Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 8 (10): 1991-1994, 2008
ISSN 1812-5654

A Numerical Investigation of the Effects of Widths and Separation of Two Soliton Pulses on Their Interaction Distance

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Abstract: The effects of pulse width and their separation on the interaction distance and interaction length are investigated. Having ignored the effect of loss and third order dispersion coefficients, we found that the interaction distance changes in a quasi-Gaussian manner as a function of two interacting pulse separation. The effects of varying pulse widths and constant pulse separation and the reverse (i.e., the constant pulse widths and varying pulse separation) on the interaction length and the interaction distance have been investigated. It is found that in either case, the interaction distance and interaction length change exponentially with pulse width.

Key words: Soliton, nonlinear Schrödinger's equation, numerical simulation, elastic collision, communication technology

INTRODUCTION

The optical Solitons has undergone three decades of research, for it's promising application to the future of communication technology. Solitons arise from a balance between the inherent dispersion and nonlinear properties of optical fibers. In fact optical Solitons in fibers are found to be a balance between group velocity dispersion and Kerr nonlinearity. Numerical simulations and experiments have shown that Solitons can propagate an extended distance without distortion, so they may become an ideal massage carrier in long distance communication systems. The exact investigation of Solitons behavior demands for the solutions of the governing equation. Propagation of Solitons is described by nonlinear Schrödinger equation (NLSE) (Iizuka, 2002; Taha and Ablowitz, 1984). These Soliton waves have been investigated analytically and numerically by many researchers (Hasegawa and Kodama, 1995; Agrawal, 2001; Ablowitz and Clarkson, 1991). Soliton interaction is one of the most exciting areas of research in nonlinear dynamics. The unusual features of collisions in systems described by the Korteweg-de vries (kdv) (Korteweg and de Vries, 1895) equation were the starting point of these intensive studies. An undesirable effect of nonlinearity is to cause mutual interaction (Chu and Desem, 1983) between pulses if they are launched close together. The interaction between two launched Solitons into a fiber is important not only from a practical point of view but also it illustrates the practical-like behavior of the Solitons. It is well known by now that such interaction can result in bandwidth reduction by a factor of 10 (Chu and Desem, 1985) Solitons travel down the fiber. The interaction distance is affected by some parameters. In this study, we investigate the effect of initial Solitons' separation and their widths on the interaction distance and interaction length with regards to minimizing such interaction using computer simulation.

NONLINEAR SCHRÖDINGER EQUATION

Several wave equations that exhibit Solitons are for instance Korteweg-de vries (Kdv) and nonlinear Schrödinger equation (NLSE). To model Soliton pulse propagation in optical fibers, we have solved the dynamic equation known as nonlinear Schrödinger equation using basic numerical methods.

If U(z,t) denotes the complex amplitude of a pulse traveling along an optical fiber, then its evolution is governed by the NLSE in the form of:

$$\frac{\partial U}{\partial z} + i\frac{\beta}{2}\frac{\partial^2 U}{\partial t^2} - \frac{\beta_1}{6}\frac{\partial^3 U}{\partial t^3} = i\gamma|U|^2 U - \frac{\alpha}{2}U \quad (1)$$

where, z is the distance along the direction of propagation, t is the time, β and β_1 are the second and third order group velocity dispersion, γ is a constant that qualifies the nonlinear phenomena and α is absorption coefficient. To solve Eq. 1, the third order dispersion and the absorption coefficient are ignored for simplicity and hence the NLS equation becomes:

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$$\frac{\partial U}{\partial z} = -i\frac{\beta}{2}\frac{\partial^2 U}{\partial t^2} + i\gamma|U|^2 U \quad (2)$$

Equation 2 allows an exact N-order temporal Soliton solution for the special case when the initial condition U(t) is given by:

$$U(z=0,t) = U(t) = NE_0 \,\mathrm{Sech}(t/t_0) \quad (3)$$

where, N is an integer, E_0 is the pulse amplitude and t_0 is the pulse width. The relationship between the amplitude and the width of a Soliton is given by:

$$t_0^2 E_0^2 = \frac{|\beta|}{\gamma} \quad (4)$$

and that the intensity $|U(z,t)|^2$ in N-Soliton is periodic in z direction with the period of:

$$L = \frac{\pi}{2}\frac{t_0^2}{|\beta|} \quad (5)$$

NUMERICAL SOLUTION OF NLSE

If the initial condition U(t) doesn't correspond to a N-Soliton, then a numerical approach is necessary for a full understanding of the propagation. We have chosen the direct explicit finite difference method (Chang and Morris, 2005) to simulate the propagation of Solitons. The explicit method is perhaps the simplest algorithm based on finite difference to solve the NLSE. Assuming that the initial conditions of the pulse launched is given by the temporal shape U(z = 0,t) = U(t) where the time extends in the range $t_i \le t \le t_f$, then we aim to determine the function U(z,t) which is the spatiotemporal evolution of the pulse. At first, we discretize Eq. 2 via the introduction of the shorthand notation:

$$U_k^j = U(z_j, t_k) \quad (6)$$

where, z_j = jh and $t_k = t_i + (k-1)\tau$, where j = 0, 1, 2, ... and k = 1, 2, ..., K denote the spatial and temporal indices, respectively. The finite difference grid consists of perpendicular lines that run parallel to z and t axes. This (z,t) space-time plane is represented by the grid matrix shown in Fig. 1. The space and time increments are denoted by h and τ, respectively. The boundary conditions are $U_1^j = 0$ and $U_K^j = 0$ and so the grid spacing is $\tau = \frac{(t_f - t_i)}{k-1}$.

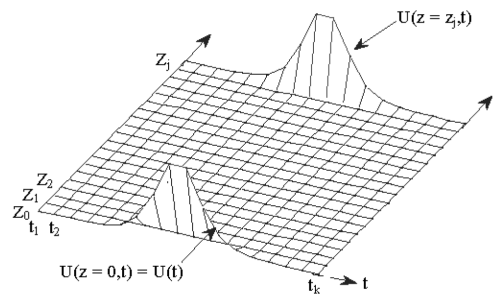

Fig. 1: Grid matrix showing the (z,t) place-time plane

To discrete Eq. 2, we approximate the spatial derivative by a first order two-point difference and the time derivative by a second order centered difference (Lopez *et al.*, 2005). By doing so, we obtain Eq. 7

$$\frac{U_k^{j+1} - U_k^j}{h} = -i\frac{\beta}{2}\left(\frac{U_{k+1}^j - 2U_k^j + U_{k-1}^j}{\tau^2}\right) + i\gamma\left|U_k^j\right|^2 U_k^j \quad (7)$$

Rearranging for U_k^{j+1} we obtain:

$$U_k^{j+1} = -\alpha(U_{k+1}^j - 2U_k^j + U_{k-1}^j) + c\left|U_k^j\right| U_k^j + U_k^j \quad (8)$$

where, $\alpha = i\beta h/2\tau^2$ and c= iγh. In direct explicit method we can compute U at level j+1 in terms of the values of U at level j. Equation 8 can be applied to the internal grid points k = 2, ..., K-1. If we express the values of U_K^j as a column vector $U^j = \left[U_2^j, \ldots, U_{k-1}^j\right]^T$, we can rewrite Eq. 8 in a matrix form as:

$$U^{j+1} = AU^j \quad (9)$$

where, A is a tridiagonal square matrix of size (K-2)*(K-2) given by:

$$A = \begin{bmatrix} 1+2\alpha+c\left|U_2^j\right|^2 & -\alpha & 0 & \cdots \\ -\alpha & 1+2\alpha+c\left|U_3^j\right|^2 & -\alpha & \cdots \\ 0 & \ddots & \ddots & \ddots \\ \cdots & 0 & -\alpha & 1+2\alpha+c\left|U_{K-1}^j\right|^2 \end{bmatrix} \quad (10)$$

Equation 9 is the basic propagating equation for solving the NLSE numerically with the direct explicit method and then for simulating temporal Solitons.

NUMERICAL CALCULATIONS

The evolution of the fundamental Soliton is given by

$$U(z,t) = E_0 \,\mathrm{sech}\left(\frac{t}{t_0}\right)\exp(i\gamma E_0{}^2 z/2) \quad (11)$$

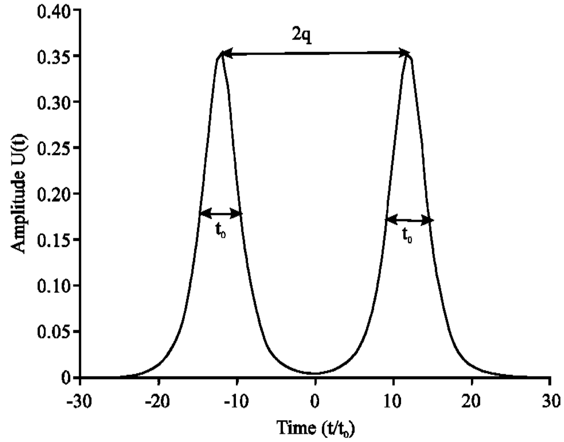

Fig. 2: Two Soliton pulses with initial separation 2q and pulse width t_0

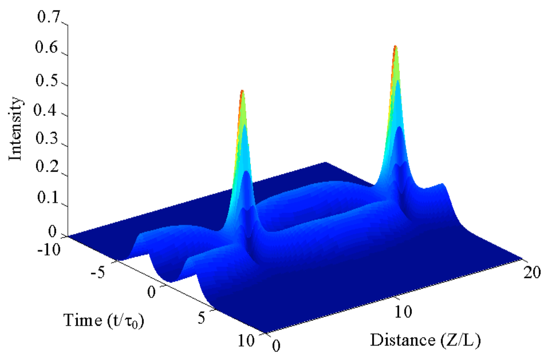

Fig. 3: The time evolution of two interacting Solitons with t_0 = 2 ps over 75 periods

To observe the Solitons interaction, we launch two fundamental Solitons initially separated by a time 2q from each other in the form of:

$$U(t) = E_0 \operatorname{sech}(\frac{t-q}{t_0}) + QE_0 \operatorname{sech}(\frac{Q(t+q)}{t_0})\exp(i\phi) \quad (12)$$

where, Q is the relative amplitude and ϕ is the initial phase differences. Figure 2 shows two Soliton pulses with initial separation 2q and pulse width t_0.

We take Q = 1 and ϕ = 0 to see the Solitons interaction since Solitons of equal amplitude and phase can collide and attract each other (Haus Hermann, 1993). Figure 3 shows time evolution of two interacting Solitons. In our calculations, the time interval is $t\in[-5t_0,5t_0]$ and the pulses are let to propagate until at least 75 Soliton periods. The typical physical constants are β = $-1.5ps^2$ km^{-1} and $\gamma = 3W^{-1}km^{-1}$. The numerical parameters are K = 150, t_0 = 2ps and the propagation step size is h = L/400.

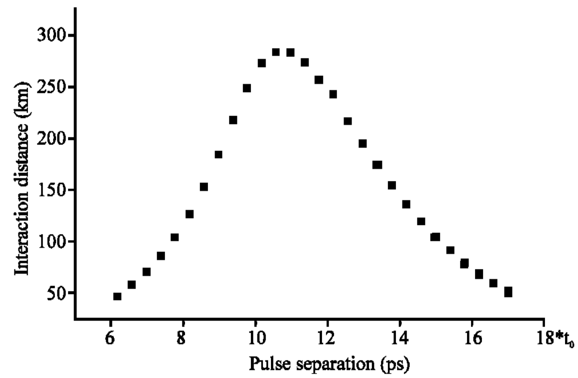

Fig. 4: The interaction distance v.s. pulse separation for two s of constant width. The interaction distance gets its maximum value at a initial Soliton separation equal to 10.6 times the pulse width

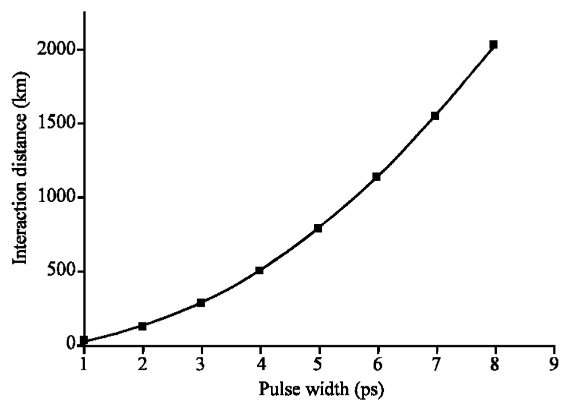

Fig. 5: The interaction distance vs pulse width with q = $4t_0$. The interaction distance increases by increasing the pulse width

According to the numerical results, keeping the pulse width constant, two Solitons collide each other as a function of pulse separation. This behavior is shown in Fig. 4. The interaction distance gets its maximum value at a initial Soliton separation equal to 10.6 times the pulse width.

Another numerical result has been obtained by varying the pulse width t_0 while the pulse separation is kept (q = $4t_0$). In this case the interaction distance changes in a parabolic manner (Fig. 5).

Now we keep the pulse separation constant, say 2q = 16 ps and vary the pulse width t_0. In this case, the interaction distance decreases. In another words two s collide each other in a closer distances (Fig. 6).

Another interesting result is the interaction length with which we mean the length that two Solitons interact with each other. Figure 7 shows that the interaction length increases as the pulse width increases.

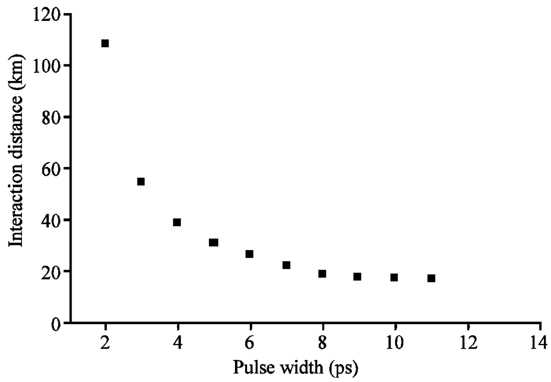

Fig. 6: The interaction distance v.s. pulse width for Solitons with constant pulse separation of 16 ps. The interaction distance decreases by increasing the pulse width with q = 16

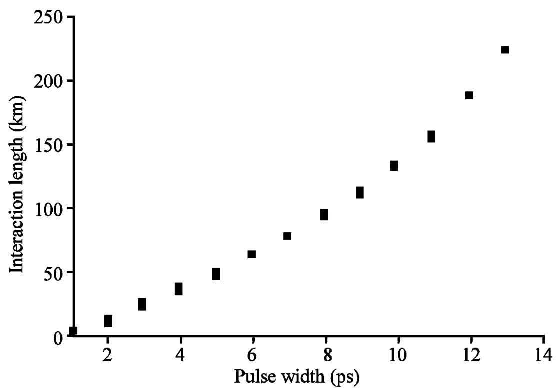

Fig. 7: The interaction length v.s. pulse width for Solitons of constant pulse separation 16 ps. The interaction length increases as the pulse width is increasing

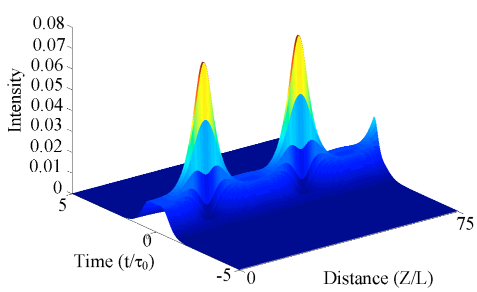

Fig. 8: The time evolution of two interacting Solitons with t_0 = 8 ps over 75 periods

Figure 8 shows the time evolution of two interacting Solitons which has been shown in Fig. 3. In this case, the pulse width is t_0 = 8ps instead of t_0 = 2ps. Comparing these two figures, we see that the interaction distance decreases while the interaction length increases by increasing the Soliton pulse width.

CONCLUSION

In this research, we studied Soliton interaction and its relation with Soliton separation, pulse width and also interaction length. According to numerical results, interaction distance increases by increasing Soliton separation and gets it's maximum value at about Soliton separation 10.6 times the pulse width. Keeping the pulse separation constant, an increase in pulse width causes a decrease in interaction distance and an increase in interaction length. Since the pulse width is a measure of the quantity of data communication, therefore Solitons with large pulse width increases Solitons interaction which is not desirable. Thus in Soliton communication systems, the best value for pulse separation is 10.6 times the pulse width and the optimised Soliton pulse width is it's minimum value.

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