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Retailer's Economic Order Quantity Model under Trade Credit Period Depending on the Order Quantity Without Calculus

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Abstract: In this study, the restrictive assumptions of the trade credit period independent of the order quantity and the retailer's unit selling price equaled to the purchasing price per unit are relaxed to fit real business situations. In 1985, Goyal considered the retailer's inventory replenishment problem under trade credit period independent of the order quantity and the retailer's unit selling price and the purchasing price per unit were equal. This article investigates the retailer's inventory problem under trade credit period dependent of the order quantity and the retailer's unit selling price not necessarily equals to the purchasing price per unit within the Economic Order Quantity (EOQ) framework. In addition, we use an easy and simple arithmetic-geometric mean inequality approach to determine the retailer's optimal ordering policy under minimizing the annual total relevant cost. This approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. Finally, numerical examples are given to illustrate the proposed model and its optimal solution.

Key words: Inventory, EOQ, trade credit, arithmetic-geometric mean inequality approach

INTRODUCTION

In a real world, the supplier often makes use of the trade credit policy to promote their commodities. Goyal (1985) is frequently cited when the inventory systems under conditions of trade credit are discussed. Khouja and Mehrez (1996) investigated the effect of four different supplier credit policies on the optimal order quantity within the EOQ framework. Chung (1998) developed an efficient decision procedure to determine the economic order quantity under condition of permissible delay in payments. Teng (2002) assumed that the selling price was not equal to the purchasing price to modify Goyal's model (1985). Chung and Huang (2003) investigated this issue within EPQ (economic production quantity) framework and developed an efficient solving procedure to determine the optimal replenishment cycle for the retailer. Huang and Chung (2003) investigated the inventory policy under cash discount and trade credit. Huang (2004) adopted alternative payment rules and, assumed finite replenishment rate, to investigate the buyer's inventory problem. Huang (2006) extended Huang (2003) to develop retailer's inventory policy under retailer's storage space limited. Recently, Huang (2007) incorporated Chung and Huang (2003) and Huang (2003) to investigate retailer's ordering policy.

Goyal (1985) implicitly makes the following assumptions:

- Supplier credit policy offered to the retailer where credit terms are independent of the order quantity. That is, whatever the order quantity is small or large, the retailer can take the benefits of payment delay.
- The unit selling price and the unit purchasing price are assumed to be equal. However, in practice, the unit selling price is not lower than the unit purchasing price in general.

According to the above arguments, this article will adopt the following assumptions to modify the Goyal's model (1985).

- To encourage retailer to order large quantity, the supplier may give the longer trade credit period for a large order quantity.
- The selling price per unit and the unit purchasing price are not necessarily equal to match the practical situations. This viewpoint can be found in Teng (2002).

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Hence, in this study, we not only incorporate the above assumptions (i) and (ii) to modify the Goyal's model (1985), but also provide an easy-to-understand and simple-to-apply arithmetic-geometric mean inequality approach without using derivatives to obtain the optimal replenishment cycle time. This approach could therefore be used easily to introduce the basic inventory theories to younger students who lack the knowledge of calculus. Finally, numerical examples are given to illustrate the proposed model and its optimal solution.

MODEL FORMULATION

Here, we want to develop the inventory model under trade credit period to take the order quantity into account. The following notation and assumptions will be used to develop our inventory model.

Notation:

Q	=	Order quantity
D	=	Annual demand
α	=	The fraction of trade credit period, $0 \le \alpha < 1$
A	=	Cost of placing one order
c	=	Unit purchasing price
s	=	Unit selling price
h	=	Unit stock holding cost per year excluding interest charges
I_p	=	Interest charges per \$ investment in inventory per year
I_e	=	Interest which can be earned per \$ per year
T	=	The cycle time in years
M	=	The trade credit period in years depending on the order quantity = α T
TVC(T)	=	The annual total relevant cost when T > 0
T*	=	The optimal cycle time of TVC(T)
Q*	=	The optimal order quantity = DT*.

Assumptions:

- Demand rate is known and constant
- Shortages are not allowed
- Time period is infinite
- Replenishments are instantaneous
- The trade credit period is dependent of the order quantity. That is, $M = \alpha T$ $(0 \le \alpha < 1)$
- During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. When the account is settled, the retailer pays off all units sold and keeps his/her profits and starts paying for the higher interest charges on the items in stock
- $s \ge c$

The annual total relevant cost consists of the following elements.

- Annual ordering cost = A/T
- Annual stock holding cost (excluding interest charges) = DTh/2
- Cost of interest charges for the items kept in stock per year $= \frac{cI_pD(T-M)^2}{2T} = \frac{cI_pD(T-\alpha T)^2}{2T} = \frac{cI_pD(1-\alpha)^2T^2}{2T}$
- Interest earned per year $= \frac{DM^2sI_e}{2T} = \frac{D(\alpha T)^2sI_e}{2T} = \frac{D\alpha^2T^2sI_e}{2T}$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

TVC(T) = ordering cost + stock-holding cost + interest payable-interest earned

The research show that the annual total relevant cost, TVC(T), is given by

$$TVC(T) = \frac{A}{T} + \frac{DTh}{2} + \frac{cI_pD(1-\alpha)^2T^2}{2T} - \frac{D\alpha^2T^2sI_e}{2T} \tag{1}$$

THEORETICAL RESULT

We here use an easy and simple arithmetic-geometric-mean-inequality approach (Horn and Johnson, 1985) to obtain the optimal cycle time that minimizes the annual total relevant cost. The arithmetic-geometric mean inequality is as follows. For any two real positive numbers, say a and b, the arithmetic mean a+b/2 is always greater than or equal to the geometric mean $\sqrt{ab}$. Namely,

$$a+b/2 \ge \sqrt{ab}$$

The equation holds only if a = b.

To minimize the annual total relevant cost, we can rewrite TVC(T) in (1) as follows:

$$TVC(T) = \frac{A}{T} + \frac{DT[h + c(1-\alpha)^2I_p - s\alpha^2I_e]}{2} \tag{2}$$

By using the arithmetic-geometric mean inequality, we can easily obtain that

$$TVC(T) = \frac{A}{T} + \frac{DT[h + c(1-\alpha)^2I_p - s\alpha^2I_e]}{2}$$

$$\ge 2\sqrt{\frac{A}{T}.\frac{DT[h + c(1-\alpha)^2I_p - s\alpha^2I_e]}{2}} = \sqrt{2AD[h + c(1-\alpha)^2I_p - s\alpha^2I_e]} \tag{3}$$

Table 1: The optimal cycle time and optimal order quantity with various values of α and s

Let A = \$100/order, D = 5000 units/year, c = \$30/unit, h = \$5/unit/year, Ip = \$0.15/\$/year and Ie = \$0.1/\$/year.

	$\alpha = 0.1$			$\alpha = 0.4$			$\alpha = 0.7$		
s = \$/unit	T*	Q*	TVC(T*)	T*	Q*	TVC(T*)	T*	Q*	TVC(T*)
40	0.06818	340.9	2933.4	0.08179	408.9	2445.4	0.10775	538.8	1856.1
50	0.06822	341.1	2931.7	0.08290	414.5	2412.5	0.11635	581.7	1719.0
60	0.06826	341.3	2930.0	0.08407	420.3	2379.1	0.12739	636.9	1570.0

When the equality

$$\frac{A}{T} = \frac{DT[h + c(1-\alpha)^2 I_p - s\alpha^2 I_e]}{2} \tag{4}$$

holds, TVC(T) has a minimum. Hence the optimal value of T for TVC(T) (say T*) can be determined by (4), namely:

$$T^* = \sqrt{\frac{2A}{D[h + c(1-\alpha)^2 I_p - s\alpha^2 I_e]}} \text{ if } h + c(1-\alpha)^2 I_p - s\alpha^2 I_e > 0 \tag{5}$$

Therefore, the optimal order quantity Q* is

$$Q^* = DT^* \sqrt{\frac{2AD}{h + c(1-\alpha)^2 I_p - s\alpha^2 I_e}} \tag{6}$$

NUMERICAL EXAMPLES

To illustrate the theoretical result obtained in this article, let us apply the proposed method to efficiently solve the following numerical examples. The optimal cycle time and optimal order quantity are shown in Table 1.

CONCLUSIONS

The purpose of this study adopts an easy and simple arithmetic-geometric mean inequality approach to investigate the effect of trade credit period depending on the order quantity and the retailer's unit selling price not necessarily equaled to the purchasing price per unit within the Economic Order Quantity (EOQ) framework. Using this approach presented in this article, we can find the optimal cycle time and optimal order quantity without using differential calculus.

From the final numerical examples, we can obtain following managerial insights. The retailer will order more quantity to take the benefits of the longer trade credit period as possible when the fraction of trade credit period is higher. In addition, the retailer will order larger quantity to take the benefits of the longer trade credit period when the larger the differences between the unit selling price and the purchasing price per item.

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