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Computerized Analysis of Discrete MRR Optimization for Constrained Due-Date

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Abstract: This study not only applies Material Removal Rate (MRR) into the objective function mathematically, but also implements calculus of variations to resolve the dynamic machining control problem comprehensively. In addition, the optimal solution of the Machining Project Control (MPC), model is proposed and the decision criteria to determine the optimal solution are also recommended. Moreover, the computerized analyses with a numerical simulation to compare with the traditional machining model are fully prepared. This study definitely contributes the applicable approach to dynamic control of the material removal rate and provides the efficient tool to concretely optimize the cost of a machining project for operation research engineers in today's machining industry with profound insight.

Key words: Optimal control, material removal rate, calculus of variation, operations research

INTRODUCTION

Machining conditions of a cutting tool have been the most critical variables in machining operation. The cutting speed, feed rate and depth of cut were considered as three factors of input cutting parameters (Lan *et al*., 2002, 2008). To calculate the optimum cutting conditions is the objective for production (Meng *et al*., 2000). Koren *et al*. (1991) have described several methods to be used under stepwise constant variation in feed, speed, or depth of cut, but none is practically applicable when two or more cutting conditions are changed. Therefore, the method of controlling cutting conditions with fixed material removal rate has been introduced (Balazinski and Ennajimi, 1984; Choudhury and Appa Rao, 1999). In most studies on this viewpoint, the material removal rate is fixed because of the expensive observation of the control. Nevertheless, through the computer-integrated interface to program the machining feed rate on modern computer numerical controlled (CNC) machines with fixed cutting speed and depth of cut, the material removal rate is capable of being dynamically controlled (Balazinski and Songmene, 1995; Lan *et al*., 2008).

In addition, the tool life is a critical parameter of the machining process (Davim and Conceicao Antonio, 2001; Lan *et al*., 2008). Novak and Wilkund (1996) proposed a suitable implementation to predict tool life and Lee *et al*. (1992) proposed a method of optimal control to ensure maximum tool life. Meng *et al*. (2000) also provided a modified Taylor tool life equation to minimize tool cost. As a matter of fact, the maximum tool life or the minimum tool cost will not guarantee the minimal cost of a machining operation. Besides, the various tool checking periods for a tool change from different operators will decrease the productivity and increase the cost during the machining project significantly. In order to manage the consumption of tools well, a fixed tool life is practically considered into the machining project. However, the production period is certainly related to the order quantity of a project. For the convenience of project scheduling, the production period is also proposed to be determinable and then introduced as fixed into the study.

Moreover, the cost to machine each part is a function of the machining time (Jung and Ahluwalia, 1995). As the marginal cost of production is a linear increasing function of production rate (Kamien and Schwartz, 1991; Lan *et al*., 2008) the marginal cost of machining operation is also considered to be a linear function of the material removal rate in this study. It is that the higher machining rate results higher operational cost such as machine maintenance, loading-unloading and machine depreciation costs.

Although, several time series modeling on the control of machining process are mentioned (Kim *et al*., 1996; Yeh and Lan, 2002), none is capable to achieve minimum cost. They are all emphasizing on the maximal tool usage or minimal tool cost. Actually, the production cost and the production period of a machining project are mostly concerned problems confronting the manufacturing industry. Besides, the need of operating CNC machines efficiently to obtain the required payback is even greater in the case of rough machining, since a greater amount of material is removed thus increasing possible savings (Meng *et al*., 2000). With the reasons above, there is an economic need to control the material removal rate of rough machining operation for a machining project.

Hence, the necessity of finding the optimal solution to reach the minimum cost of a machining project with fixed tool life and production period is absolutely arising.

ASSUMPTIONS AND NOTATIONS

Before formulating the problem, several assumptions and notations are to be made. They are described as follows:

Assumptions

- The machining project is a continuous rough turning operation with only one type of tool and it is assigned to one machine only.
- Each tool performs the same fixed length of cutting time (tool life) before replacement.
- The upper limit of material removal rate is generated from the maximum cutting conditions suggested in the handbook and the fixed tool life is derived from the Taylor's tool life equation (DeGarmo *et al*., 1997) with these maximal conditions. Thus, no tool will break before this fixed tool life even with the upper limit of material removal rate.
- The total material removal amount of the project is proportionally distributed to the number of tools consumed for the project in order to assure the consistent quality of all products.
- There is no chattering and scrapping of parts occurs during the whole manufacturing process.
- The time required for a tool change is relatively short to the tool life and it is neglected.
- The chip from cutting and the finished parts are held in the machine until a tool change and then shipped to other department from manufacturing immediately at the tool change.
- The marginal cost of operation is considered to be a linear function of the material removal rate (Lan *et al*., 2008).

Notations

a : Average volume of material machined per unit part.
B : Upper limit of material removal rate.
$bx'(t)$: Marginal operation cost per fully consumed tool at the material removal rate $x'(t)$; where b is a constant.
$by'(t)$: Marginal operation cost per partially consumed tool at the material removal rate $y'(t)$; where b is a constant.
c : Overall holding cost per unit chip machined per unit time in the machine, where $c = h_1 + h_2/a$
c_1 : Labor cost per unit machine per unit time; including production and queuing
c_s : Tool cost per unit tool; including cost of a tool and set-up cost
h_1 : Chip holding cost per unit chip per unit time
h_2 : Finished part holding cost per unit finished part per unit time
Q : Production quantity of the machining project
T : Production period of the project with quantity Q
t_x : Queuing time before machining for a fully consumed tool
t_y : Queuing time before machining for a partially consumed tool
$\bar{t}$: Fixed tool life for each tool
$[A]^+$: No. of tools required for the machining project, where $A = aQ/L\bar{t}$.

Decision functions

$x(t)$: Cumulative volume of material machined for a fully consumed tool during $[t_x, t]$.
$X'(t)$: Material removal rate at time t for a fully consumed tool.
$Y(t)$: Cumulative volume of material machined for a partially consumed tool during $[t_y, t]$.
$Y'(t)$: Material removal rate at time t for a partially consumed tool.

MODEL FORMULATION

Comparing to the productivity of traditional machining model (fixed MRR), it is necessary for MPC Model to competitively satisfy $L\bar{t} \le x(\bar{t}) \le B\bar{t}$. In addition, for the machining quality of all products, the lower limit of material removal rate is applied to determine the number of tools required for the production project. Therefore, with $A = aQ/L\bar{t}$ and $T = \bar{t}A$, the tools required for production and the production period for project scheduling is then achieved and proposed as given for the model.

In this study,

$$\int_{t_x}^{\bar{t}} \left[bx'^2(t) + h_1 x(t) + \frac{h_2}{a} x(t) \right] dt$$

and

$$\int_{t_y}^{(1-[A]^+ + A)\bar{t}} \left[by'^2(t) + h_1 y(t) + \frac{h_2}{a} y(t) \right] dt$$

denote the machining cost during $[t_x, \bar{t}]$ for a fully consumed tool and during $[t_y, (1-[A]^+ + A)\bar{t}]$ for a partially consumed tool, respectively. Thus, the objective function for the machining project is constructed as below.

$$\underset{x,y}{\text{Min}}\left\{\begin{array}{l}(A-Z)\int_{t_x}^{\bar{t}}\left[bx'^2(t)+h_1x(t)+\frac{h_2}{a}x(t)\right]dt+\\ \int_{t_y}^{Z\bar{t}}\left[by'^2(t)+h_1y(t)+\frac{h_2}{a}y(t)\right]dt\\ +c_1T+(A-Z+1)c_s\end{array}\right\}$$

Set $c = h_1 + h_2/a$ as the overall holding cost per unit chip per unit time. Therefore, the MPC Model and its constraints are formulated and described as below.

$$\text{MPC}\begin{cases}\underset{x,y}{\text{Min}}\left\{\begin{array}{l}(A-Z)\int_{t_x}^{\bar{t}}\left[bx'^2(t)+cx(t)\right]dt+\\ \int_{t_y}^{Z\bar{t}}\left[by'^2(t)+cy(t)\right]dt+c_1T+(A-Z+1)c_s\end{array}\right\}\\ \text{s.t. } x(\bar{t})=\frac{aQ}{A},\ y(Z\bar{t})=Z\frac{aQ}{A},\ 0\le x'(t), y'(t)\le B\\ \quad x(t_x)=y(t_y)=0,\ 0\le t_x\le\bar{t}\text{ and } 0\le t_y\le Z\bar{t}\\ \text{where } Z=1-[A]^{+}+A,\ 0<Z\le 1\end{cases}$$

OPTIMAL SOLUTION

Let (x^*,y^*) be the optimal solution of MPC Model and (t_{x^*}, t_{y^*}) be the optimal queuing time for a fully and a partially consumed tools, respectively. Assume that the time interval $[t_x,\tilde{t}]$ and $[t_y,\tilde{\tilde{t}}]$ are the maximal subintervals of $[0,\bar{t}]$ and $[0Z\bar{t}]$, respectively to satisfy Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992).

There are two possible situations to be discussed in this study.

Situation 1: $x^{*\prime}(t)$ $(y^{*\prime}(t))$ will not touch B before $\bar{t}$ $(Z\bar{t})$
The optimal solution for Situation 1 is shown as follows:

$$x^{*\prime}(t)=\frac{c}{2b}(t-t_{x^*}) \tag{1}$$

$$y^{*\prime}(t)=\frac{c}{2b}(t-t_{y^*}) \tag{2}$$

$$x^*(t)=\frac{c}{4b}(t-t_{x^*})^2 \tag{3}$$

$$y^*(t)=\frac{c}{4b}(t-t_{y^*})^2 \tag{4}$$

$$t_{x^*}=\bar{t}-\sqrt{\frac{4abQ}{Ac}} \tag{5}$$

$$t_{y^*}=Z\,\bar{t}-\sqrt{\frac{4abQ}{Ac}Z} \tag{6}$$

The detail is described in Appendix A.

Here, one PROPERTY is proposed and shown as follow:

PROPERTY: If the line $Y = x^{*\prime}(t)$ $(Y = y^{*\prime}(t))$ touches the line $Y = B$, two lines should overlap to be $Y = B$ from the touch point $\tilde{t}$ $(\tilde{\tilde{t}})$ to the end point $\bar{t}$ $(Z\bar{t})$.

The proof of PROPERTY is discussed in Appendix B.

Situation 2: $x^{*\prime}(t)$ $(y^{*\prime}(t))$ touches B at time $\tilde{t}$ $(\tilde{\tilde{t}})$ before $\bar{t}$ $(Z\bar{t})$; where $\tilde{t}<\bar{t}$ $(\tilde{\tilde{t}}<Z\bar{t})$.

The optimal solution for Situation 2 is shown as follows:

$$\tilde{t}=\frac{bB}{c}+\bar{t}-\frac{aQ}{AB} \tag{7}$$

$$\tilde{\tilde{t}}=\frac{bB}{c}+Z\bar{t}-Z\frac{aQ}{AB} \tag{8}$$

$$t_{x^*}=\bar{t}-\frac{aQ}{AB}-\frac{bB}{c} \tag{9}$$

$$t_{y^*}=Z\bar{t}-Z\frac{aQ}{AB}-\frac{bB}{c} \tag{10}$$

$$x^*(t)=\begin{cases}\frac{c}{4b}(t-t_{x^*})^2, & \text{for } t\in[t_{x^*},\tilde{t})\\ B(t-\tilde{t})+\frac{bB^2}{c}, & \text{for } t\in[\tilde{t},\bar{t}]\end{cases} \tag{11}$$

$$y^*(t)=\begin{cases}\frac{c}{4b}(t-t_{y^*})^2, & \text{for } t\in[t_{y^*},\tilde{\tilde{t}})\\ B(t-\tilde{\tilde{t}})+\frac{bB^2}{c}, & \text{for } t\in\left[\tilde{\tilde{t}},Z\bar{t}\right]\end{cases} \tag{12}$$

The detail is described in Appendix C.

Decision criteria: From Eq. 1 and 3, the maximum values of $x^{*\prime}(t)$ and $x^*(t)$ are found at $t = \bar{t}$ when $t_{x^*} = 0$, the following criteria are made.

If $aQ/A \le B\bar{t}/2$, $x^{*\prime}(t)$, will not reach the upper limit B before $\bar{t}$.

If $aQ/A > B\bar{t}/2$, $x^{*\prime}(t)$, will reach the upper limit B before $\bar{t}$.

In addition, from Eq. 2 and 4, the maximum value of $y^{*\prime}(t)$ and $y^*(t)$ are found at $t = Z\bar{t}$ when $t_{y^*} = 0$ and the criteria are then obtained as below.

If $aQ/A \le B\bar{t}/2$, $y^{*\prime}(t)$, will not reach the upper limit B before $Z\bar{t}$.

If $aQ/A > B\bar{t}/2$, $y^{*\prime}(t)$, will reach the upper limit B before $Z\bar{t}$.

It is noticed that the criteria for $x^{*\prime}(t)$ and $y^{*\prime}(t)$ are exactly identical. This denotes that $x^{*\prime}(t)$ and $y^{*\prime}(t)$ will both either never reach B before $\bar{t}$ ($Z\bar{t}$), or touch B before $\bar{t}$ ($Z\bar{t}$). Therefore, when $aQ/A \le B\bar{t}/2$, both $x^{*\prime}(t)$ and $y^{*\prime}(t)$ will not reach the upper limit B; the optimal solution is Situation 1. When $aQ/A > B\bar{t}/2$, both $x^{*\prime}(t)$ and $y^{*\prime}(t)$ will reach the upper limit B; the optimal solution is Situation 2.

COMPUTERIZED EXAMINATION

For a specific turning operation, there are ranges for cutting conditions suggested in the machining handbook. Therefore, there must exist a maximum material removal rate U and a minimum material removal rate L derived from the maximum and the minimum cutting conditions respectively. Each material removal rate between U and L can feasibly be selected as the upper limit B. From the well-known Taylor's expression of the tool life (DeGarmo *et al*., 1997), it is then modified to be $B\times\bar{t}^n = k$ when the cutting speed and depth of cut are selected fixed. Therefore, the fixed tool life $\bar{t}$ for each feasible upper limit is then obtained. However, for every selected feasible upper limit, there exist a lowest cost for the MPC Model and the traditional machining model with a fixed MRR between U and L is also possible to be optimal. To find the minimum cost solution among all the possibilities, a computer program written in MATLAB to analyze the problem is then developed. The concept of the flow chart is described as follows:

Q, a, b, c, U, L, n, k, c_1 and c_s should be given before the following algorithm

Initialize B = L

Step 1: Compute $\bar{t}$, A, Z, T for MPC,
then compute $A_m = aQ/B\bar{t}$, Z_m, T_m for traditional.

Step 2: Compute

$$O_b = A_m bB^2\bar{t} + \frac{A_m cB}{2}\bar{t}^2 + c_1 T_m + (A_m - Z_m + 1)c_s$$

Step 3: Plot $C(B, \frac{O_b}{Q})$ and $P(B, T_m)$, then go to Step 4.

Step 4: If $aQ/A > B\bar{t}/2$, go to Step 6; otherwise go to Step 5.

Step 5: Compute t_x, t_y; then compute

$$\bar{O}_b = (A - Z)\int_{t_x}^{\bar{t}}\left[bx'^2(t) + cx(t)\right]dt + \int_{t_y}^{Z\bar{t}}\left[by'^2(t) + cy(t)\right]dt + c_1T + (A - Z + 1)c_s$$

Go to Step 7.

Step 6: Compute $\tilde{t}, \tilde{\tilde{t}}, t_x, t_y$; then compute

$$\bar{O}_b = (A - Z)\left\{\begin{array}{l}\int_{t_x}^{\tilde{t}}\left[bx'^2(t) + cx(t)\right]dt + \\ \int_{\tilde{t}}^{\bar{t}}\left[bB^2 + c(x(\tilde{t}) + B(t - \tilde{t}))\right]dt\end{array}\right\} + \int_{t_y}^{\tilde{\tilde{t}}}\left[by'^2(t) + cy(t)\right]dt + \int_{\tilde{\tilde{t}}}^{Z\bar{t}}\left[bB^2 + c(y(\tilde{\tilde{t}}) + B(t - \tilde{\tilde{t}}))\right]dt + c_1T + (A - Z + 1)c_s$$

Go to Step 7.

Step 7: Plot $\bar{C}(B, \frac{\bar{O}_b}{Q})$ and $\bar{P}(B, T)$, then go to Step 8.

Step 8: If $B \ge U$, stop the program;
otherwise, set B = B+0.025, as initialized, return to Step 1.

NUMERICAL ANALYSIS

The example referred to a rough turning operation of specific shafts from a machining company in Taipei is studied. The machining process is assigned to a CNC lathe with FANUC controller. All data provided are converted and listed as follows:

Q = 1000 parts, a = 3.375 in^3, b = 0.150 dollars-min/in^3, c = 0.100 dollars/in^3, U = 10.0 in^3/min, L = 6.0 in^3/min, n = 0.2, k = 12.0, c_1 = 0.350 dollars/min and c_s = 12.0 dollars.

From Fig. 1, it is observable that the MPC Model is superior and less costly than the traditional machining model for the whole allowable MRR range. In addition, as the selected upper limit B increases, the production cost will decrease with the MPC Model, while it increases for the traditional model. Therefore, the MPC Model is the optimal solution for production and the maximum MRR generated from machining handbook is the optimal upper limit B for the minimum cost. Moreover, with the three different production quantities in Fig. 1, it is noted that the production cost per unit product of the MPC Model will slightly decrease for each feasible machining speed selected, while the cost per product for the traditional model stays. Thus, when the production quantity increases, the MPC Model is much more competitive in minimizing the production cost.

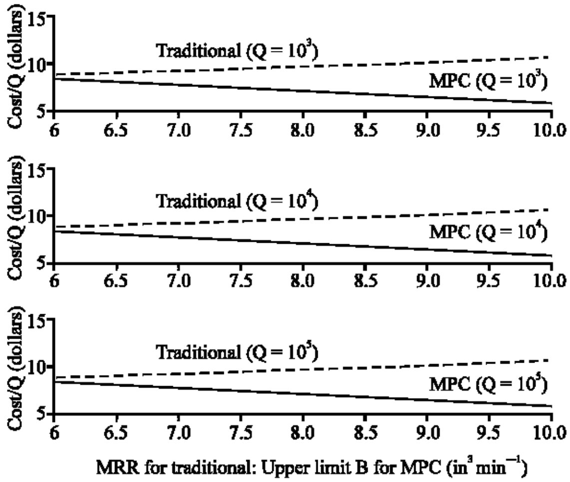

Fig. 1: Analysis for production cost per unit part

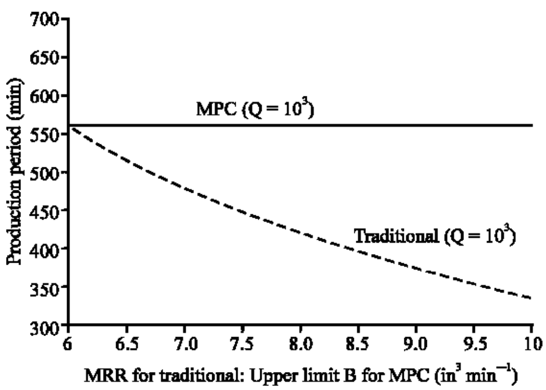

Fig. 2: Analysis for production period

From Fig. 2, it is also observed that the production period for the MPC Model is longer than the traditional model for the whole range. Because that the fixed tool life is derived from the maximum cutting conditions suggested in the handbook and the number of tools required is generated from the minimum cutting conditions suggested in the handbook; the production period for the MPC Model is then less competitive with this aspect. However, when there is a need for the production period to be shorter than proposed in the model, the production period for the MPC Model can always be possibly considered within $aQ/L \le T \le aQ/B$. Thus, the required tools for production will become reduced and the tool cost for production will also be minimized. Besides, this will neither change the optimal solution nor the competition of the MPC Model, but fortunately increase the flexibility in production period for the MPC Model.

CONCLUSIONS

The tool life, tool cost, operational cost, holding cost, production period, production quantity, average material removal per unit part machined and upper limit are considered simultaneously to determine the optimal control of material removal rate and queuing time for the machining project. This is an extremely hard-solving and complicated issue. However, the problem becomes concrete and solvable through the MPC Model.

In addition, the three characteristics from the optimal solution of MPC Model are illustrated as follows: First, from the optimal solution of material removal rates, $x^{*\prime}(t)$ and $y^{*\prime}(t)$ are strictly increasing linear functions of t before reaching upper limit B. Second, by PROPERTY described before, if the material removal rate, $x^{*\prime}(t)$ or $y^{*\prime}(t)$, touches the upper speed limit B; the optimal material removal rate will stay to be upper limit B. Third, with the maximum values of $x^{*\prime}(t)$, $y^{*\prime}(t)$, $x^*(t)$ and $y^*(t)$; it is found that $x^{*\prime}(t)$ and $y^{*\prime}(t)$ will both either never reach B before $\bar{t}$ ($Z\bar{t}$), or touch B before $\bar{t}$ ($Z\bar{t}$).

Moreover, from the computerized analyses and numerical simulation with the MATLAB program, the three remarks are then provided. First, the MPC Model is the optimal solution for a machining project and the maximum allowable MRR from the handbook is the optimal upper limit B for the MPC Model. Second, when the production quantity increases, the MPC Model is much more competitive in minimizing the production cost. Third, when it is acquired for the production period to be shorter than proposed, the production period for the MPC Model is always possible to be scheduled within $aQ/L \le T \le aQ/B$. With these remarks above, the application flexibility for the MPC Model is significantly extended. Thus, the production planning, production cost estimating and even the contract negotiation can be further approached with this study.

The material removal rate is an important control factor of a machining project and the control of machining rate is also critical for production planners. This study not only delivers the idea of controlling the material removal rate to the machining technology, but also leads a machining project towards to achieve minimal cost. Future researches with the modeling of dynamic optimization on multi-tool machining process and multi-project production control are absolutely encouraged. In sum, this study surely generates a reliable and applicable concept of machining control to the techniques and also provides a better and practical solution to this field.

Appendix A: The optimal solution for Situation 1

Suppose that the material removal rate $x^{*\prime}(t)$ ($y^{*\prime}(t)$)

will never reach the upper limit B before tool life $\bar{t}$. From Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992), it is derived that

$$c = \frac{d}{dt} 2bx^{*\prime}(t)$$
$$c = \frac{d}{dt} 2by^{*\prime}(t)$$

There exists k_1 and $\bar{k}_1$ to satisfy

$$x^{*\prime}(t) = \frac{c}{2b} t + k_1 \quad \forall t \in [t_{x^*}, \bar{t}] \tag{A1}$$

$$y^{*\prime}(t) = \frac{c}{2b} t + \bar{k}_1 \quad \forall t \in [t_{y^*}, Z\bar{t}] \tag{A2}$$

Integrating Eq. A1 and A2 with t, it is obtained that

$$x^*(t) = \frac{c}{4b} t^2 + k_1 t + k_2 \quad \forall t \in [t_{x^*}, \bar{t}] \tag{A3}$$

$$y^*(t) = \frac{c}{4b} t^2 + \bar{k}_1 t + \bar{k}_2 \quad \forall t \in [t_{y^*}, Z\bar{t}] \tag{A4}$$

With the transversality conditions for free t_x and t_y (Kamien and Schwartz, 1991; Chiang, 1992), then

$$cx^*(t_{x^*}) = bx^{*\prime 2}(t_{x^*}) \tag{A5}$$

$$cy^*(t_{y^*}) = by^{*\prime 2}(t_{y^*}) \tag{A6}$$

Using Eq. A5, A6 and the boundary conditions, $x(t_x) = 0$ and $y(t_y) = 0$, it is derived that

$$x^{*\prime}(t_{x^*}) = 0 \quad \text{and} \quad y^{*\prime}(t_{y^*}) = 0$$

From Eq. A1 and A2, it is then found

$$k_1 = -\frac{c}{2b} t_{x^*} \tag{A7}$$

$$\bar{k}_1 = -\frac{c}{2b} t_{y^*} \tag{A8}$$

Using Eq. A3, A4, A7 and A8, $x(t_{x^*})$ and $y(t_{y^*}) = 0$; we have

$$k_2 = \frac{c}{4b} t_{x^*}^2 \tag{A9}$$

$$\bar{k}_2 = \frac{c}{4b} t_{y^*}^2 \tag{A10}$$

Applying Eq. A7, A8, A9 and A10 into Eq. A1, A2, A3 and A4; $x^{*\prime}(t)$, $y^{*\prime}(t)$, $x^*(t)$ and $y^*(t)$ are then obtained.

With the boundary conditions, $x(\bar{t}) = aQ/A$ and $y(Z\bar{t}) = Z\, aQ/A$, hence t_{x^*} and t_{y^*} are derived.

Appendix B: The proof of PROPERTY.

Proof: From Eq. 1, $x^{*\prime}(t)$ is a strictly increasing linear function of t. And it holds for any subinterval during $[0,\bar{t}]$ satisfying $0 \le x^{*\prime}(t) \le B$. Therefore, $x^{*\prime}(t)$ in the time interval $[\tilde{t}, \bar{t}]$ cannot exist because it contradicts to be a decreasing linear function of t, the PROPERTY of $x^{*\prime}(t)$ is verified. In addition, the PROPERTY of $y^{*\prime}(t)$ can also be derived with the same proof.

Appendix C: The optimal solution for Situation 2.

Before touching the upper limit, Eq. 1, 2, 3 and 4 are satisfied for this situation either.

Using the transversality condition for free end point $\tilde{t}$ and $\tilde{\tilde{t}}$ (Kamien and Schwartz, 1991; Chiang, 1992), it is derived that

$$bx^{*\prime 2}(\tilde{t}) + cx^*(\tilde{t}) - x^{*\prime}(\tilde{t}) 2bx^{*\prime}(\tilde{t}) = 0 \tag{C1}$$

$$by^{*\prime 2}(\tilde{\tilde{t}}) + cy^*(\tilde{\tilde{t}}) - y^{*\prime}(\tilde{\tilde{t}}) 2by^{*\prime}(\tilde{\tilde{t}}) = 0 \tag{C2}$$

With Eq. 3, 4, C1, C2 and PROPERTY; we have

$$x^*(\tilde{t}) = \frac{c}{4b}(\tilde{t} - t_{x^*})^2 = \frac{bB^2}{c} \tag{C3}$$

$$y^*(\tilde{\tilde{t}}) = \frac{c}{4b}(\tilde{\tilde{t}} - t_{y^*})^2 = \frac{bB^2}{c} \tag{C4}$$

In addition, from boundary conditions,

$$x(\bar{t}) = \frac{aQ}{A}, \quad y(Z\bar{t}) = Z\frac{aQ}{A}$$

and PROPERTY, it is found that

$$\frac{bB^2}{c} = \frac{aQ}{A} - (\bar{t} - \tilde{t})B \tag{C5}$$

$$\frac{bB^2}{c} = Z\frac{aQ}{A} - \left(Z\bar{t} - \tilde{\tilde{t}}\right)B \tag{C6}$$

By Eq. C3, C4, C5 and C6, t_{x^*}, t_{y^*}, $\tilde{t}$ and $\tilde{\tilde{t}}$ can be determined.

From Eq. 3, 4, PROPERTY, $x(\bar{t}) = aQ/A$ and $y(Z\bar{t}) = Z\, aQ/A$; $x^*(t)$ and $y^*(t)$ are then obtained.

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