification graphs and observed direct hydrographs are shown in Fig. 5.

Considering Table 3 and 4, average values of efficiency criteria in SLRC model are greater than average values of criteria in Nash and SCS models. With a view to the fact that Nash model has two degrees of freedom, calibration result must be better because a two-parameter model may have better fitness on the observed data. Also having two parameters, can increase more dependency on the accuracy of the determined values of the parameters and existing error in determined parameters in calibration step can decrease accuracy of the model in the

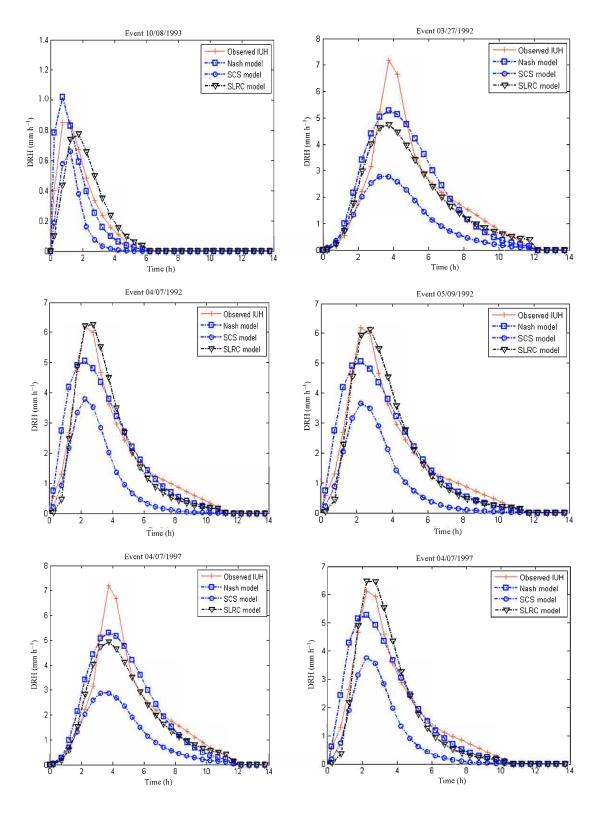


Fig. 4: Calibration process graphs of Nash, SCS and SLRC models

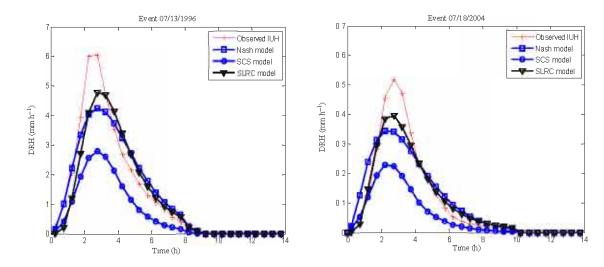


Fig. 5: Verification process graphs of Nash, SCS and SLRC models

verification step. Considering the result of SCS model shows that, use of bad and inadequate modeling of physical parameters in SCS model causes untrustworthy result.

According to the results presented in Fig. 4 and 5, in spite of considering just one calibrated parameter in the SLRC model, it can properly detect the rising limbs of the hydrographs which are usually depended on the storm properties as well as recession limbs which are more related to the watershed geomorphological factors. The existence of both geomorphological and calibrated parameters in the model formulation may lead to this capability.

In high discharges, a watershed usually shows linear behavior (Pilgrim, 1976) and non-linear models may have weak performance in such situation. However, taking advantage from Nash equation as a linear routing model may help to the SLRC model in order to catch the peak discharges more appropriately, as indicated by RAE<sub>p</sub> index in Table 3 and 4.

#### CONCLUDING REMARKS

On the basis of obtained results, distinguished properties of SLRC model are:

- SLRC model is a geomorphological model which is on the basis of watershed physical properties, covers area with different properties and also considers them in modeling.
- Formulation only depending on a single parameter.
- SLRC model is a semi-distributed model and against the lumped models such as Nash's model, also can consider hydrological condition of the interior parts

- of the watershed, therefore this is realized that capability of SLRC model is more than the other classic model such as Nash and SCS models and also it has ability to simulate rainfall-runoff properly.
- Accompaniment of a non-linear distributed routing model (i.e., kinematics wave) by a linear lumped rainfall-runoff model (i.e., Nash's model) gives a suitable ability to the SLRC model in order to be a reliable runoff routing model. Particularly for watersheds which have variant geomorphological properties such as Ammameh watershed which the variation of the land elevation is more perceptible in it (from 1900 to 3868 m).

Proposed model in this paper was considered in a watershed which was divided on the basis of same interval lines, but this model could be used for watersheds which were divided into sub-watersheds on the basis of joint point in the drainage network.

It is also suggested to contemplate the seasonality and temporal effects accompanied by physical and geomorphological factors in the formulation of the presented model. This accompaniment and using more storms data may extend the model abilities and efficiencies; this proposal can be considered as a new research plan for the future work.

# REFERENCES

Agirre, U., M. Goñi, J.J. López and F.N. Gimena, 2005. Application of a unit hydrograph based on subwatershed division and comparison with Nash's instantaneous unit hydrograph. Catena, 64 (2-3): 321-332.

- Bandaragoda, C., D.G. Tarboton and R. Woods, 2004. Application of TOPNET in the distributed model intercomparison project. J. Hydrol., 298 (1-4): 178-201.
- Boyd, M.J., 1978. A storage-routing model relating drainage basin hydrology and geomorphology. Water Resour. Res., 14 (15): 921-928.
- Boyd, M.J., D.H. Pilgrim and I. Cordery, 1979. A storage routing model based on catchment geomorphology. J. Hydrol., 42 (3-4): 209-230.
- Chow, V.T., D.R. Maidment and L.W. Mays, 1988. Applied Hydrology. McGraw-Hill, New York, USA.
- Dooge, J.C.I., 1959. A general theory of the unit hydrograph theory. J. Geophys. Res., 64 (2): 241-256.
- Gupta, V.K., E. Waymire and C.T. Wang, 1980. A representation of an instantaneous unit hydrograph from geomorphology. Water Resour. Res., 16 (5): 855-862.
- Hsieh, L.S. and R.Y. Wang, 1999. A semi-distributed parallel-type linear reservoir rainfall-runoff model and its application in Taiwan. Hydrol. Processes, 13 (8): 1247-1268.
- Jeng, R.I. and G.C. Coon, 2003. True form instantaneous unit hydrograph of linear reservoirs. J. Irrig. Drain. Eng. ASCE., 129 (1): 11-17.
- Jenson, S.K. and J.O. Domingue, 1988. Extracting topographic structure from digital elevation data. Photogrametic Engineering and Remote Sensing, 54 (11): 1593-1600.
- Karnieli, A.M., M.H. Diskin and L.J. Lane, 1994. CELMOD 5-A semi-distributed cell model for conversion of rainfall into runoff in semi-arid watersheds. J. Hydrol., 157 (1-4): 61-85.
- López, J.J., F.N. Gimena, M. Goñi and U. Agirre, 2005. Analysis of a unit hydrograph model based on watershed geomorphology represented as a cascade of reservoirs. Agric. Water Manage., 77: 128-143.
- Maidment, D.R., J.F. Olivera, A. Calver, A. Eatherall and W. Fraczek, 1996. A unit hydrograph derived from a spatially distributed velocity field. Hydrol. Processes, 10 (6): 831-844.

- Maidment, D.R., 2002. Arc Hydro: GIS for Water Resources. ESRI Press, Redlands, CA.
- Nash, J.E., 1957. The form of the Instantaneous Unit Hydrograph. IASH Publication, 45 (3-4): 114-121.
- Nash, J.E. and J.V. Sutcliffe, 1970. River flow forecasting through conceptual models I: A discussion of principles. J. Hydrol., 10 (3): 282-290.
- Nourani, V. and A. Mano, 2007. Semi-distributed flood runoff model at sub continental scale for south western Iran. Hydrol. Processes, 21 (23): 3173-3180.
- Nourani, V., P. Monadjemi and V.P. Singh, 2007. Liquid analog model for laboratory simulation of rainfallrunoff process. J. Hydrol. Eng. ASCE., 12(3): 246-255.
- Olivera, F. and D.R. Maidment, 1999. Geographic information systems based spatially distributed model for runoff routing. Water Resour. Res., 35 (4): 1155-1164.
- Pilgrim, D.H., 1976. Travel times and nonlinearity of flood runoff from tracer measurements on a small watershed. Water Resour. Res., 12 (3): 487-426.
- Rodriguez-Iturbe, I. and J.B. Valdés, 1979. The geomorphologic structure of hydrology response. Water Resour. Res., 15 (6): 1409-1420.
- Rosso, R., 1984. Nash model relation to Horton order ratios. Water Resour. Res., 20 (7): 914-920.
- Singh, V.P., 1988. Hydrologic Systems. Vol. I. Rainfall-Runoff Modeling. Prentice-Hall, Englewood Cliffs.
- Singh, V.P. and D.A. Woolhiser, 2002. Mathematical modeling of watershed hydrology. J. Hydrol. Eng., ASCE., 7 (4): 270-292.
- Wang, G.T. and S. Chen, 1996. A linear spatially distributed model for a surface rainfall-runoff system. J. Hydrol., 185 (1-4): 183-198.
- Yue, S. and M. Hashino, 2000. Unit hydrographs to model quick and slow runoff components of stream flow. J. Hydrol., 227 (1-4): 195-206.