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Application of Wave Equation in Pile Foundation Using Homotopy Perturbation Method and Variational Iteration Method

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Abstract: In this research, wave mechanic is applied for the analysis of piles during impact driving force. In particular, the governing second order differential equation of waver equation is solved with two approximate methods, namely Homotopy Perturbation Method (HPM) and Variational Iteration Method (VIM). The results obtained here clearly show, that the variational iteration method, is capable of solving wave equation in pile foundations subjected to impact driving load. The comparisons of the results reveal that these two methods are very effective, convenient and quite accurate to systems of differential equation.

Key words: Wave equation, pile foundation, homotopy perturbation method, variational iteration method

INTRODUCTION

Wave equation is usually used to investigate bearing capacity resistance of pile foundation.

There are many reasons a geotechnical engineer would recommend a deep foundation over a shallow foundation, but some of the common reasons are very large design loads, a poor soil at shallow depth, or site constraints. There are different terms used to describe different types of deep foundations including piles, drilled shafts, caissons and piers. Wave mechanics have been employed for the analysis of piles during impact driving for last forty years. Over the last decades several analytical/approximate methods have been developed to solve linear and nonlinear ordinary and partial differential equations. Some of these techniques include Variational Iteration Method (VIM) (He, 1999b, 2006a; Ganji *et al*., 2007; Ganji and Sadighi, 2007; Momani and Odibat, 2007), homotopy perturbation method (HPM) (He, 1999a, 2003, 2006b; Ganji and Sadighi, 2006; Ganji *et al*., 2007; Rafei and Ganji, 2006; Choobbasti *et al*., 2008; Barari *et al*., 2008) etc.

Linear and nonlinear phenomena play an important role in various fields of science and engineering. Most models of real-life problems are still very difficult to solve. Therefore, approximate analytical solutions such as homotopy-perturbation method were introduced.

This method is the most effective and convenient ones for both linear and nonlinear equations.

Perturbation method is based on assuming a small parameter. The majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all and the approximate solutions obtained by the perturbation methods, in most cases, are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion on which the small parameter should exists. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Recently, He (1999b) proposed a variational iteration method based on the use of restricted variations and correction functionals which has found a wide application for the solution of nonlinear ordinary and partial differential equations. This method does not require the presence of small parameters in the differential equation and provides the solution (or an approximation to it) as a sequence of iterates. The method does not require that the nonlinearities be differentiable with respect to the dependent variable and its derivatives.

In this research we will apply the homotopy perturbation method and variational iteration method to wave equation in piles during impact driving.

BASIC IDEA OF HOMOTOPY-PERTURBATION METHOD

To explain this method, let us consider the following function:

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$$A(u) - f(r) = 0, \quad r \in \Omega \tag{1}$$

With the boundary conditions of:

$$B(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma, \tag{2}$$

where, A, B, f(r) and Γ are a general differential operator, a boundary operator, a known analytical function and the boundary of the domain Ω, respectively.

Generally speaking the operator A can be divided in to a linear part L and a nonlinear part N(u). Equation 1 can therefore, be written as:

$$L(u) + N(u) - f(r) = 0, \tag{3}$$

By the homotopy technique, we construct a homotopy $v(r, p):\Omega\times[0, 1]\to R$ which satisfies

$$H(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0, \quad p \in [0,1], r \in \Omega, \tag{4}$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p\left[N(v) - f(r)\right] = 0, \tag{5}$$

where, $p \in [0, 1]$ is an embedding parameter, while u_0 is an initial approximation of Eq. 1, which satisfies the boundary conditions. Obviously, from Eq. 4 and 5 we will have:

$$H(v,0) = L(v) - L(u_0) = 0, \tag{6}$$

$$H(v,1) = A(v) - f(r) = 0, \tag{7}$$

The changing process of p from zero to unity is just that of v(r,p) from u_0 to u(r). In topology, this is called deformation, while $L(v)-L(u_0)$ and A(v)-f(r) are called homotopy.

According to the HPM, we can first use the embedding parameter p as a small parameter and assume that the solutions of Eq. 4 and 5 can be written as a power series in p:

$$v = v_0 + pv_1 + p^2v_2 + ..., \tag{8}$$

Setting p = 1 yields in the approximate solution of Eq. 4 to:

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots, \tag{9}$$

The combination of the perturbation method and the homotopy method is called the HPM, which eliminates the drawbacks of the traditional perturbation methods while keeping all its advantage.

The series (9) is convergent for most cases. However, the convergent rate depends on the nonlinear operator A(v). Moreover, He (1999a) made the following suggestions:

- The second derivative of N(v) with respect to v must be small because the parameter may be relatively large, i.e., $p \to 1$.
- The norm of $L^{-1}\, \partial N/\partial v$ must be smaller than one so that the series converges.

BASIC IDEA OF VARIATIONAL ITERATION METHOD

To clarify the basic ideas of VIM, we consider the following differential equation:

$$Lu + Nu = g(t) \tag{10}$$

where, L is a linear operator, N is a nonlinear operator and g(t) is a homogeneous term.

According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\left(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\right)d\tau \tag{11}$$

where, λ is a general lagrangian multiplier which can be identified optimally via the variational theorem. The subscript n indicates the nth approximation and u_n is considered as a restricted variation, i.e., $\delta\, \tilde{u}_n = 0$.

THE WAVE EQUATION IN GENERAL

The classical one-dimensional wave equation is given by the formula:

$$u_{tt}(x,t) = c^2 u_{xx}(x,t) \tag{12}$$

Where:

c = Acoustic speed of pile material (m sec^{-1})
u(x,t) = Displacement of pile particle (m)
t = Time from zero point (sec)
x = Distance from pile top (m)

For longitudinal vibrations, the constant (c) is the acoustic speed of the material of the bar, given by the equation:

$$c = \sqrt{\frac{E}{\rho}} \tag{13}$$

Where:
E = Pile Young's Modulus of Elasticity (Pa)
ρ = Pile Density (kg m^{-3})

Consider Eq. 12 and assume that the bar has but one boundary at (x = 0). Further assume that the bar begins with no initial displacement or velocity, i.e.,

$$u(x,0) = f(x) = 0 \tag{14}$$

and

$$u_t(x,0) = g(x)\ 0 \tag{15}$$

Where:
f(x) = Initial or momentary displacement distribution in pile (m)
g(x) = Initial or momentary velocity distribution in pile (m sec^{-1})

Assume also that the bar is excited at the boundary in such a way that the displacement of the end of the bar can be defined as:

$$u(0,t) = f(t) = 0.0149 - 0.01339e^{-835.176t} - 0.00151e^{-396.154t} \cos(401.6678t) - 0.0293e^{-396.154t}\sin(401.6678t) \tag{16}$$

Where:
f(t) = Displacement function at pile top (m)

$$u_x(0,t) = 0 \tag{17}$$

APPLICATION OF HOMOTOPY-PERTURBATION METHOD

We consider Eq. 12 for steel pile with conditions as follows:

$$E = 200\times 10^9 \text{ (Pa)} \tag{18}$$

$$\rho = 7850 \text{ (kg m}^{-3}\text{)} \tag{19}$$

$$c=\sqrt{\frac{E}{\rho}}=5047.5 \text{ (m sec}^{-1}\text{)} \tag{20}$$

To solve Eq. 12 by means of HPM, we consider the following process after separating the linear and nonlinear parts of the equation.

A homotopy can be constructed as follows:

$$\begin{aligned} H(u,p) &= (1-p)\left(\frac{\partial^2}{\partial x^2}v(x,t) - \frac{\partial^2}{\partial x^2}v_0(x,t)\right) \\ &+ p\left(\frac{\partial^2}{\partial t^2}v(x,t) - c\frac{\partial^2}{\partial x^2}v(x,t)\right) = 0, \end{aligned} \tag{21}$$

Substituting $v = v_0 + pv_1 + \ldots$ into Eq. 21 and rearranging the resultant equation based on powers of p-terms, one has:

$$p^0 : \frac{\partial^2}{\partial x^2}v_0(x,t) = 0, \tag{22}$$

$$\begin{aligned} p^1 : &\left(\frac{\partial^2}{\partial x^2}v_1(x,t)\right) + \left(\frac{\partial^2}{\partial t^2}v_0(x,t)\right) - \\ &\left(\frac{\partial^2}{\partial x^2}v_0(x,t)\right) - c^2\left(\frac{\partial^2}{\partial x^2}v_0(x,t)\right) = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} p^2 : -&\left(\frac{\partial^2}{\partial x^2}v_1(x,t)\right) + \left(\frac{\partial^2}{\partial x^2}v_2(x,t)\right) + \\ &\left(\frac{\partial^2}{\partial t^2}v_1(x,t)\right) - c^2\left(\frac{\partial^2}{\partial x^2}v_1(x,t)\right) = 0, \end{aligned} \tag{24}$$

With the following conditions:

$$\begin{aligned} &v_0(x,0) = 0,\ \frac{\partial}{\partial t}v_0(x,0) = 0,\ v_0(0,t) = f(t) \\ &\quad = 0.0149 - 0.01339e^{-835.176t} - 0.00151e^{-396.154t} \\ &\quad \cos(401.6678t) - 0.0293e^{-396.154t} \\ &\quad \sin(401.6678t),\ \frac{\partial}{\partial x}v_0(0,t) = 0 \\ &v_i(x,0) = 0,\ \frac{\partial}{\partial t}v_i(x,0) = 0, \\ &v_i(0,t) = 0,\ \frac{\partial}{\partial x}v_i(0,t) = 0 \quad i = 1,2,\ldots\ldots. \end{aligned} \tag{25}$$

V(x,t) may be written as follows by solving the Eq. 22, 23 and 24:

$$\begin{aligned} v_0(x,t) &= 0.0149 - 0.01339e^{-835.176t} - 0.00151e^{-396.154t} \\ &\cos(401.6678t) - 0.0293e^{-396.154t}\sin(401.6678t), \end{aligned} \tag{26}$$

$$\begin{aligned} v_1(x,t) &= \frac{1}{2}[9339.8e^{(-835.18t)} - 9331.2e^{(-396.15t)} \\ &\cos(401.67t) + 351.66e^{(-396.15t)}\sin(401.67t)]x^2, \end{aligned} \tag{27}$$

$$\begin{aligned} v_2(x,t) &= 1.6667*10^{-15}x^4\begin{pmatrix} -1.62867*10^{23}e^{(-835.18t)} \\ +1.771610^{21}e^{(-396.15t)}\cos(401.67t) \\ +7.427910^{22}e^{(-396.15t)}\sin(401.67t) \end{pmatrix} \\ &+\frac{1}{2}\begin{pmatrix} 9339.8e^{(-835.18t)} - 9331.2e^{(-396.15t)}\cos(401.67t) \\ +351.66e^{(-396.15t)}\sin(401.67t) \\ +9339.8c^2e^{(-835.18t)} - 9331.2c^2e^{(-396.15t)}\cos(401.67t) \\ +351.66c^2e^{(-396.15t)}\sin(401.67t) \end{pmatrix}x^2, \end{aligned} \tag{28}$$

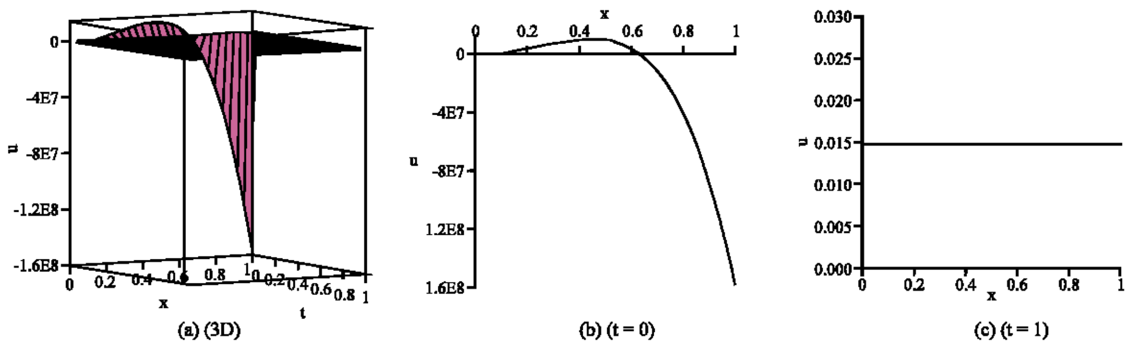


Fig. 1: Result of wave equation in pile using (HPM) method [c = 5047.5 (m sec^{-1}) and 0 < x < 1 and 0 < t < 1]

In the same manner, the rest of components were obtained using the maple package.

According to the HPM, we can conclude that:

$$u(x,t)=\lim_{p\to 1} v(x,t)=v_0(x,t)+v_1(x,t)+\ldots, \tag{29}$$

Therefore, substituting the values of $v_0(x,t)$, $v_1(x,t)$ and $v_2(x,t)$ from Eq. 26, 27 and 28 into Eq. 29 yields:

$$\begin{aligned}u(x,t)&=0.0149-0.01339e^{-835.176t}-0.00151e^{-396.154t}\\&\cos(401.6678t)-0.0293e^{-396.154t}\sin(401.6678t)\\&+\frac{1}{2}[9339.8e^{(-835.18t)}-9331.2e^{(-396.15t)}\cos(401.67t)\\&+351.66e^{(-396.15t)}\sin(401.67t)]x^2\\&+1.6667*10^{-15}x^4\begin{pmatrix}-1.62867*10^{23}e^{(-835.18t)}\\+1.771610^{21}e^{(-396.15t)}\cos(401.67t)\\+7.427910^{22}e^{(-396.15t)}\sin(401.67t)\end{pmatrix}\\&+\frac{1}{2}\begin{pmatrix}9339.8e^{(-835.18t)}-9331.2e^{(-396.15t)}\cos(401.67t)\\+351.66e^{(-396.15t)}\sin(401.67t)\\+9339.8c^2e^{(-835.18t)}-9331.2c^2e^{(-396.15t)}\cos(401.67t)\\+351.66c^2e^{(-396.15t)}\sin(401.67t)\end{pmatrix}x^2,\end{aligned}\tag{30}$$

Figure 1 shows the result of wave equation in pile foundation after substituting c and plotting for 0 < x < 1 and 0 < t < 1.

APPLICATION OF VARIATIONAL ITERATION METHOD

Here, variational iteration method is developed for solving wave equation in pile.

Consider wave equation in pile (Eq.12).

To solve Eq. 12 via VIM, one has to find the Lagrangian multiplier, which can be identified by substituting Eq. 12 into Eq. 11, upon making it stationary leads to the following:

$$\begin{aligned}1-\lambda'\big|_{\tau=t}&=0\\ \lambda\big|_{\tau=t}&=0\\ \lambda''\big|_{\tau=t}&=0\end{aligned}\tag{31}$$

Solving the system of Eq. 31, yields:

$$\lambda(\tau)=\tau-t, \tag{32}$$

And the variational iteration formula is obtained in the form:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t\left\{(\tau\text{-}t)\left(\frac{\partial^2}{\partial t^2}u_n(\tau,t)-c^2\frac{\partial^2}{\partial x^2}u_n(\tau,t)\right)\right\}dt, \tag{33}$$

Now, we assume that the initial approximation has the form:

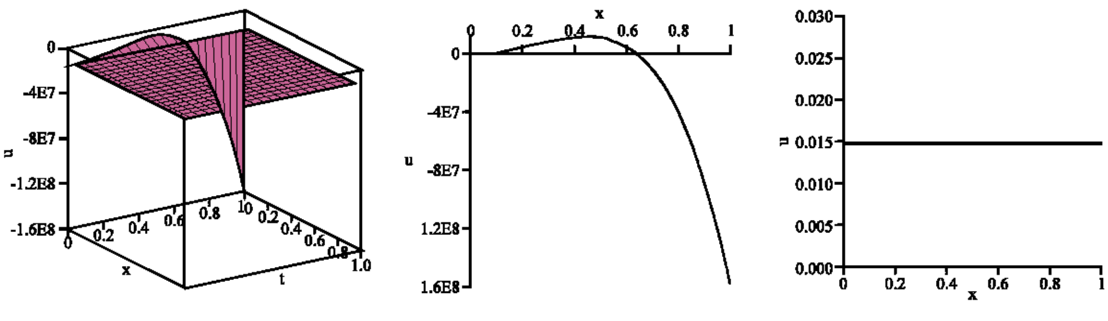
$$\begin{aligned}u_0(x,t)&=0.0149-0.01339e^{-835.176t}-0.00151e^{-396.154t}\\&\cos(401.6678t)-0.0293e^{-396.154t}\sin(401.6678t),\end{aligned}\tag{34}$$

Using the above variational formula (33), we have:

$$u_1(x,t)=u_0(x,t)+\int_0^t\left\{(\tau\text{-}t)\left(\frac{\partial^2}{\partial t^2}u_0(\tau,t)-c^2\frac{\partial^2}{\partial x^2}u_0(\tau,t)\right)\right\}dt, \tag{35}$$

Substituting Eq. 34 into Eq. 35 and after simplification, we have:

$$\begin{aligned}u_1(x,t)&=0.0149-0.01339e^{-835.176t}-0.00151e^{-396.154t}\\&\cos(401.6678t)-0.0293e^{-396.154t}\sin(401.6678t)\\&+4669.89x^2e^{-835.176t}-4669.89x^2e^{-396.154t}\cos(401.6678t)\\&+175.829x^2e^{-396.154t}\sin(401.6678t)\end{aligned}\tag{36}$$



(a) (3D) (b) (t = 0) (c) (t = 1)

Fig. 2: Result of wave equation in pile using (VIM) method [c = 5047.5 (m sec^{-1}) and 0 < x < 1 and 0 < t < 1]

In the same way, we obtain $u_2(x,t)$ as follows:

$$\begin{aligned} u_2(x,t) &= 0.0149 - 0.01339e^{-835.176t} - 0.00151e^{-396.154t} \\ &\cos(401.6678t) - 0.0293e^{-396.154t}\sin(401.6678t) \\ &+1.1897*10^{11}x^2e^{-835.176t} - 1.1886*10^{11}x^2e^{-396.154t} \\ &\cos(401.6678t)+4.4796*10^{9}x^2e^{-396.154t}\sin(401.6678t) \\ &-2.71444*10^{8}x^4e^{-835.176t} + 2.9527*10^{6}x^4e^{-396.154t} \\ &\cos(401.6678t)+1.2379*10^{8}x^4e^{-396.154t}\sin(401.6678t), \end{aligned} \tag{37}$$

Figure 2 shows results of wave equation in pile after substituting c and plotting for 0 < x < 1 and 0 < t < 1.

CONCLUSION

In this research, variational Iteration and homotopy perturbation methods have been successfully applied to find the solution of wave equation in piles foundation during impact driving. Solution of wave equation shows that the results of proposed methods are in agreement with each other. The homotopy perturbation method which was used to solve wave equation in piles during impact driving seems to be very easy. There is less computation needed in comparison with the other methods (close form solutions and numerical methods). The results obtained here clearly show, that the variational iteration method, is capable of solving wave equation in pile foundations subjected to impact driving load, with a rapid convergent successive approximation, without any restrictive assumptions or transformations that may change the physical behavior of the problem.

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