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Design and Implement of a Digital PID Controller for a Chaos Synchronization System by Evolutionary Programming

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Abstract: This study is concerned with the design of a digital Proportional Integral Derivative (PID) controller for synchronization of a continuous chaotic model. The Evolutionary Programming Algorithm (EPA) has been considered as a useful technique for finding global optimization solutions for certain complicated functions in recent years. Therefore, in this research, we attempt to use the EP algorithm in digital PID control design for deriving optimal or near optimal digital PID control gains such that a performance index between the master and slave Sprott chaotic circuits is minimized. A numerical and experimental result exemplifies the synchronization procedure.

Key words: Proportional Integral Derivative (PID) controller, Evolutionary Programming Algorithm (EPA), optimal, chaos synchronization, Sprott circuit

INTRODUCTION

Chaos synchronization has gained a lot of attention among scientists from variety of research fields over the last few years (Chen and Dong, 1998). Chaos synchronization can be applied in the vast areas of physics and engineering science, especially in secure communication (Kocarev and Parlitz, 1995; Murali and Lakshmanan, 1998). The idea of synchronizing two identical chaotic systems was first introduced by Carroll and Pecora (1991). In the most synchronization approaches used for the continuous-time chaotic systems, master-slave or drive-response formulism is employed (Pecora and Carroll, 1990). Let us call a particular chaotic system the master (drive) and another one the slave (response). The goal is to synchronize the slave (response) system behavior to the master (drive) one. In order to achieve the synchronization, a nonlinear controller that obtains signals from the master and slave systems and manipulates the slave system should be designed. Recently, many control methods have been developed to achieve chaos synchronization between two identical chaotic systems with different initial conditions (Yau *et al*., 2005, 2006; Yau, 2004; Lin *et al*., 2005). However, in the earlier research, none discusses how to obtain optimal or near optimal digital controller to synchronize continuous chaotic systems according to a specified performance index. As we know, the Evolutionary Programming Algorithm (EPA) has been considered as a useful technique for finding the global optimization solutions for certain complicated functions and also has been applied to solve difficult problems in the field of control engineering (Cao, 1997). Generally speaking, the EP algorithm for global optimization contains four parts: initialization, mutation, competition and reproduction. Furthermore, a Quasi Random Sequence (QRS) is used to generate an initial population for EP algorithm to avoid causing clustering around an arbitrary local optimum. On the other hand, a great majority of industrial processes are still controlled by means of proportional-integral-derivative (PID) controller due to its simplicity in architecture and acceptable performance. However, it is extremely difficult to find the optimal set of PID gains for nonlinear dynamical systems. Since the PID controller gains play an important role in determining the behavior of the dynamical system, many tuning schemes for linear systems have been proposed in the literature (Chien *et al*., 1952; Cameron and Seborg, 1983; David *et al*., 2006). The objective of this study is to present a simple but effective digital PID controller to implement the mutual synchronization of two identical Sprott chaotic circuits. The EP algorithm is used for determining the optimal control gains of digital PID controller. An optimization problem is then well defined and an EP algorithm is presented to solve the optimization problem such that the cost function of master-slave

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system is minimized as possibly. The numerical and experimental results are used to demonstrate the proposed controller in this research.

PROBLEM FORMULATION

The one type of Sprott circuits is defined by David *et al.* (2006):

$$\dddot{x} + 0.6\ddot{x} + \dot{x} = -1.2x + 2\text{sign}(x) \tag{1}$$

where, the dots above the variable x means time derivatives (first, second and third) and sign $(\cdot)$ is the sign function. A state representation of system (1) can be obtained by defining $x_1 = x$, $x_2 = \dot{x}$ and $x_3 = \ddot{x}$. Now define a master system, denoted with the subscript m, given in the state form:

$$\begin{aligned} \dot{x}_{1m} &= x_{2m} \\ \dot{x}_{2m} &= x_{3m} \\ \dot{x}_{3m} &= -1.2x_{m1} - x_{m2} - 0.6x_{m3} + 2\cdot\text{sign}(x_{m1}) \\ y_m &= x_{1m} \end{aligned} \tag{2}$$

Also define a slave system with the same form, denoted with the subscript s:

$$\begin{aligned} \dot{x}_{1s} &= x_{2s} \\ \dot{x}_{2s} &= x_{3s} + u \\ \dot{x}_{3s} &= -1.2x_{s1} - x_{s2} - 0.6x_{s3} + 2\cdot\text{sign}(x_{s1}) \\ y_s &= x_{1s} \end{aligned} \tag{3}$$

Now let us define the state errors between the master system and slave system as:

$$e_1 = x_{1s} - x_{1m},\ e_2 = x_{2s} - x_{2m},\ e_3 = x_{3s} - x_{3m} \tag{4}$$

One main objective of this work is to present a simple but effective PID controller based on EP algorithm to achieve the synchronization of two identical chaotic systems with different initial conditions. The term u in (3) is a PID controller obtained via EP algorithm to guarantee the synchronization performance. The procedure to determine the PID controller u is to first define the output error signal $y_e = y_s - y_m$, then the transfer function of a digital PID controller, from input e(z) to output u(z) in z-domain as shown in Fig. 1, is generally given by:

$$u(z) = k_p y_e(z) + k_i \frac{T}{2}\frac{z+1}{z-1} y_e(z) + k_d \frac{1}{T}\frac{z-1}{z} y_e(z) \tag{5}$$

where, T is the sampling time, k_p is the proportional gain, k_i is the integral time constant and k_d is the derivative time constant.

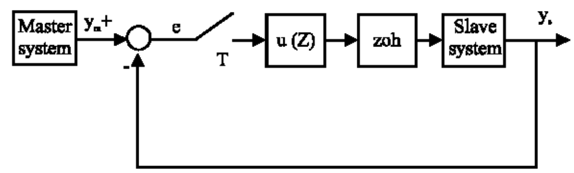

Fig. 1: The digital controlled system

In general, for a controller design, the performance criterion or objective function can be defined according to our desired specifications. Two kinds of performance criteria usually considered in the EP algorithm are the Integrated Squared Error (ISE) and the integrated absolute error (IAE). In this study, the IAE index is used as the Objective Function (OF), which is given as:

$$OF \equiv IAE = \sum_{k=1}^{k_f} |E(kT)| \tag{6}$$

where, $E(k)T = [e_1(kT)\ e_2(kT)\ e_3(kT)]$ and $|\cdot|$ is the Euclidean norm of a vector and k is referred to as the sampling time point and k_f is the total numbers of sampling. In the following, based on using EP algorithm, we will develop a tuning method for the digital PID controller with optimal gain parameters to minimize the objective function score (6).

SOLVING THE OPTIMIZATION PROBLEM VIA EPA

Here, an extended EP algorithm is proposed to obtain the digital PID controller with optimal gain parameters to minimize the following Objective Function (OF) score (6). Let g be the continuously differentiable matrix-valued function defined for $g \in S$, where $S = \{g \in R^3 | 0 \le g_i \le M_i,\ I = 1,2,3\}$, M_i is the searching space and is bounded. The optimization problem involves finding $g^* = [k_p^*, k_i^*, k_d^*] \in S$ such that the OF performance index of the system is minimized. More accurately, the optimization problem (P1) is stated mathematically as:

(P1): To find $g^* \in S$ such that

$$OF \equiv IAE = \sum_{k=1}^{k_f} |e_k|,\ \text{for } g^* \in S \tag{7}$$

is minimized.

Based on the results shown in Cao (1997), an extended EP algorithm for solving the above optimization problem is described as following steps:

Step 1: Generate an initial population $p_0 = [p_1, p_2, K, pN]$ of size N by randomly initializing each 3-dimensional solution vector $p_i \in S$, i = 1,2,..., N, according to the Quasi Random Sequence (QRS).

Step 2: Calculate the fitness score (objective function) $f_i = f(p_i)$ for every p_i, i = 1,2,K, N, where

$$f_i(p_i) = OF = \sum_{k=1}^{k_f} |E(kT)| \quad (8)$$

Step 3: Mutate every pi, i = 1,2,..., N, based on the statistics to double the population size from N to 2N and generate p_i+N in the following:

$$p_{i+N,j} = p_{i,j} + N\left(0, \beta\frac{f_i}{f\Sigma}\right), \forall j = 1,2,3 \quad (9)$$

where, p_i, j denotes the jth element of the ith individual,

$$N(0, \beta\frac{f_i}{f_\Sigma})$$

represents a Gaussion random variable with a mean zero and variance

$$\beta\frac{f_i}{f_\Sigma}, f_\Sigma$$

is the sum of all fitness scores and β is a parameter to scale

$$\frac{f_i}{f_\Sigma}.$$

Step 4: Calculate the fitness score f_i+N for every pi, N, i = 1,2,..., N, by using Eq. 8. By the stochastic competition process, pi, i = 1,2,..., N, competes with p_j, j = N+1,...,2N randomly each against the other. If $f_i < f_j$, p_i is the winner and survives; otherwise, p_j is the winner and p_i is replaced by p_j. After the competition process, we select the N winners for the next generation and let the individual with the minimum objective function in the winners be p_1.

Step 5: If the value f_Σ converges to a minimum value, then $g^* = p_1$ is the global optimum value and $g^* = [k_p^*, k_i^*, k_d^*]$ such that the OF performance index of the system is minimized as possibly. Otherwise, return to Step 3.

SIMULATION AND EXPERIMENTAL RESULTS

The initial value conditions (x_{1m}, x_{2m}, x_{3m}) = (0.1, 0.1, 0.1) and (x_{1s}, x_{2s}, x_{3s}) = (-1, -1,-1) are used in this numerical example. From Fig. 2, it can be seen that the master system exists a complex trajectory in the phase plane. Figure 3 reveals that the corresponding maximum Lyapunov exponent has a positive value and thus it can be inferred that the master system is in a state of chaotic motion (Chen and Dong, 1998).

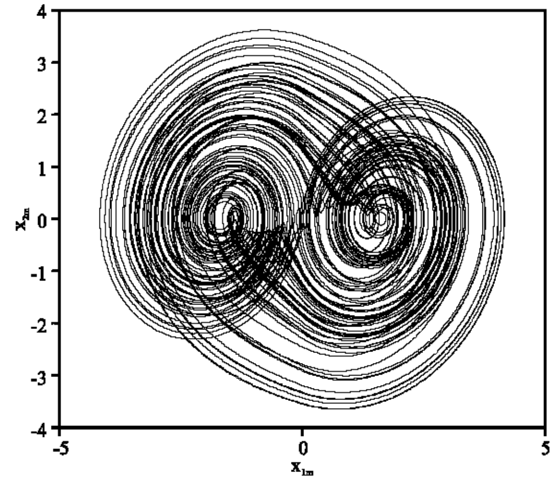

Fig. 2: The phase plane trajectory of a Sprott circuit

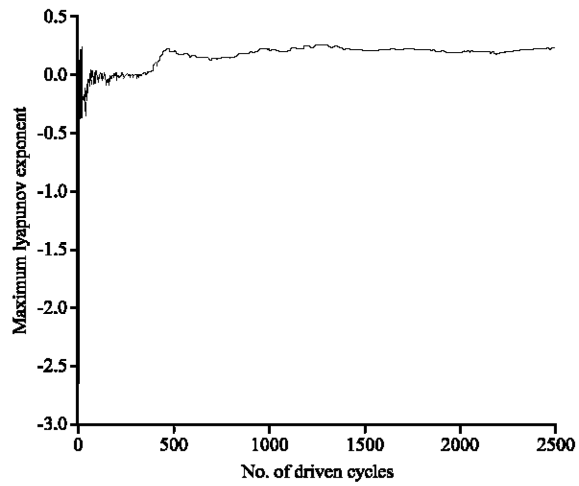

Fig. 3: The maximum Lyapunov exponent of x_{1m} plot as a function of the number of driving cycles

We solved the optimization problem (P1) with N = 70 and β = 0.01 by using the control toolbox of Matlab and Simulink. According to the proposed EP algorithm, we generated $P_0 = [p_1, p_2, \ldots, p_{70}]$ according to the QRS. It can be easily observed from Fig. 4 that it converges after about 100 iterations and its final value of IAE is f(z*) = 0.1623. Correspondingly, the PID control gains are $z^* = (k_p^*, k_i^*, k_d^*) = (14.257, 30, 10.385)$. The trajectories of k_p, k_i and k_d during the evolutionary procedure are also shown in Fig. 5. For reference, the output response using the resulting PID control gains z* is then shown in Fig. 6. The numerical simulation results show that the proposed PID controller via EP algorithm is viable for synchronization of chaotic systems.

In order to verify the proposed PID controller in practical system, an electronic circuit implementing Eq. (2-3) is shown in Fig. 7, shows the experimental results when these circuits are connected in a master/slave configuration. The control was implemented on an industrial computer with a sampling rate of 2000 Hz and

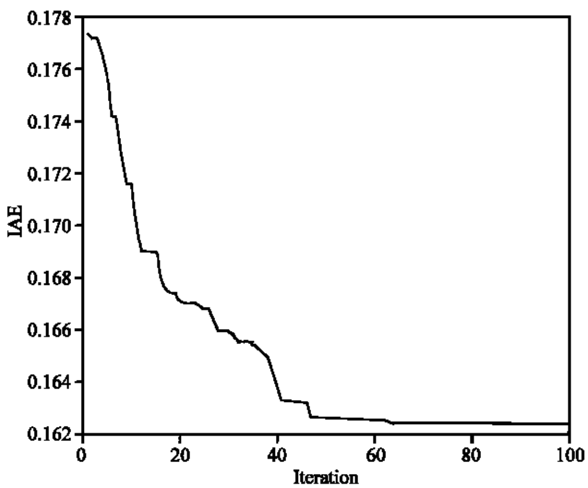

Fig. 4: Convergence curve of IAE

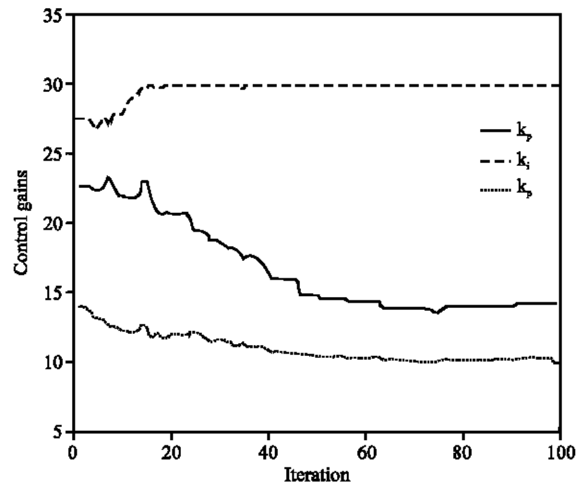

Fig. 5: Trajectories of k_p, k_i and k_d

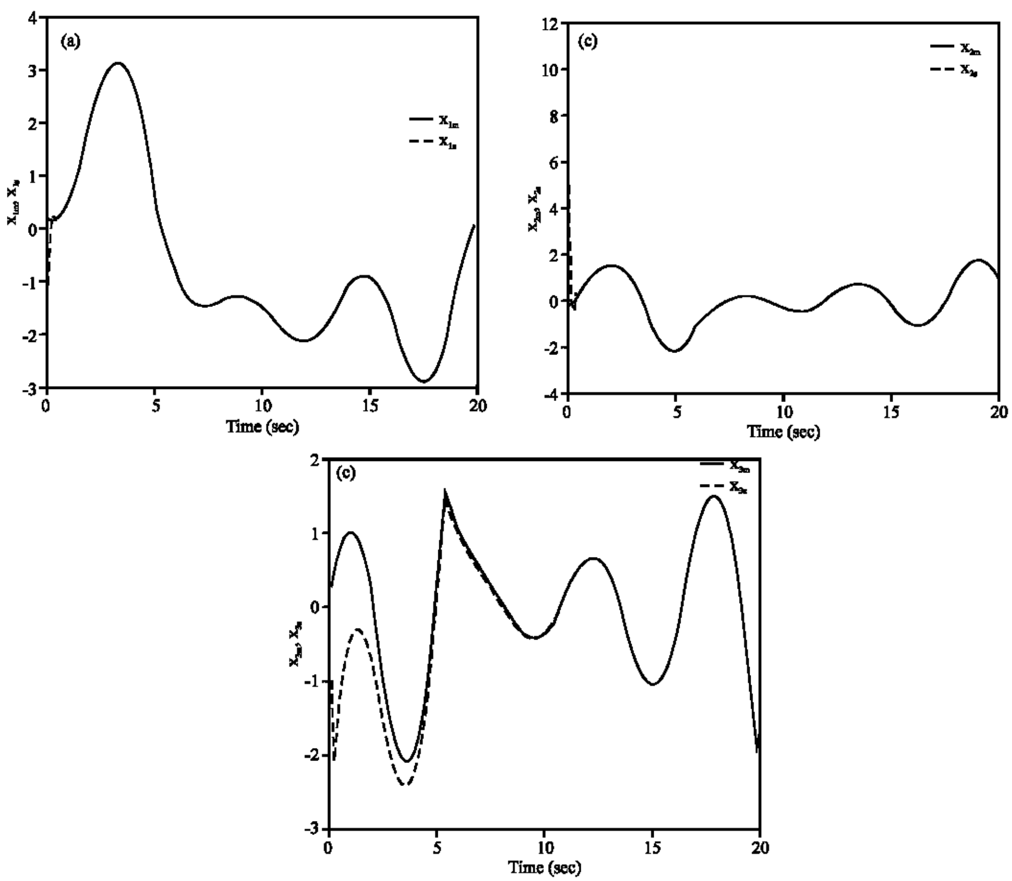

Fig. 6: Numerical time responses of chaos synchronization in Sprott circuits: master and slave system outputs are x_{1m}, x_{2m}, x_{3m} (——) and x_{1s}, x_{2s}, x_{3s} (- - -), respectively

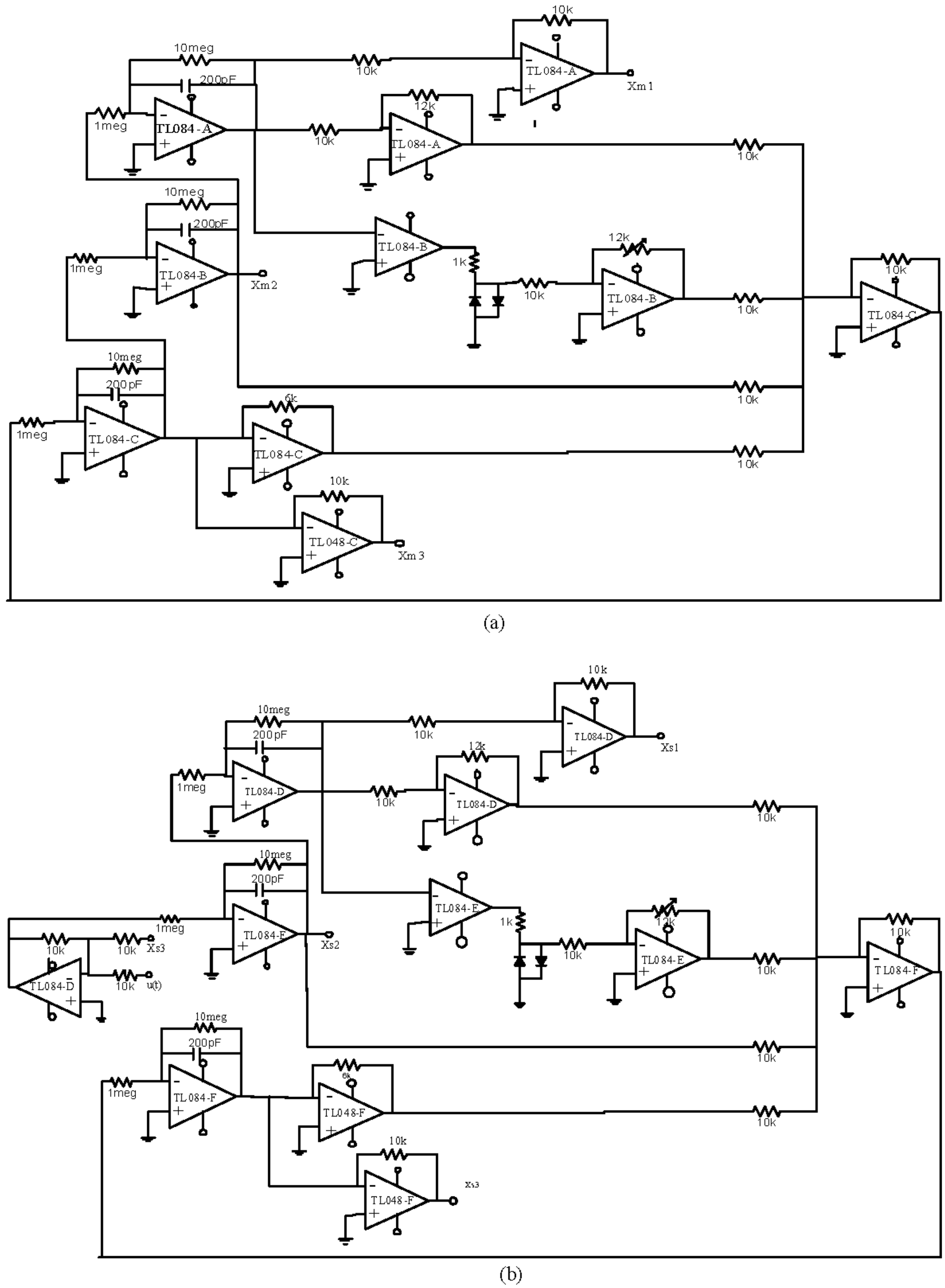

Fig. 7: Master (a) and slave (b) Sprott circuits

shown in Fig 8. From Fig. 9, it can be seen that the slave circuit response is synchronized to the master circuit response after the control is action at t = 15 sec. The experimental results of error dynamics in Fig. 10 show a convergence to a very small synchronization error and a continuous control input is obtained.

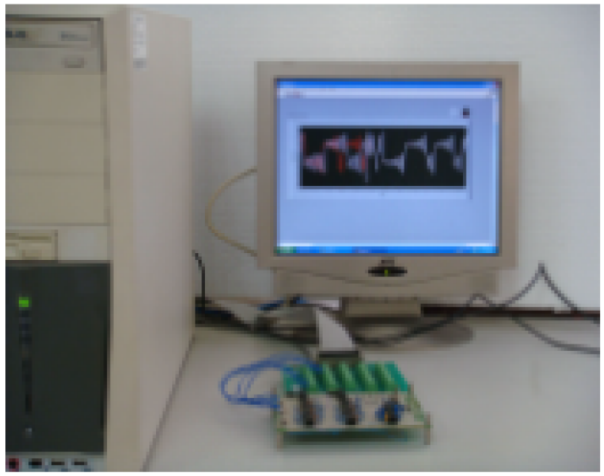

Fig. 8: A photograph of the proposed Sprott chaos synchronization system

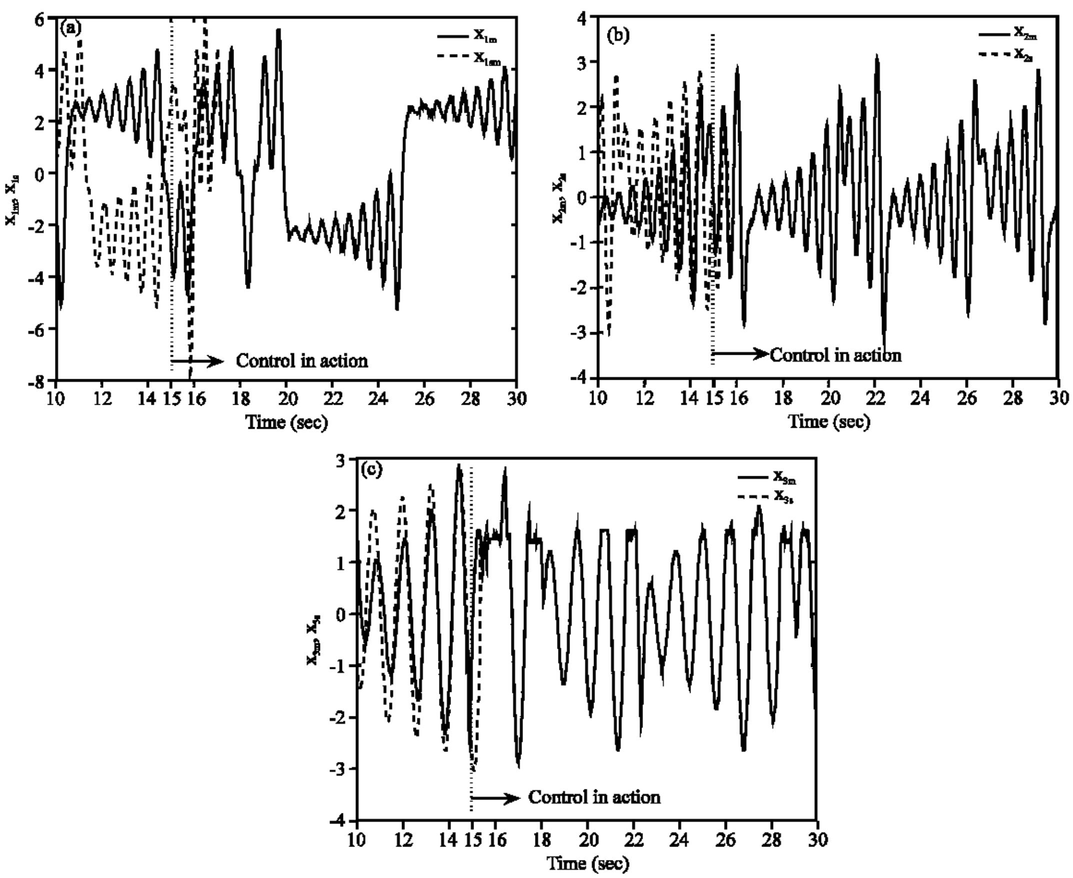

Fig. 9: Experimental time responses of chaos synchronization in Sprott circuits: master and slave system outputs are x_{1m}, x_{2m}, x_{3m} (——) and x_{1s}, x_{2s}, x_{3s} (- - -), respectively

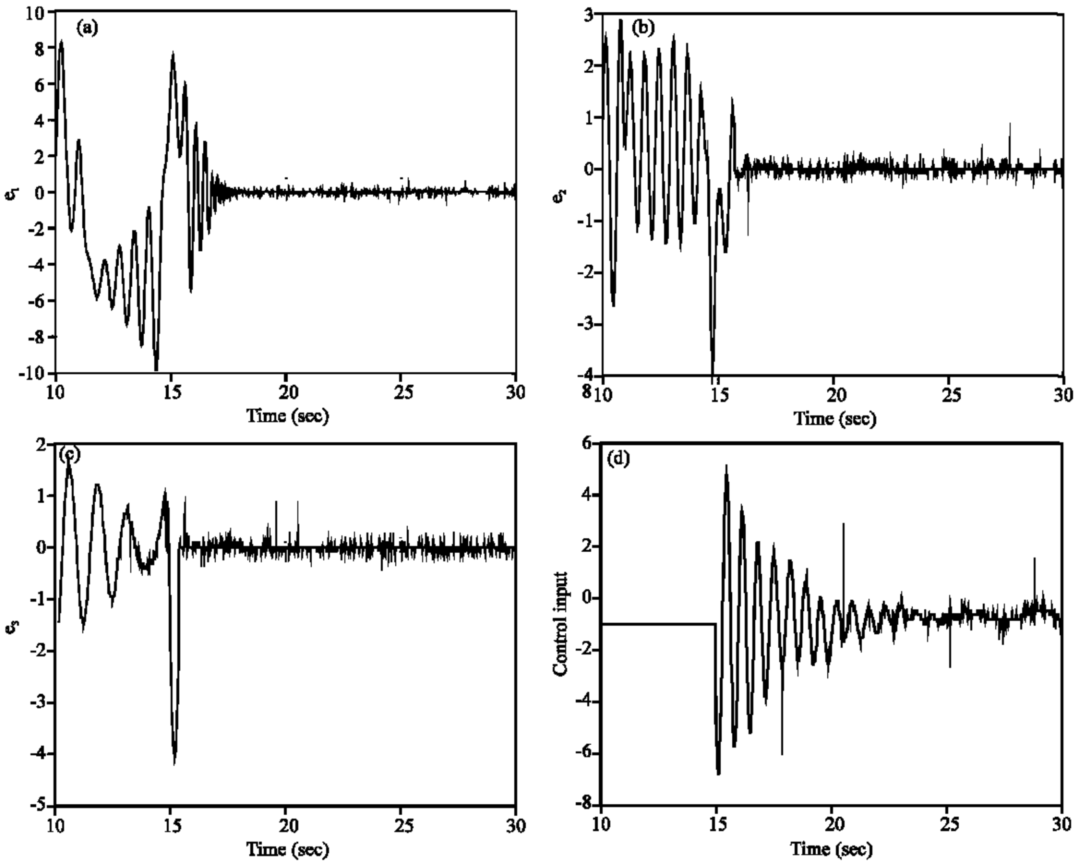

Fig. 10: Experimental results. Syncronization errors e_1, e_2, e_3 and control signal (u)

CONCLUSION

In this study, using the evolutionary programming algorithm, a simple and effective digital PID controller has been proposed for synchronization of two Sprott circuits. Three gains of digital PID controller can be directly obtained by solving a specified optimization problem as defined above by performing Steps 1-5. Compared with the existing reports for chaotic synchronization, the proposed digital PID controller based on EP algorithm is not only effective but also simple in architecture to implement in a digital base controller.

REFERENCES

Cameron, F. and D.E. Seborg, 1983. A self-tuning controller with a PID structure. Int. J. Control, 38 (3): 401-417.

Cao, Y.J., 1997. Eigenvalue optimization problems via evolutionary programming. Electron. Lett., 33 (8): 642-653.

Carroll, T.L. and L.M. Pecora, 1991. Synchronizing chaotic circuits. IEEE Trans. Circ. Syst., 38 (4): 453-456.

Chen, G. and X. Dong, 1998. From chaos to order: Methodologies perspectives and applications. World Scientific, Singapore.

Chien, K.L., J.A. Hrones and J.B. Reswick, 1952. On the automatic control of generalized passive systems. Trans. ASME., 74 (2): 175-185.

David, I., R. Almeida, J. Alvarez and J.G. Barajas, 2006. Robust synchronization of Sprott circuits using sliding mode control. Chaos Solitons Fractals, 30 (1): 11-18.

Kocarev, L. and U. Parlitz, 1995. General approach for chaotic synchronization with application to communication. Phys. Rev. Lett., 74 (25): 5028-5031.

Lin, J.S., J.J. Yan and T.L. Liao, 2005. Chaotic synchronization via adaptive sliding mode observers subject to input nonlinearity. Chaos Solitons Fractals, 24 (1): 371-381.

Murali, K. and M. Lakshmanan, 1998. Secure communication using a compound signal from generalized synchronizable chaotic systems. Phys Lett., A 241 (6): 303-310.

Pecora, L.M. and T.L. Carroll, 1990. Synchronization in chaotic systems. Phys. Rev. Lett., 64 (4): 821-824.

Yau, H.T., 2004. Design of adaptive sliding mode controller for chaos synchronization with uncertainties. Chaos Solitons Fractals, 22 (2): 341-347.

Yau, H.T., J.S. Lin and J.J. Yan, 2005. Synchronization control for a class of chaotic systems with uncertainties. Int. J. Bifurc. Chaos, 15 (7): 1-12.

Yau, H.T., C.L. Kuo and J.J. Yan, 2006. Fuzzy sliding mode control for a class of Chaos. Int. J. Nonlinear Sci. Numerical Simulation, 17 (3): 333-338.