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An Approximate Solution for Consolidation of Inelastic Clays under Rectangular Cyclic Loading

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Abstract: In this study, the consolidation of inelastic clays under cyclic loading is investigated. For simple calculations, an approximate semi analytical method is presented. Presented method is based on a time transformation method and superimposing rule to solve the consolidation problem of inelastic soils under cyclic loading. Changes of material properties are applied in the solution by modification of loading and unloading duration by introducing a time transformation. A set of continuous static loads in specified times used instead of cyclic loading based on the superimposing rule. More than 20 laboratory tests are performed in order to verify the presented method. Accuracy of the presented method as a function of material properties and loading conditions is investigated.

Key words: Consolidation, inelastic clay, approximate solution, laboratory test

INTRODUCTION

Since the settlement of the saturated fine grain soil layers is a time dependent phenomena, nonlinear and inelastic behavior of soil and cyclic loading during the consolidation process would affect the rate and amount of the settlement at any particular time.

Rectangular cyclic loading can be produced by structures such as oil tanks, silos and reservoirs.

In the case of the Normally Consolidated (NC) inelastic clays under cyclic loadings, consolidation computation will be more complicated and the solution for consolidation partial differential equation will be more difficult too. There are solution charts for consolidation of normally consolidated clays that can be used for calculations (Fox, 1999).

A semi analytical method presented by Baligh and Levadoux (1978) suggests a simplified solution for such problems for inelastic clays that is useful just for long period cyclic loads. Favaretti and Soranzo (1995) presented a simplified consolidation theory in cyclic loading condition based on superimposing rule for elastic soils. Rahal and Vuez (1998) presented solutions for such problems about linear and sinusoidal loadings for elastic soils. Lekha *et al*. (2003) have studied on one-dimensional consolidation of soils under static and cyclic loading with varying compressibility and permeability. Also analytical solutions for linear finite- and small-strain one-dimensional consolidation are presented by Morris (2002, 2005).

All of the mentioned studies were based on the Terzaghi's consolidation theory. Recently Cai *et al*. (2007) presented a semi-analytical solution for such problems based on the Gibson's equations considering the variation of the void ratio in the depth of the clay layer.

Precisely analytical solution for consolidation of inelastic clays under cyclic loading leads to complicated mathematical formulations; therefore, application of approximate solutions for complicated cases is unavoidable (Jeng *et al*., 2006). In this research, an approximate solution is presented based on a semi-analytical method. This approximate solution is based on assumptions same as ones presented by Baligh and Levadoux (1978) to solve such problem. Presented method employs the superimposing rule along with a time transformation to solve the problem. In the consolidation process, change of the soil properties from NC to OC states and vice versa during loading and unloading is applied in the calculation by a time transformation. A set of continuous static loads is used instead of the time dependant cyclic load according to superimposing rule.

INELASTIC BEHAVIOR OF CLAY UNDER CYCLIC LOAD

Inelastic behavior of clays under a cycle of load can be considered using the bilinear model that is shown in Fig. 1. Since the coefficient of compressibility and

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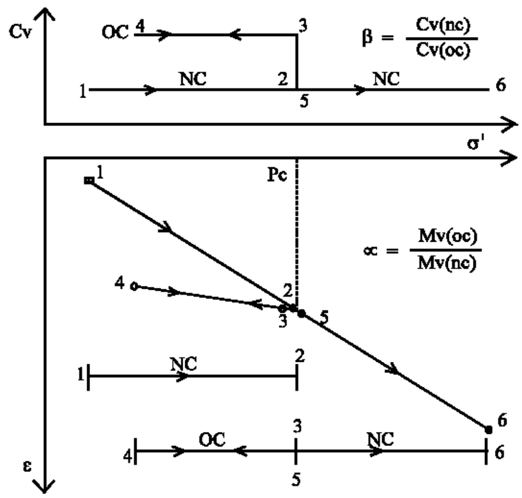

Fig. 1: Inelastic behavior of clay under cyclic loading

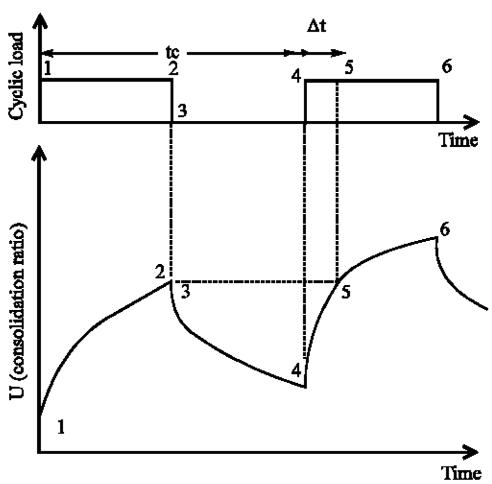

Fig. 2: Consolidation of inelastic clay under cyclic load

permeability of inelastic clay changes during the loading and unloading cycles, the coefficient of consolidation (c_v) is a function of these parameters and can be changed during each cycle of loading. In this study we assumed that the c_v has a constant value during the state of NC and or OC and changes suddenly when the soil body changes from NC to OC states or vice versa.

At first half cycle of loading, the soil body is in NC conditions and stress path is according to 1-2 route as shown in Fig. 1.

During the unloading half cycles, the soil body is in OC conditions and the stress path is according to 3-4 route in Fig. 1 and 2.

After the first cycle, the stress path will be according to 4-5-6 route in Fig. 1 and 2 in the following loading half cycles. Position of node 5 shows the amount of the precompression pressure (Pc). The value of Pc increases by the number of cycles and reaches to a constant value which is called as the steady-state condition.

Since in the loading half cycles the value of c_v changes next to point 5 according to bilinear model in Fig. 1 and 2, one of the main objectives of this problem is to determine the position of point 5 in the loading half cycles.

TIME TRANSFORMATION

Based on compatibility of fluid outflow and volume change of soil body, governing partial differential equation for one-dimensional consolidation phenomena under a time dependent loading is:

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} + \frac{\partial p}{\partial t} \tag{1}$$

The term $\partial p/\partial t$ is the effect of the time dependent loading and c_v is the consolidation coefficient.

In Terzaghis' consolidation theory, the value of the applied load and c_v assumed to be constant during the consolidation process. In this study, time dependent loading is adopted by superimposing rule, which is introduced in Fig. 3.

Analytical solutions for the one-dimensional consolidation differential equation with uniform initial pore water pressure based on Terzaghi's consolidation theory are according to Eq. 2 and 3 for pore water pressure distribution and degree of consolidation, respectively.

$$u(t) = \sum_{m=0}^{\infty} \frac{2u_0}{M} \exp(-M^2.T) \sin(\frac{M.z}{H})$$
$$T = \frac{c_v t}{H_d^2}, \quad M = \frac{(2m+1)\pi}{2} \tag{2}$$

And

$$U(t) = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp(-M^2.T) \tag{3}$$

where, u and U are pore water pressure and degree of consolidation, respectively.

It can be seen in Eq. 2 and 3 that the effect of the material properties and time is introduced in solutions by the time factor (T). Since the time factor, T, is a linear degree one homogeneous function of c_v and t, it means that equal changes of both factors have same changes on results. Therefore, according to Eq. 4 changes of c_v can be applied to t.

$$T = \frac{(kc_v)t}{H_d^2} = \frac{c_v\ (kt)}{H_d^2} = \frac{c_v\ t'}{H_d^2}, \quad t' = kt \tag{4}$$

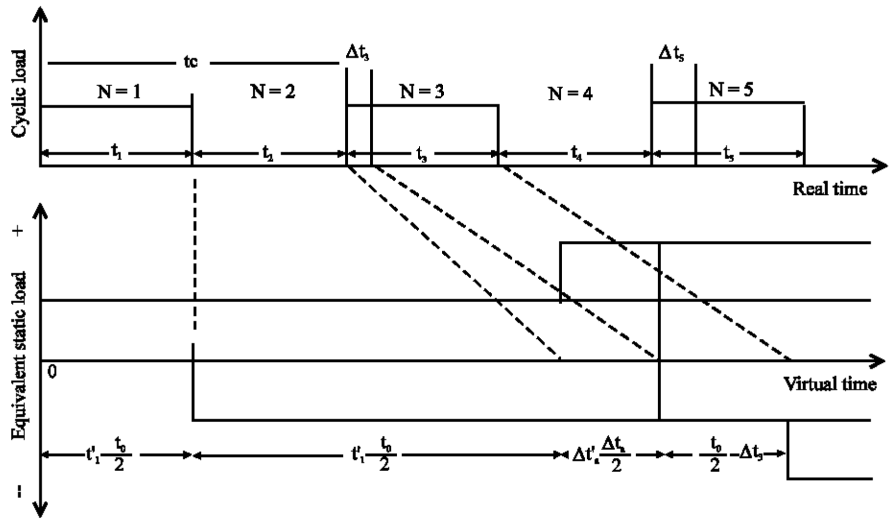

Fig. 3: Cyclic loading adapted by superimposing rule in the transformed time space

Where:

- t = Real time variable
- t' = Transformed time variable and
- k = Can be any factor

Equation 4 can be used to establish the time transformation relationships.

During the unloading half cycles the soil body is in OC conditions and the value of c_v is different than its value in NC condition. Therefore equivalent time for unloading half cycles can be as follows:

$$t'_N = \frac{t_c}{2\beta}, \quad N = 2,4,6,.. \tag{5}$$

Where:

- t_c and β = Introduced in Fig. 2 and Fig. 1, respectively
- N = Number of the half cycle

After the first full cycle (a loading and an unloading half cycles), in the flowing loading half cycles the soil is in OC conditions until the average degree of consolidation reaches to previous maximum degree of consolidation, which is equal to its value at end of last half cycle of loading (according to point 5 in Fig. 1 and 2).

In Fig. 3, Δt_N is the first portion of each loading half cycle that the soil body is in OC conditions (according to route 4-5 in Fig. 1 and 2) and afterward becomes NC.

Figure 3, shows a rectangular cyclic loading system which is adapted in the transformed time space by a set of continuous loads based on the superimposing rule.

Equation 6 can be used to calculate $\Delta t'_N$.

$$\sum_{n=1}^{N-1}(-1)^{n+1}U(t'_{i,n-1}+\Delta t'_N) = Uc_{N-2} \qquad t'_{i,n-1} = \sum_{i=n}^{N-1} t'_i \tag{6}$$

where, the $\Delta t'_N$ in the transformed time space is related to Δt_N ($\Delta t'_N = \Delta t_N/\beta$). At the end of Δt_N, (according to point 5 in Fig. 2 in each loading half cycle) the soil body changes from OC to NC condition. The average degree of consolidation at end of $\Delta t'_N$ must be equal to its previous maximum value.

The left Eq. 6 is the degree of consolidation at the end of $\Delta t'_N$ based on the superimposing rule. The right side of Eq. 6 is the maximum degree of consolidation at the end of the previous loading half cycle.

APPROXIMATE METHOD FOR VIRTUAL TIME

The exact value of Δt_N can be calculated by Eq. 6 based on a step-by-step procedure.

The following sections describe an approximate method to calculate the value of Δt_N supposing that the duration of unloading half cycle is long enough, so that the pore water pressure remained from the last cycle vanishes. This is similar to assumption used by Baligh and Levadoux (1978) for determination of the first portion of each half cycle of loading that the soil is in OC conditions.

They presented an approximate method to calculate $\Delta t'_N$ considering the effect of the change of the c_v during the loading half cycle based on an equivalent coefficient of consolidation for each loading half cycles as follows:

$$\Delta T'_N = \beta T'_{N-2}, \quad N = 3,5,7,... \tag{7}$$

where, T' is equivalent time factor.

Therefore equivalent time factor for loading half cycles will be:

$$T'_N = \Delta T'_N + \frac{T_c}{2} - \beta \Delta T'_N, \quad N = 3,5,7,... \tag{8}$$

Extending Eq. 8 results (Baligh and Levadoux, 1978):

$$T'_N = \frac{T_c}{2\beta}\left[1-(1-\beta)^{\frac{N+1}{2}}\right], \quad N = 3,5,7,... \tag{9}$$

where, N is number of half cycle.

Equation 9 can not be used to append the effect of the previous cycles in the calculation; because there is not any support to assume a constant value for c_v in all of the loading cycles in real time space. Different cycles with different c_v can't be composed.

Based on the method described in the previous section, the value of c_v is constant in the transformed time space. Therefore, different cycles can be composed in transformed time space.

By assuming a constant value for c_v, Eq. 9 can be rewritten in the transformed time space as follows:

$$t'_N = \frac{t_c}{2\beta}\left[1-(1-\beta)^{\frac{N+1}{2}}\right], \quad N = 3,5,7,... \tag{10}$$

In the method presented by Baligh and Levadoux (1978), the effect of the pore water pressure remained from the previous cycle was neglected in the solution and each cycle of load acts independently. This is true only when the period of each cycle is very long. In that method, only the effect of the increment of pre-compressing stress has considered in each cycle of loading. If the duration of unloading half cycle be shorter than the time require for disappearing the negative pore water pressure, the negative pore water pressure remains at the beginning of the next cycle and produces an error.

By using Eq. 10 in the transformed time space, the superimposing rule can be used to include the effect of the remained pore water pressure from previous cycles.

By using approximate equivalent times for loading and unloading half cycles, pore water pressure distribution and degree of consolidation under cyclic load can be calculated by:

$$u_{cN} = (-1)^N \sum_{n=1}^{N} (-1)^n u(t'_{nN}), \quad t'_{nN} = \sum_{i=n}^{N} t'_i \tag{11}$$

and

$$U_{CN} = (-1)^N \sum_{n=1}^{N} (-1)^n U(t'_{nN}) \tag{12}$$

In the above equations, t'_i can be calculated by Eq. 5 and 10 for unloading and loading half cycles, respectively.

Equation 13 can be used to calculate the amounts of the settlement at the end of loading and unloading half cycles as follows:

$$\begin{aligned} S_N &= U_{cN}.m_v.p.H, \quad N = 1,3,5,.. \\ S_N &= S_{N-1} + (U_{cN}-U_{cN-1}).m_v\alpha.P.H, \quad N = 2,4.. \end{aligned} \tag{13}$$

In this equations, S_N is settlement at end of loading or unloading half cycle, Uc_N is degree of consolidation under cyclic loading obtained from Eq. 12, m_v is compressibility coefficient in NC condition, p is cyclic load intensity, H is the clay stratum thickness and α is the ratio of the compressibility coefficient in OC conditions to its value in NC conditions.

In the presented method, approximate equivalent times for loading half cycles can be longer than its real values. By using these larger values, the effect of the previous cycles applied by the superimposing rule can be more than its real effect. This is because of the neglected pore water pressure at the beginning of the loading half cycles, which decreases the require time to reach to point 5 in Fig. 2. Application of superimposing rule causes to decrease this error. Meanwhile in the presented approximate method the consolidation process would complete faster than exact solution.

Original method (Baligh and Levadoux, 1978) offers a lower bound solution for this problem while the presented method offers an upper bound solution.

VERIFICATION OF THE PRESENTED METHOD

In Fig. 4 and 5 the results of the presented approximate method and original method (Baligh and Levadoux, 1978) are compared by the result of the finite difference method.

In the Fig. 4 the normalized error is:

$$N\,Err = \max\left|\frac{Uc_{ap}(t)-Uc_{numerical}(t)}{Uc_{numerical(steady\text{-}state)}}\right|, \quad t \in [0,\infty] \tag{10}$$

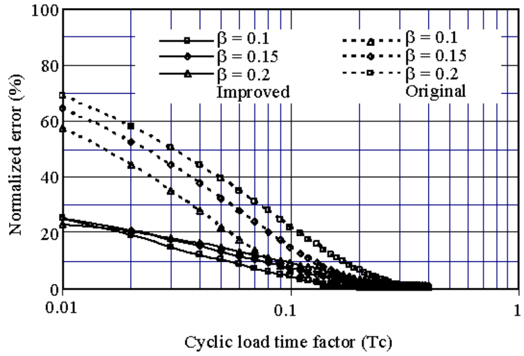

Fig. 4: Effect of the loading period on the results of the presented and original methods

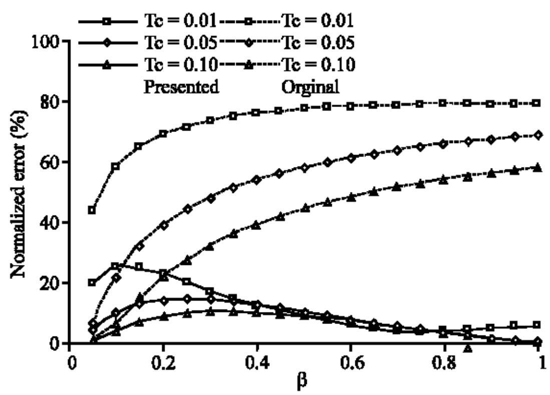

Fig. 5: Effect of β on the results of the presented and original methods

where, U_{cap} is the value of the degree of consolidation by either original or modified method. $Uc_{numerical}$ is the degree of consolidation at any time and $Uc_{numerical\ (steady\text{-}state)}$ is the final value of the degree of consolidation calculated by the numerical method. Tc, is the time factor for a complete cycle of loading.

Figure 5 shows the normalized error vs. loading time factor Tc.

It can be seen that, the results of the both methods have some errors but it is obvious that error for proposed approximate method is less than the original one. Above calculations have done for several values of β and loading time factor. For both methods, errors decrease as the loading period increases. It can be seen that, the results of the proposed method is more accurate for all loading periods comparing to the original method. It also can be seen that the original method has higher error for high frequency cyclic loads.

It can be seen in Fig. 5 that in the original method increasing the value of β causes the value of the estimated error to increases, so it is not suitable for large

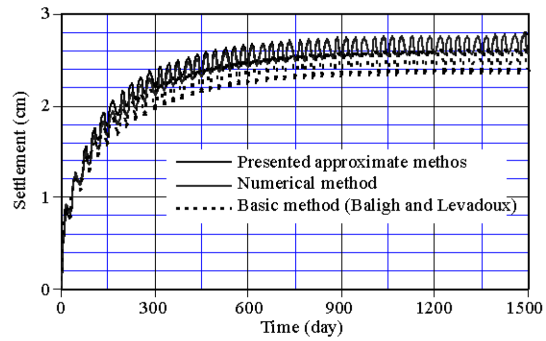

Fig. 6: Comparing the result of the presented method by the results of the original and numerical methods

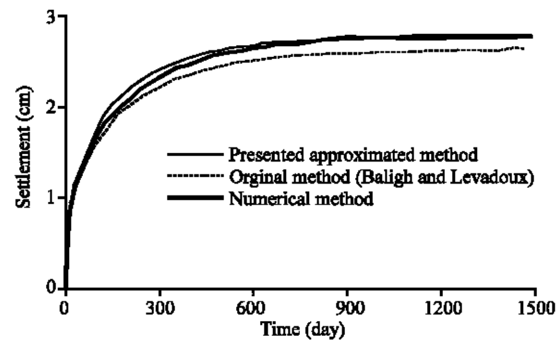

Fig. 7: Comparing the result of presented method by original method

values of β; but the presented method becomes more accurate as the values of β increases. β refers to inelastic behavior of material, its value change from 0 to 1 for a rigid to a prefect plastic material, respectively.

It can be seen in Fig. 5 that the presented method can be used for both elastic and inelastic materials.

As an example, the consolidation of an oil tank foundation under a cyclic load with period of one month and intensity of 96 kPa is considered. The thickness of clay layer is 5.7 m with double drainage system. Consolidation and compressibility coefficients are 0.027648 $m^2\ day^{-1}$ and 0.00006 $m^2\ kN^{-1}$, respectively (Baligh and Levadoux, 1978).

Figure 6 shows the consolidation settlements resulted from the presented method along with the numerical and original methods.

In Fig. 7, it can be seen that the results of the suggested method are closer to the results of the numerical method comparing to the results of the original method. Also the final value of the settlement calculated by the presented method is coincide to that one calculated by the numerical method.

Table 1: Laboratory tests specifications

Pc (kPa)	ΔP (kPa)	t_c (min)	H_0 (cm)	H_d (cm)	m_v $(10^4 kPa)^{-1}$	c_v (cm^2/min)	β	α	T_c	T_c/β
275	25	60	1.3845	0.6915	1.2712	0.0012	0.095	0.090	0.1506	1.58
400	100	60	1.3815	0.7111	1.1763	0.0021	0.100	0.110	0.2492	2.49
450	50	20	1.372	0.678	2.0408	0.0005	0.070	0.100	0.0218	0.31
200	50	30	1.4304	0.7092	3.3557	0.0014	0.100	0.100	0.0823	0.82
200	50	60	1.4156	0.7016	3.5321	0.0014	0.100	0.100	0.1682	1.68
300	75	20	1.379	0.6822	2.8233	0.0014	0.105	0.110	0.0606	0.58
300	75	40	1.38	0.683	2.7053	0.0014	0.105	0.110	0.1209	1.15
300	50	60	1.367	0.6793	2.4872	0.0008	0.100	0.100	0.1040	1.04
700	100	60	2.724	1.3565	0.8076	0.0015	0.090	0.090	0.0489	0.54
650	100	60	2.8398	1.4143	0.7923	0.0021	0.093	0.090	0.0618	0.67
600	60	30	2.5926	1.2940	0.5946	0.0011	0.065	0.110	0.0197	0.30
600	60	60	2.6362	1.3157	0.6006	0.0011	0.065	0.110	0.0381	0.59
500	62.5	20	2.7789	1.3861	0.7773	0.0015	0.080	0.100	0.0156	0.20
500	62.5	40	2.7695	1.3815	0.7626	0.0015	0.080	0.100	0.0314	0.39
400	50	30	2.826	1.4104	0.7502	0.0029	0.095	0.095	0.0437	0.46
400	50	60	2.837	1.4158	0.7755	0.0029	0.095	0.095	0.0868	0.91
1400	200	30	2.9	1.4463	0.2586	0.0096	0.090	0.090	0.1377	1.53
1400	200	20	2.92	1.4563	0.2568	0.0096	0.090	0.090	0.0905	1.01
1600	100	30	2.885	1.441	0.2080	0.0026	0.090	0.090	0.0377	0.42
600	100	60	2.905	1.451	0.2065	0.0026	0.090	0.090	0.0744	0.83

LABORATORY TESTS

A series of laboratory tests have performed to investigation the consolidation of inelastic clays under cyclic loading. Tests were preformed in one-dimensional consolidation apparatus. Specifications of 20 specimens are shown in Table 1.

Value of Tc/β is a factor for estimating the frequency of the cyclic load (Baigh and Levadoux, 1978). Tc is the time factor for a loading cycle and can be calculated same as Eq. 2.

The value Tc/β is 1 and larger for a relatively long period cyclic load and 0.02 and smaller for a short period cyclic load.

Each test includes 3 stages. At the first stage a static load was applied in order to obtain the consolidation and compressibility coefficients in normally consolidated conditions; then the actual testing was performed by a cyclic load with intensity of ΔP and at third stage, when the cyclic consolidation reached to steady-state condition, the soil was unloaded and when its rebounding was completed, a static load was applied in order to determine c_v and m_v in OC conditions and the maximum settlement under static load. The effect of the consecutive cyclic loads on the settlement of normally consolidated clays is investigated experimentally by Yildirım and Ersan (2006).

Figure 8 shows the result of the test number 13. The value of Tc/β is 0.2 in test number 13 which is relatively rapid condition. It can be seen that the results of presented method and basic method presented by Baligh and Levadoux (1978) have some errors. However, the final value of settlement calculated by the presented method is closer to the laboratory results. Maximum error

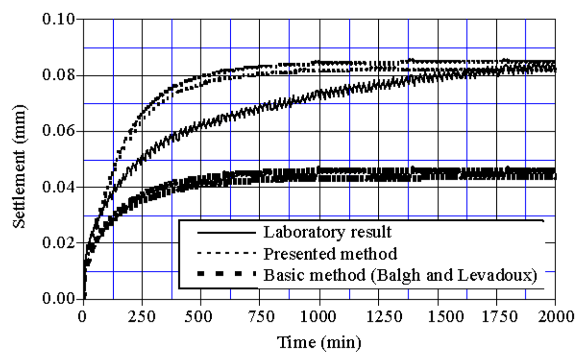

Fig. 8: Comparing the result of the presented and basic methods with lab results

for calculated settlement by presented approximate method is about 25% and its value for basic method is about 50%. However, errors for presented method are still less than of basic method.

The results of the test number 15 are shown in Fig. 9.

The maximum error of the basic method presented by Baligh and Levadoux (1978) is about 20% and it is about 13% for presented method. It can be seen that the errors of the presented method decrease as the time increases but the errors of the basic method increase.

Figure 10 shows laboratory test results along with the presented and basic methods for test number 1.

It is noticeable that the result of approximate method and basic method (Baligh and Levadoux, 1978) are approximately same as laboratory results.

As previously mentioned Tc/β for test numbers 13, 15 and 1 are 0.2, 0.46 and 1.58, respectively. It can be seen that the errors are decreasing as Tc/β increase. The higher value of Tc/β means slower loading or lower frequency.

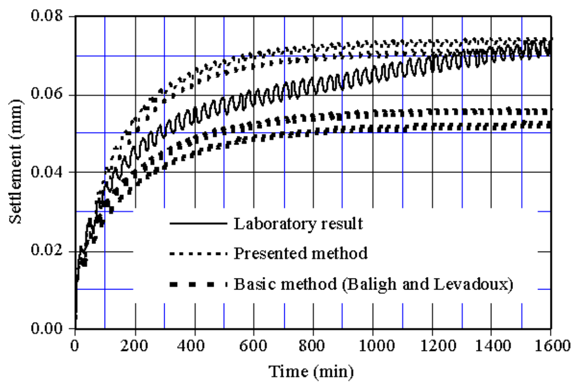

Fig. 9: Comparing the result of the presented and basic methods with lab results

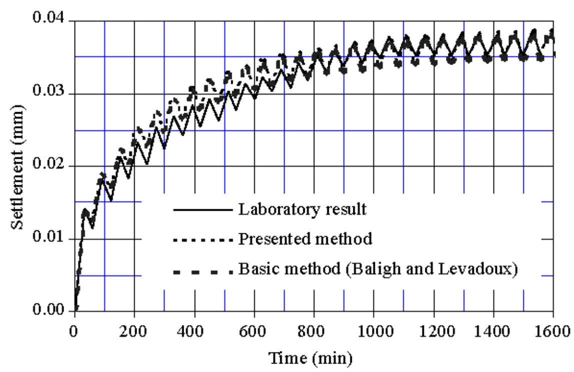

Fig. 10: Comparing the result of the presented and basic methods with lab results

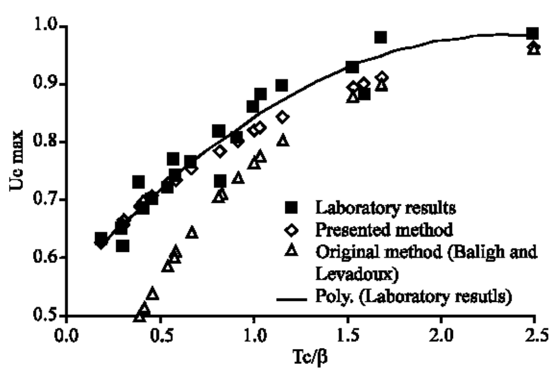

Fig. 11: Results of the presented and basic methods along with laboratory tests

For easy comparing, just the final value of the degree of consolidation obtained from laboratory tests is plotted. Also in this Fig. 11, the results of the presented method and basic method presented by Baligh and Levadoux (1978) are shown.

It should be noted that the final results of presented approximate and numerical methods are coincide. It can be seen that for long period loads, all of three methods have reasonable results.

CONCLUSION

In this study, consolidation of inelastic clays under cyclic loads has investigated. A new method based on the superimposing rule and a time transformation is presented. Superimposing rule had been used previously for consolidation of elastic clays under cyclic loading. Time transformation is used to apply the effect of the change of the material properties under cyclic loading. Application of the time transformation caused the superimposing rule to be applicable for consolidation inelastic clays under cyclic loading. The results of the presented method verified by the results of the numerical method and laboratory tests.

The presented method is based on Terzaghi's theory of consolidation. It has a simplified formulation based on physical aspects of the consolidation phenomena. It can be used for consolidation of both elastic and inelastic clays under relatively rapidly cyclic loads. The final values of settlements calculated by the presented method are in a good correspondence with the results of the laboratory tests and the numerical method.

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