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A Mathematical Approach for Sprinkler Irrigation Drop Distribution on Soil Surface

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Abstract: The objectives of this study were to develop an equation and derive equation for different boundary conditions of sprinkler irrigation drop falling and to stop runoff from soil surface. Irrigation scheduling is the process related to when, how much water to apply to a soil. The irrigation method concerns how that desired water depth is applied to the field. The uniformity of water distribution depends on an irrigated field and efficiency of on-farm water application. Conclusions point out on the laterals must install parallel to field slope contours for controlling runoff, erosion and on-farm water application.

Key words: Mathematical, model, drop, distribution

INTRODUCTION

Irrigation scheduling requires knowledge on crop water requirements (evapotranspiration) and yield responses to water, the constraints specific to each irrigation method and irrigation equipment, the limitation relative to the water supply system and the financial and economic implications of the irrigation practice (Heermann, 1996). To improve the irrigation method requires the consideration of the factor influencing the hydraulic processes, the water infiltration and the uniformity of water application to entire field (Hlavec, 1992). The consideration of all these aspects makes irrigation management a complex decision making and field practice process (Pereira, 1999).

Sprinkler irrigation is suitable for most crops. It is also adaptable to nearly all irrigable soils, because sprinklers are available for a wide range of discharge capacities. Where soils have low water-holding capacity and shallow-rooted crops are to be irrigated, lighter and more frequent irrigations are required. Fixed and continuously moving systems are both suitable for such applications. The flexibility of present sprinkler equipment and its efficient control of water application make the method almost universally applicable. Its usefulness for most topographic conditions is subject only to limitations imposed by land use capability and economics. It can be adapted to most climatic conditions where irrigated agriculture is practical (Keller and Bliesner, 1990; Gencoglan *et al.*, 2005).

The sprinkler irrigation method can be operated with application rate higher than the steady state infiltration rate. In general, this type of operation is implemented for stationary sprinkler systems. The sprinkler application rate will be lower than the infiltration rate immediately after irrigation commences and all the water applied infiltrates into the soil. As time passes, the infiltration rate decreases and becomes less than the sprinkler application rate.

When irrigation continues after this point, runoff occurs (James, 1988; James and Larson, 1976), although the amount of runoff will step depend on the amount of water that can accumulate in small surface depressions and on the slope.

This research aims at approaching mathematical models which could contribute to the achievement of higher irrigation performances when drop distributions and surface runoff on soil.

ALGEBRAIC MODEL FORMULATION

Here, an algebraic model formulation develop the effective drop distribution. For this reason, precipitation of drop from sprinkler nozzle and wetted area on soil must be detected well for distribution uniformity.

In sprinkler irrigation, in the wetted area drops, wetted distribution determine as a function ($D(\rho)$) which in symmetric to axis (Fig. 1). Wetted distribution in circle shape with a r radius. If distribution boundary as given below;

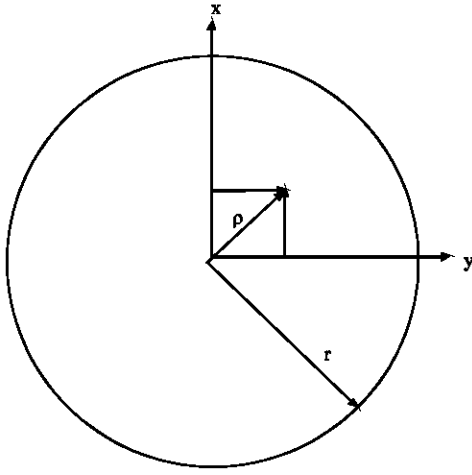


Fig. 1: Under ideal condition irrigation drop distribution

$$D(\rho) = \begin{cases} f(\rho), & \rho \leq r \\ 0, & \rho > r \end{cases} \quad (1)$$

Meaning, the drop from nozzle constitutes parallel and equal distance lines in wetting circle. Some of irrigation water infiltrates to soil but some of water accumulate soil surface. Accumulate water will be same quantity between two lines (band). In this case wetted distribution can be described by Eq. 2:

$$\rho(x) = \sqrt{x^2 + y^2} \quad (2)$$

x and y is coordinates points. S(x) is the volume of water between two lines. Water volume is described by Eq. 3:

$$S(x) = 2 \int_0^y D(\rho) dy = 2 \int_0^y f(\rho) dy \quad (3)$$

as it is known:

$$x^2 + y^2 = r^2 \Rightarrow y = \sqrt{r^2 - x^2} \quad (4)$$

adding Eq. 4 into Eq. 3 resulting S(x) Eq. 5:

$$S(x) = 2 \int_0^{\sqrt{r^2 - x^2}} f(\sqrt{x^2 + y^2}) dy \quad (5)$$

Band length for unit width is. Since irrigation distribution (g(x)) is described by Eq. 6:

$$g(x) = \frac{S(x)}{1} = \frac{\int_0^{\sqrt{r^2 - x^2}} f(\sqrt{x^2 + y^2}) dy}{\sqrt{r^2 - x^2}} \quad (6)$$

Irrigation distribution will work different boundary conditions. There are some scenarios to occur:

Scenario 1

Suppose the wetted drop distribution $f(\rho) = a$, g(x) is determining Eq. 7:

$$g(x) = \frac{1}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} a dy = \frac{a}{\sqrt{r^2 - x^2}} y \Big|_0^{\sqrt{r^2 - x^2}} = \frac{a\sqrt{r^2 - x^2}}{\sqrt{r^2 - x^2}} = a \quad (7)$$

A result shows that drop distribution is uniform.

Scenario 2

Suppose the wetted drop distribution $f(\rho) = b\rho$, g(x) determining Eq. 8:

$$\begin{aligned} g(x) &= \frac{\int_0^{\sqrt{r^2 - x^2}} f(\rho) dy}{\sqrt{r^2 - x^2}} \\ &= \frac{1}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} b\rho dy = \frac{b}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} \sqrt{x^2 + y^2} dy \\ &= \frac{b}{\sqrt{r^2 - x^2}} \left[\frac{y}{2} \sqrt{x^2 + y^2} + \frac{x^2}{2} \ln(y + \sqrt{x^2 + y^2}) \right]_0^{\sqrt{r^2 - x^2}} \\ &= \frac{b}{\sqrt{r^2 - x^2}} \left(\frac{r\sqrt{r^2 - x^2}}{2} + \frac{x^2}{2} \ln \frac{\sqrt{r^2 - x^2} + r}{x} \right) \\ &= \frac{br}{2} + \frac{bx^2}{2\sqrt{r^2 - x^2}} \ln \frac{\sqrt{r^2 - x^2} + r}{x} \end{aligned} \quad (8)$$

Scenario 3

Suppose the wetted drop distribution $f(\rho) = c\rho^2$, g(x) determining Eq. 9

$$\begin{aligned} g(x) &= \frac{1}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} c(\sqrt{x^2 + y^2})^2 dy \\ &= \frac{c}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} (x^2 + y^2) dy \end{aligned}$$

$$\begin{aligned}
 &= \frac{c}{\sqrt{r^2 - x^2}} \left(x^2 y + \frac{y^3}{3} \right) \Big|_0^{\sqrt{r^2 - x^2}} \\
 &= \frac{c}{\sqrt{r^2 - x^2}} \left[x^2 \sqrt{r^2 - x^2} + \frac{(\sqrt{r^2 - x^2})^3}{3} \right] \quad (9) \\
 &= \frac{c}{\sqrt{r^2 - x^2}} \left[x^2 \sqrt{r^2 - x^2} + \frac{(r^2 - x^2) \sqrt{r^2 - x^2}}{3} \right] \\
 &= c \left(x^2 + \frac{r^2 - x^2}{3} \right) = \frac{c(2x^2 + r^2)}{3}
 \end{aligned}$$

Scenario 4

Suppose the wetted drop distribution is parabolic $f(\rho) = a + b\rho + c\rho^2$, $g(x)$ determining Eq. 10:

$$\begin{aligned}
 g(x) &= \frac{1}{\sqrt{r^2 - x^2}} \int_0^{\sqrt{r^2 - x^2}} (a + b\rho + c\rho^2) d\rho \\
 &= a + \frac{br}{2} + \frac{bx^2}{2\sqrt{r^2 - x^2}} \ln \frac{\sqrt{r^2 - x^2} + r}{x} + \frac{c(2x^2 + r^2)}{3} \quad (10)
 \end{aligned}$$

The result of Eq. 10 is means irrigation drops distribute as equal to soil surface.

By considering interaction between irrigation water and soil surface while irrigation nozzle circling given in Fig. 2.

For this case boundary conditions given Eq. 11:

$$D(\rho) = \begin{cases} f(\rho), & \rho \geq r \\ 0, & \rho < r \end{cases} \quad (11)$$

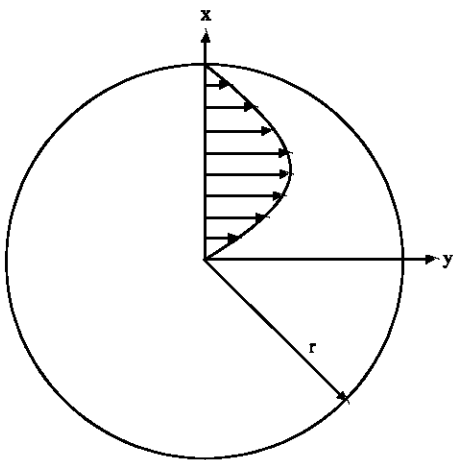


Fig. 2: Irrigation drop distribution with sprinkle nozzle moving on y axis

In this case, water source point moves from y axis. Since $S(x)$ (volume of water) be formed small water volumes on coordinates x and y , thus Eq. 12 can be shown as,

$$g_k(x) = 2 \int_0^y f(\rho) dy = 2 \int_0^{\sqrt{r^2 - x^2}} f(\sqrt{x^2 + y^2}) dy \quad (12)$$

There are some other cases that in Eq. 11 boundary condition.

Scenario 5

Suppose the wetted drop distribution is $f(\rho) = a + b\rho + c\rho^2$, by using Eq. (12), Eq. 13 is presented as;

$$\begin{aligned}
 g_{kk}(x) &= 2 \int_0^{\sqrt{r^2 - x^2}} (a + b\rho + c\rho^2) dy \\
 &= (2a + br) \sqrt{r^2 - x^2} + bx^2 \ln \frac{\sqrt{r^2 - x^2} + r}{x} \quad (13) \\
 &\quad + \frac{(4cx^2 + 2cr^2) \sqrt{r^2 - x^2}}{3}
 \end{aligned}$$

Scenario 6

Suppose starting drop distribution $D(\rho)$ as an n degree polynomial, it occurs such $f(\rho) = \sum_{i=0}^n a_i \rho^i$ as

when by looking at the first four term of the Eq. ($f(\rho) = a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 + \dots$) can be written as Eq. 14, 15.

$$\begin{aligned}
 g_{kp}(x) &= 2 \int_0^{\sqrt{r^2 - x^2}} (a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 + \dots) dy \\
 &= (2a_0 + a_1r) \sqrt{r^2 - x^2} + a_1x^2 \ln \frac{\sqrt{r^2 - x^2} + r}{x} \\
 &\quad + \frac{(4a_2x^2 + 2a_2r^2) \sqrt{r^2 - x^2}}{3} \\
 &\quad + 2a_3 \left[\frac{r(2r^2 + 3x^2) \sqrt{r^2 - x^2} + 3x^4 \ln \frac{\sqrt{r^2 - x^2} + r}{x}}{8} \right] \\
 &\quad + 2a_4 \left[\frac{(8x^4 + 4x^2r^2 + 3r^4) \sqrt{r^2 - x^2}}{15} \right] + \dots \quad (14)
 \end{aligned}$$

or

$$\begin{aligned}
 g_{kp}(x) &= \int_0^{\sqrt{r^2-x^2}} (a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3 + a_4\rho^4 + \dots) dy \\
 &= g_{kk}(x)_{a=a_0, b=a_1, c=a_2} \\
 &+ 2a_3 \left[\frac{r(2r^2 + 3x^2)\sqrt{r^2 - x^2} + 3x^4 \ln \frac{\sqrt{r^2 - x^2} + r}{x}}{8} \right] \\
 &+ 2a_4 \left[\frac{(8x^4 + 4x^2r^2 + 3r^4)\sqrt{r^2 - x^2}}{15} \right] + \dots
 \end{aligned} \tag{15}$$

According to case results, irrigation distribution function depends on x and r values.

RESULTS AND DISCUSSION

The cases show that in different boundary conditions with different scenarios. When the field separates equal apart lines (band) movement of sprinkler irrigation nozzle has to be y axis. Derivations show that starting drop distribution $f(\rho)$ is not dependent and irrigation distribution ($g(x) = \text{constant}$) is becomes a constant.

Irrigating in cropping season, soil surface preparation and laterals must be installed parallel to slope contours.

CONCLUSIONS

Irrigation distribution in different boundary conditions were discussed with an algebraic model formulation, the main scenario was when infiltration becomes constant surface flow starts. As it is known,

infiltration does not stop but it works with surface flow together. There must be some other scenarios to get more accurate results, such as field slope and precipitations (rain).

As results show, another conclusion is, before cropping and irrigation in order to decrease surface flow field must be tilled parallel to contours.

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