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Explicit Solution of Non-Linear Fourth-Order Parabolic Equations via Homotopy Perturbation Method

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Abstract: In this study, a powerful analytical method, termed homotopy perturbation method is utilized for finding explicit solutions of non-linear fourth-order parabolic equations. In order to manifest the capability of proposed approach, five illustrating examples have been presented and solved. The obtained solutions, in comparison to those of exact solutions admit a remarkable accuracy. A clear conclusion can be drawn from the numerical results that the HPM provides highly accurate solutions for these kinds of nonlinear differential equations.

Key words: Parabolic equations, homotopy perturbation method, non-linear partial differential equations

INTRODUCTION

Most scientific problems are inherently nonlinear as their functional equations are nonlinear such as parabolic equations. The nonlinear parabolic equations arise in various fields of mechanics, physics, statistics and material science, for instance: transverse vibrations of uniform flexible beams (Arshad *et al*., 2005), optimization of the trade-off between noise removal and edge preservation that may minimize a cost functional (You and Kaveh, 2000), the epitaxial growth of nanoscale thin films (Belinda *et al*., 2003) and waves of the steady natural convection in a vertical fluid layer (Tang and Christov, 2007). We know that most of these types of equations do not have analytical solution as these are functioning nonlinear. Therefore, those should be solved using numerical techniques, although; perturbation methods had been used for the analytical solutions of some especial cases before. In the numerical methods, stability and convergence should be considered so as to avoid divergence or inappropriate results. In the perturbation methods, we should exert the small parameter in the equation. Therefore, finding the small parameter and exerting it into the equation are difficulties of this method. Since there are some limitations with the common perturbation method, also because the basis of the common perturbation method is upon the existence of a small parameter, developing this method for different applications is very difficult.

Indeed, many different methods have been recently introduced some ways to eliminate the small parameter, such as the variational iteration method (He, 1998, 1999, 2000; Khatami *et al*., 2008), the homotopy perturbation method (He, 2003, 2005, 2006; Tolou *et al*., 2008) and the Exp-function method (He and Wu, 2006; Mahmoudi *et al*., 2008). In this letter, we will apply He's homotopy perturbation method in order to drive the explicit solution of fourth-order parabolic equations analytically. In order to evaluate the benefits of proposed method, five illustrating examples have been used. Wazwaz (2001, 2002) obtained exact solution of these equations and (Biazar and Ghazvini, 2007) previously used variational iteration method to solve these equations. This research is motivated to extend these works by implementing homotopy perturbation method that shall be more efficient. The results of homotopy perturbation method have been compared and verified with those of exact solutions from the study of Wazwaz (2001, 2002).

The fourth-order parabolic partial differential equation with variable coefficients reads as following (Wazwaz, 2001, 2002):

$$\frac{\partial^2 u}{\partial t^2}+\mu(x,y,z)\frac{\partial^4 u}{\partial x^4}+\lambda(x,y,z)\frac{\partial^4 u}{\partial y^4}+\eta(x,y,z)\frac{\partial^4 u}{\partial z^4}=g(x,y,z) \qquad (1)$$

where, μ (x, y, z), λ (x, y, z) and η (x, y, z) are positive. Subject to the initial conditions (Wazwaz, 2002):

$$u(x,y,z,0)=f_0(x,y,z) \qquad \frac{\partial u}{\partial t}(x,y,z,0)=f_0(x,y,z)$$

And the boundary conditions (Wazwaz, 2002):

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$$u(a,y,z,t)=g_0(y,z,t) \qquad u(b,y,z,t)=g_1(y,z,t)$$
$$u(x,a,z,t)=k_0(x,z,t) \qquad u(x,b,z,t)=k_1(x,z,t)$$
$$u(x,y,a,t)=h_0(x,y,t) \qquad u(x,y,b,t)=h_1(x,y,t)$$
$$\frac{\partial^2 u}{\partial x^2}(a,y,z,t)=\bar{g}_0(y,z,t) \qquad \frac{\partial^2 u}{\partial x^2}(b,y,z,t)=\bar{g}_1(y,z,t)$$
$$\frac{\partial^2 u}{\partial y^2}(x,a,z,t)=\bar{k}_0(x,z,t) \qquad \frac{\partial^2 u}{\partial y^2}(x,b,z,t)=\bar{k}_1(x,z,t)$$
$$\frac{\partial^2 u}{\partial z^2}(x,y,a,t)=\bar{h}_0(x,y,t) \qquad \frac{\partial^2 u}{\partial z^2}(x,y,b,t)=\bar{h}_1(x,y,t)$$

where, the functions f_i, g_i, k_i, h_i, $\bar{g}_i, \bar{k}_i, \bar{h}_i$, i = 0.1 are continuous.

THE BASIC IDEA OF HPM

To illustrate the basic idea of this method, we consider the following nonlinear differential equation:

$$A(u)-f(r)=0, \quad r\in\Omega \tag{2}$$

Considering the boundary conditions of:

$$B(u,\partial u/\partial n)=0, \quad r\in\Gamma \tag{3}$$

where, A is a general differential operator, B a boundary operator, f (r) a known analytical function and Γ is the boundary of the domain Ω.

The operator A can be divided into two parts of L and N, where L is the linear part, while N is a nonlinear one. Eq. 2 can, therefore, be rewritten as:

$$L(u)+N(u)-f(r)=0 \tag{4}$$

By the homotopy technique, we construct a homotopy as $v(r,p):\Omega\times[0,1]\to\Re$ which satisfies:

$$H(v,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)]=0, \quad P\in[0,1], \ r\in\Omega \tag{5}$$

where, $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of Eq. 4 which satisfies the boundary conditions. Obviously, considering Eq. 4 we will have:

$$\begin{cases} H(v,0)=L(v)-l(u_0)=0 \\ H(v,1)=A(v)-f(r)=0 \end{cases} \tag{6}$$

The changing process of p from zero to unity is just that of v (r, p) from u_0 (r) to u (r). In topology, this is called deformation and $L(v)-L(u_0)$ and A (v)−f (r) are called homotopy. According to HPM, we can first use the embedding parameter p as small parameter and assume that the solution of Eq. 5 can be written as a power series in p:

$$v=v_0+pv_1+p^2v_2+...+p^nv_n \tag{7}$$

Setting p = 1 results in the approximate solution of Eq. 5:

$$u=\lim_{p\to 1} v=v_0+v_1+v_2+.... \tag{8}$$

The combination of the perturbation method and the homotopy method is called the HPM, which lacks the limitations of the traditional perturbation methods, although this technique has full advantages of the traditional perturbation techniques. The series (8) is convergent for most cases. However, the convergence rate depends on the nonlinear operator A(v).

IMPLEMENTATION OF HPM

In order to illustrate the solution procedure and to show the capability of the method, five examples of different kind of fourth-order non-linear parabolic partial differential equations is presented here.

Example one: Consider the following one dimensional, variable coefficient fourth-order parabolic partial differential equations (Wazwaz, 2002).

$$\frac{\partial^2 u}{\partial t^2}+\left(\frac{1}{x}+\frac{x^4}{120}\right)\frac{\partial^4 u}{\partial x^4}=0 \qquad \frac{1}{2}<x<1, t>0 \tag{9}$$

Subject to the initial conditions:

$$u(x,0)=0 \qquad \frac{\partial u}{\partial t}(x,0)=1+\frac{1}{120}x^5 \tag{10}$$

And the boundary conditions:

$$u\left(\frac{1}{2},t\right)=\left(1+\frac{0.5^5}{120}\right)\sin t \qquad u(1,t)=\left(\frac{121}{120}\right)\sin t$$
$$\frac{\partial^2 u}{\partial x^2}\left(\frac{1}{2},t\right)=\frac{1}{6}\left(\frac{1}{2}\right)^3\sin t \qquad \frac{\partial^2 u}{\partial x^2}(1,t)=\frac{1}{6}\sin t$$

Substituting Eq. 7 and 9 into Eq. 5, after some simplification and substitution and rearranging based on powers of p-terms we have:

$$p^0\begin{cases} \dfrac{\partial^2 u_0(x,t)}{\partial t^2}=0 \\ \dfrac{\partial u_0(x,0)}{\partial t}=1+\dfrac{1}{120}x^5 \qquad u_0(x,0)=0 \end{cases} \tag{11a}$$

$$p^1\begin{cases} \dfrac{\partial^2 u_1(x,t)}{\partial t^2}+\dfrac{\dfrac{\partial^4 u_0(x,t)}{\partial t^4}}{x}+\dfrac{1}{120}x^4\left(\dfrac{\partial^4 u_0(x,t)}{\partial t^4}\right)=0 \\ \dfrac{\partial u_1(x,0)}{\partial t}=0 \qquad u_1(x,0)=0 \end{cases} \tag{11b}$$

$$p^2\begin{cases}\dfrac{\partial^4 u_1(x,t)}{\partial x^4}+\dfrac{\partial^2 u_2(x,t)}{\partial t^2}+\dfrac{1}{120}x^4(\dfrac{\partial^4 u_1(x,t)}{\partial x^4})=0\\ \dfrac{\partial u_2(x,0)}{\partial t}=0 \qquad u_2(x,0)=0\end{cases} \tag{11c}$$

Accuracy of solution shall give rise as n in Eq. 7 and power of p increasing. Solving Eq. 11 subject to boundary conditions will result in:

$$u_0(x,t)=(1+\frac{1}{120}x^5)t \tag{12a}$$

$$u_1(x,t)=(1+\frac{1}{120}x^5)(\frac{-t^3}{3!}) \tag{12b}$$

$$u_2(x,t)=(1+\frac{1}{120}x^5)(\frac{t^5}{5!}) \tag{12c}$$

So on substituting Eq. 12 into Eq. 7 gives the approximate solution in the following form:

$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)=(1+\frac{1}{120}x^5)(t-\frac{t^3}{3!}+\frac{t^5}{5!}) \tag{13}$$

Driving Eq. 7 for n>2 gives

$$u(x,t)=\lim_{n\to\infty}u_n(x,t)=(1+\frac{1}{120}x^5)\sin t \tag{14}$$

While this is the same as the exact solution presented (Wazwaz, 2001, 2002).

Example two: Consider the following parabolic equation (Wazwaz, 2002).

$$\frac{\partial^2 u}{\partial t^2}+(\frac{x}{\sin x}-1)\frac{\partial^4 u}{\partial x^4}=0 \qquad 0<x<1 \quad t>0 \tag{15}$$

Subject to the following initial conditions

$$u(x,0)=x-\sin x \qquad \frac{\partial u(x,0)}{\partial t}=-(x-\sin x)$$

And the boundary conditions of

$$u(0,t)=0 \quad u(1,t)=e^{-t}(1-\sin 1) \quad \frac{\partial^2 u}{\partial x^2}(0,t)=0 \quad \frac{\partial^2 u}{\partial x^2}(1,t)=e^{-t}\sin 1$$

As stated in example one, Increasing the n in Eq. 7, give rise to the accuracy of the solution.

Substituting v(r) from Eq. 7 and 9 into Eq. 5, after some simplification and substitution and rearranging based on powers of p-terms up to second orders of p, we have:

$$p^0\begin{cases}\dfrac{\partial^2 u_0(x,t)}{\partial t}=0\\ \dfrac{\partial u_0(x,0)}{\partial t}=-x+\sin x \qquad u_0(x,0)=x-\sin x\end{cases} \tag{16a}$$

$$p^1\begin{cases}\dfrac{\partial^2 u_1(x,t)}{\partial t^2}-\dfrac{\partial^4 u_0(x,t)}{\partial x^4}+\dfrac{x(\dfrac{\partial^4 u_0(x,t)}{\partial x^4})}{\sin x}=0\\ \dfrac{\partial u_1(x,0)}{\partial t}=0 \qquad u_1(x,0)=0\end{cases} \tag{16b}$$

$$p^2\begin{cases}\dfrac{-\partial^4 u_1(x,t)}{\partial x^4}+\dfrac{\partial^2 u_2(x,t)}{\partial t^2}+\dfrac{x(\dfrac{\partial^4 u_1(x,t)}{\partial x^4})}{\sin x}=0\\ \dfrac{\partial u_2(x,0)}{\partial t}=0 \qquad u_2(x,0)=0\end{cases} \tag{16c}$$

In the same manner, the rest of component can be obtained in order to obtain better approximation.
The solution of set of Eq. 16 gives:

$$u_0(x,t)=(x-\sin x)(1-t) \tag{17a}$$

$$u_1(x,t)=(x-\sin x)(\frac{t^2}{2!}-\frac{t^3}{3!}) \tag{17b}$$

$$u_2(x,t)=(x-\sin x)(\frac{t^4}{4!}-\frac{t^5}{5!}) \tag{17c}$$

Substituting the Eq. 17a-c into Eq. 7 gives the approximate solution for n = 2 as:

$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)=(x-\sin x)(1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}) \tag{18}$$

Therefore if we continue,

$$u(x,t)=\lim_{n\to\infty}u_n(x,t)=(x-\sin x)e^{-t} \tag{19}$$

This is as the exact solution that has been obtained previously by Wazwaz (2001, 2002).

Example three: Now we solve the following one dimensional non-homogeneous fourth-order equation (Wazwaz, 2002).

$$\frac{\partial^2 u}{\partial t^2}+(1+x)\frac{\partial^4 u}{\partial x^4}=(x^4+x^3-\frac{6}{7!}x^7)\cos t \qquad 0<x<1 \quad t>0 \tag{20}$$

In the same manner to previous examples by implementing HPM to Eq. 20 we have:

$$p^0\begin{cases}\dfrac{\partial^2 u_0(x,t)}{\partial t^2}=0\\ \dfrac{\partial u_0(x,0)}{\partial t}=0 \qquad u_0(x,0)=\dfrac{6}{7!}x^7\end{cases} \tag{21a}$$

$$p^1\begin{cases}(x+1)(\dfrac{\partial^4 u_0(x,t)}{\partial x^4}-\cos t x^3)+\dfrac{\partial^2 u_1(x,t)}{\partial t^2}+\dfrac{6}{7!}x^7\cos t=0\\ \dfrac{\partial u_1(x,0)}{\partial t}=0 \qquad u_1(x,0)=-\dfrac{6}{7!}x^7(x^3+x^4)\end{cases} \tag{21b}$$

$$p^2\begin{cases}\dfrac{\partial^2 u_2(x,t)}{\partial t^2}+(x+1)(\dfrac{\partial^4 u_1(x,t)}{\partial x^4})\\ \dfrac{\partial u_2(x,0)}{\partial t}=0 \qquad u_2(x,0)=-x^4+24x-x^3+24\end{cases} \tag{21c}$$

$$p^3\begin{cases}(x+1)\dfrac{\partial^4 u_2(x,t)}{\partial x^4}+\dfrac{\partial^2 u_3(x,t)}{\partial t^2}=0\\ \dfrac{\partial u_3(x,0)}{\partial t}=0 \qquad u_3(x,0)=-24-24x\end{cases} \tag{21d}$$

Thus,

$$u_0(x,t)=\frac{6}{7!}x^7 \tag{22a}$$

$$u_1(x,t)=-\frac{6}{7!}x^3(1-\cos t)(-\frac{6}{7!}x+x^4-\frac{6}{7!})-\frac{1}{2}x^3t^2(1-x) \tag{22b}$$

$$u_2(x,t)=\frac{1}{2}(1+x)(-2x^3+2\cos t x^3+t^2x^3-48\cos t+48+2t^4-24t^2) \tag{22c}$$

$$u_3(x,t)=(x+1)(24\cos t-t^4+12t^2-24) \tag{22d}$$

$$u_4(x,t)=u_5(x,t)=...=0 \tag{22e}$$

And so,

$$u(x,t)=u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)=\frac{6}{7!}x^7\cos t \tag{23}$$

Equation 23 is as the exact solution that has been obtained before (Wazwaz, 2002).

Example four: Consider the fourth-order parabolic equation in two space variables (Wazwaz, 2001).

$$\frac{\partial^2 u}{\partial t^2}+2(\frac{1}{x^2}+\frac{x^4}{6!})\frac{\partial^4 u}{\partial x^4}+2(\frac{1}{y^2}+\frac{y^4}{6!})\frac{\partial^4 u}{\partial y^4}=0 \qquad \frac{1}{2}<x,y<1 \quad t>0 \tag{24}$$

And the initial conditions are:

$$\frac{\partial u(x,y,0)}{\partial t}=2+\frac{x^4}{6!}+\frac{y^4}{6!} \qquad u(x,y,0)=0$$

Also, boundary conditions are:

$$u(\frac{1}{2},y,t)=(2+\frac{0.5^4}{6!}+\frac{y^6}{6!})\sin t \qquad u(1,y,t)=(2+\frac{1}{6!}+\frac{y^6}{6!})\sin t$$

$$\frac{\partial^2 u(\frac{1}{2},y,t)}{\partial x^2}=\frac{0.5^4}{24}\sin t \qquad \frac{\partial^2 u(1,y,t)}{\partial x^2}=\frac{1}{24}\sin t$$

$$\frac{\partial^2 u(x,\frac{1}{2},t)}{\partial x^2}=\frac{0.5^4}{24}\sin t \qquad \frac{\partial^2 u(x,1,t)}{\partial x^2}=\frac{1}{24}\sin t$$

Afterwards, implementing HPM to Eq. 24 result in:

$$p^0\begin{cases}\dfrac{\partial^2 u_0(x,y,t)}{\partial t^2}=0\\ \dfrac{\partial u_0(x,y,0)}{\partial t}=2+\dfrac{1}{6!}x^6+\dfrac{1}{6!}y^6 \qquad u_0(x,y,0)=0\end{cases} \tag{25a}$$

$$p^1\begin{cases}\dfrac{\partial^4 u_0(x,y,t)}{\partial x^4}(\dfrac{x^4}{360}+\dfrac{2}{x^2})+\dfrac{\partial^4 u_0(x,y,t)}{\partial y^4}(\dfrac{x^4}{360}+\dfrac{2}{y^2})+\dfrac{\partial^2 u_1(x,y,t)}{\partial t^2}=0\\ \dfrac{\partial u_1(x,y,0)}{\partial t}=0 \qquad u_1(x,y,0)=0\end{cases} \tag{25b}$$

$$p^2\begin{cases}\dfrac{\partial^4 u_1(x,y,t)}{\partial x^4}(\dfrac{x^4}{6!}+\dfrac{2}{x^2})+\dfrac{\partial^4 u_1(x,y,t)}{\partial y^4}(\dfrac{y^4}{6!}+\dfrac{2}{y^2})+\dfrac{\partial^2 u_3(x,y,t)}{\partial t^2}=0\\ \dfrac{\partial u_2(x,y,0)}{\partial t}=0 \qquad u_2(x,y,0)=0\end{cases} \tag{25c}$$

Then, solving Eq. 25 we have:

$$u_0(x,y,t)=(2+\frac{x^4}{6!}+\frac{y^4}{6!})t \tag{26a}$$

$$u_1(x,y,t)=(2+\frac{x^6}{6!}+\frac{y^6}{6!})\frac{-t^3}{3!} \tag{26b}$$

$$u_2(x,y,t)=(2+\frac{x^6}{6!}+\frac{y^6}{6!})\frac{t^5}{5!} \tag{26c}$$

So on substituting Eq. 26 into Eq. 7 gives the approximate solution in the following form:

$$u(x,y,t)=u_0(x,y,t)+u_1(x,y,t)+u_2(x,y,t)=(2+\frac{x^4}{6!}+\frac{y^4}{6!})(t-\frac{t^3}{3!}+\frac{t^5}{5!}) \tag{27}$$

Thus,

$$u(x,y,t)=\lim_{n\to\infty}u_n(x,y,t)=(2+\frac{x^4}{6!}+\frac{y^4}{6!})\sin t \tag{28}$$

Equation 28 is the same as the result obtained previously (Wazwaz, 2001, 2002).

Example five: Finally, we solve the following partial differential equation in three space variables (Wazwaz, 2001).

$$\frac{\partial^2 u}{\partial t^2}+\frac{\partial^4 u}{\partial x^4}(\frac{y+z}{2\cos x}-1)+\frac{\partial^4 u}{\partial y^4}(\frac{z+x}{2\cos y}-1)+\frac{\partial^4 u}{\partial z^4}(\frac{x+y}{2\cos z}-1)=0 \tag{29}$$

Subject to the initial conditions:

$$-\frac{\partial u(x,y,z,0)}{\partial t}=u(x,y,z,0)=x+y+z-(\cos x+\cos y+\cos z)$$

And the boundary conditions:

$$u(0,y,z,t)=e^{-t}(-1+y+z-\cos y-\cos z) \qquad u(\frac{\pi}{3},y,z,t)=e^{-t}(\frac{2\pi-3}{6}+y+z-\cos y-\cos z)$$

$$u(x,0,z,t)=e^{-t}(x-1+z-\cos x-\cos z) \qquad u(x,\frac{\pi}{3},z,t)=e^{-t}(x+\frac{2\pi-3}{6}+z-\cos x-\cos z)$$

$$u(x,y,0,t)=e^{-t}(x+y-1-\cos x-\cos y) \qquad u(x,y,\frac{\pi}{3},t)=e^{-t}(x+y+\frac{2\pi-3}{6}-\cos x-\cos y)$$

$$\frac{\partial u(0,y,z,t)}{\partial x}=\frac{\partial u(x,0,z,t)}{\partial y}=\frac{\partial u(x,y,0,t)}{\partial z}=e^{-t}$$

$$\frac{\partial u(\frac{\pi}{3},y,z,t)}{\partial x}=\frac{\partial u(x,\frac{\pi}{3},z,t)}{\partial y}=\frac{\partial u(x,y,\frac{\pi}{3},t)}{\partial z}=e^{-t}(\frac{\sqrt{3}+2}{2})$$

In the same manner of previous examples, after some manipulation and rearranging based on powers of p–terms we have:

$$p^0\begin{cases}\dfrac{\partial^2 u_0(x,y,z,t)}{\partial t}=0\\ -\dfrac{\partial u_0(x,y,z,0)}{\partial t}=u_0(x,y,z,0)=x+y+z-(\cos x+\cos t+\cos z)\end{cases} \tag{30a}$$

$$p^1\begin{cases}\dfrac{\partial^4 u_0(x,y,z,t)}{\partial x^4}(\dfrac{z+y}{2\cos x}-1)+\dfrac{\partial^4 u_0(x,y,z,t)}{\partial y^4}(\dfrac{x+z}{2\cos y}-1)+\dfrac{\partial^4 u_0(x,y,z,t)}{\partial z^4}(\dfrac{x+y}{2\cos z}-1)+\dfrac{\partial^2 u_1(x,y,z,t)}{\partial t^2}=0\\ \dfrac{\partial u_1(x,y,z,0)}{\partial t}=u_1(x,y,z,0)=0\end{cases} \tag{30b}$$

$$p^2\begin{cases}\dfrac{\partial^4 u_1(x,y,z,t)}{\partial x^4}(\dfrac{z+y}{2\cos x}-1)+\dfrac{\partial^4 u_1(x,y,z,t)}{\partial y^4}(\dfrac{x+z}{2\cos y}-1)+\dfrac{\partial^4 u_1(x,y,z,t)}{\partial z^4}(\dfrac{x+y}{2\cos z}-1)+\dfrac{\partial^2 u_2(x,y,z,t)}{\partial t^2}=0\\ \dfrac{\partial u_2(x,y,z,0)}{\partial t}=u_2(x,y,z,0)=0\end{cases} \tag{30c}$$

Solving Eq. 29 subject to initially condition give:

$$u_0(x,y,z,t)=[x+y+z-(\cos x+\cos y+\cos z)]\ (1-t) \tag{31a}$$

$$u_1(x,y,z,t)=[x+y+z-(\cos x+\cos y+\cos z)]\ (\frac{t^2}{2!}-\frac{t^3}{3!}) \tag{31b}$$

$$u_2(x,y,z,t)=[x+y+z-(\cos x+\cos y+\cos z)]\ (\frac{t^4}{4!}-\frac{t^5}{5!}) \tag{31c}$$

So,

$$u(x,y,z,t)=\sum_{i=0}^{4}u_i(x,y,z,t)=[x+y+z-(\cos x+\cos y+\cos z)]\ (1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}-\frac{t^5}{5!}) \tag{32}$$

It is known that as $\sum_{n=0}^{n=\infty}(-1)^n\frac{t^n}{n!}=e^{-t}$ as $n\to\infty$; thus an exact solution is obtained which reads.

$$u(x,y,z,t) = \lim_{n\to\infty} u_n(x,y,z,t) = [x + y + z - (\cos x + \cos y + \cos z)]\ e^{-t} \quad (33)$$

And this is the exact solution (Wazwaz, 2001, 2002).

CONCLUSION

In this study, for the first time a kind of analytical method called, HPM has been successfully applied to find the solution of the parabolic equations. This method has been used for solving five examples of parabolic equations. The results show that this method provides excellent approximations to the solution of this nonlinear systems with high accuracy. It is worth pointing out that this method presents a rapid convergence for the solutions with out the difficulties that have been arisen in traditional analytical methods. As shown, the homotopy perturbation method doesn't need a small parameter. Finally, it has been attempted to show the capabilities and wide-range applications of the homotopy perturbation method.

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