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## Rainfall-Runoff Modeling Using Fuzzy Methodology

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**Abstract:** The present study investigates an illustrative evaluation of the performance of some new approaches including fuzzy rule-based modeling and fuzzy regressions analysis in rainfall-runoff modeling which is a well-known and challenging problem in hydraulic engineering. On the basis of the results obtained for Halil Rud River, central Iran, it can be suggested that the fuzzy methodology is efficiently capable of handling highly scattered data. The developed fuzzy rule-based model shows flexibility and ability in modeling an ill-defined relationship between input and output variables. The two fuzzy regression models developed seem to be more informative and illuminative.

**Key words:** Fuzzy logic, fuzzy regression, uncertainty, hydrologic modeling, Halil Rud river

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### INTRODUCTION

Water supply scheming as well as design and management of water reservoirs essentially relies on the competence of the process of hydrologic modeling adopted. For hydrologist, flood protection engineer and reservoirs operator who concern with surface water planning for an arid and semiarid region like central and southeastern Iran, where water is regarded as the essence of existence for human beings and their related economic and agricultural activities, forming a mathematical model so as to understand and simulate the relationship between rainfall and runoff events takes on all shades of meaning. The so-called Rainfall-Runoff (RR) process comprises of motion of rainfall through different terrestrial settings and its transformation into the runoff in natural or artificial canals. In the context of hydrology, immense variations of watershed attribute and precipitation patterns over time and location as well as interaction of numerous variables involved, render the RR relation a highly complicated phenomenon to be tackled and therefore make the conduction of a simple, quick and credible RR modeling program a challenge to the hydrologists. The literature contains numerous papers and contributions investigating the applicability and potentials of Artificial Neural Networks (ANN) modeling approaches in the area of RR modeling and time series forecasting

(Lachtermacher and Fuller, 1994; Hsu *et al.*, 1995; Smith and Eli, 1995; van den Boogard *et al.*, 1998; Tokar and Johnson, 1999; Zealand *et al.*, 1999; Luk *et al.*, 2000; Coulibaly *et al.*, 2000; Toth *et al.*, 2000; Deo and Thirumalaiah, 2000; Thirumalaiah and Deo, 2000; Zhang and Govindaraju, 2000; Sudheer *et al.*, 2003; Wilby *et al.*, 2003). A comprehensive review on the application of ANNs to hydrology can be found in ASCE Task Committee on Application of Artificial Neural Networks in Hydrology (Govindaraju, 2000 a, b) and in Maier and Dandy (2000). Having utilized as alternative tools in constructing nonlinear system theoretic models of the hydrological processes, ANNs act as fundamentally semi-parametric regression estimators and can almost approximate any measurable function up to an arbitrary degree of accuracy (Hornik, 1991). Although ANNs have confirmed many promising results in hydrologic modeling and water resources simulation, their utilization needs for special attention in certain cases viz., when the data are noisy and show a large scatter over measured ranges. In such cases the performance of all neural network structures seems to be of equal quality and relatively unsatisfactory. This thus poses definite limitations on their application for an accurate and suitable RR modeling program. Besides, choice of optimal network structure, transfer functions or network type and training algorithm, which devours considerable amounts

of data in order to find the patterns embedded in the system, require relatively great computing efforts and trial and error cycles as well, making the modeling procedure time-consuming and tedious. To eliminate these restrictions, a fuzzy methodology approach seems to be practical and promising. Zadeh (1965) introduced the theory of fuzzy sets as an extension for conventional set theory where the membership of an object to a set is restricted to one or zero. A fuzzy set, however, is defined by assigning a membership grade from the interval of [0, 1] for every article that its belonging to the set is the subject of the question. Such a definition is very suggestive because it allows the representation of concepts of interest as qualitative; that is, fuzzy methodology is capable of dealing with vague or imprecise inputs from designers and human experts who describe each system variable with some linguistic terms, such as high, low, nearly 10, etc. These linguistic terms can mathematically be represented by designating suitable membership functions and be manipulated on the basis of fuzzy set theory. Indeed, fuzzy sets theory has led to evolution of two distinct basic territories in modern mathematics; while fuzzy arithmetic deals with fuzzy numbers and their generalized arithmetical operations such as fuzzy regression on the basis of Zadeh's extension principle (Zimmermann, 1996), fuzzy logic employs fuzzy sets and their associated membership functions as foundations for carrying out the process of approximate reasoning, resembling a human expert's manner. Fuzzy methodology has proved quite useful applications in different disciplines such as decision-making problems (Zimmermann, 1996), mechanical design (Wood and Antonsson, 1989), control engineering (Nguyen *et al.*, 2003), etc.

Fuzzy models exhibit a number of advantages compared to global nonlinear models, such as neural networks. The model structure is easy to comprehend and is in some cases interpretable. Integration of knowledge of diverse types including statistical data and empirical knowledge can be managed within the model structure with ease. Moreover, fuzzy models have effectively substantiated tolerance for input data being imprecise i.e., represented as intervals (fuzzy numbers) or vague i.e., depicted as fuzzy sets.

The present study investigates the ability of fuzzy regression and fuzzy logic in modeling RR process for Hali Rud River watershed, central Iran. The predicted results are in good agreement with measured data.

### FUZZY SETS

Membership functions serve as the foundations for any fuzzy approach program, demonstrating the degree of belonging of a given object or value from the spectrum of possible objects or values over a universe of discourse to an arbitrary fuzzy subset; the meticulous act of their selection is considered to be application dependent and subjective. They can be specified via either human expert's knowledge or automated techniques such as data mining tools and methodologies.

The membership degree of article  $x$  to fuzzy set  $A$  is designated by  $A(x)$  or by  $\mu$  for short. The closer the magnitude of  $A(x)$  to unity (zero), the higher (less) membership degree of  $x$  in  $A$ . There exist some standard forms of membership functions that are suggested for engineering purposes. Of simple ones, the triangular membership function shown in Fig. 1 is nothing more than an arrangement of three points creating a triangle. An arbitrary fuzzy set with a triangular membership function can then be denoted by  $\tilde{A} = T(p, q, r)$  where  $p, q$  and  $r$  correspond to minimum, mean and maximum values of the parameter of interest, respectively. Due to its smoothness, symmetry and owning nonzero values for the input range, Gaussian membership function shown in Fig. 1 is also a popular operator for designating a fuzzy set and specified by two parameters  $\sigma$  and  $c$  as follows:

$$A(x) = \exp\left(-\frac{(x - c)^2}{2\sigma^2}\right) \tag{1}$$

where,  $c$  and  $\sigma$  are mean value and standard deviation of the Gaussian function, respectively. A fuzzy set with a Gaussian membership function can then be denoted as  $\tilde{A} = G(\sigma, c)$ .

As Fig. 1 shows, fuzzy sets do not own crisp and clearly defined boundaries, keeping values with only a partial grade of membership. If membership grades encompass a central value or range in which the

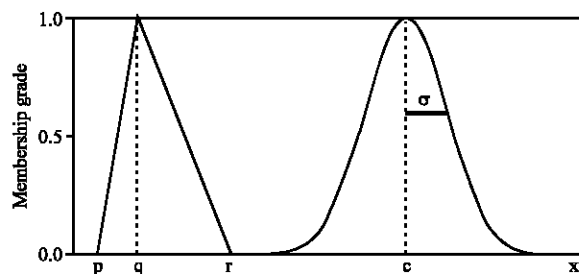


Fig. 1: Triangular and Gaussian membership functions

membership grade reaches to unity the fuzzy set is referred to as a normal fuzzy set. As an example consider a vague description of high cost. A system experts picks out a membership grade to express to what degree he believes a given value from all of the possible ones, for example the cost of 3000 dollars, can be classified as high. Now, consider the inexact phrase of around 14. Somewhat different from a vague concept modeling situation where the central value(s) can be uncertain and the assignment of membership grades is subjected to the expert's judgment, the entire range of a word number or a fuzzy number is determined by an interval whose normal value is that central value, 14 here and the membership grades are continuously decreased by approaching two sides of the interval where memberships finally become zero. Basically, fuzzy numbers can be considered as a special class of fuzzy sets showing some specific properties (Zimmermann, 1996). The uncertainty in determining the true and precise values of model inputs or outputs can be casted as appropriate fuzzy numbers.

**DATA AND AREA OF STUDY**

Halil Rud River flows in Jazmoriyan basin, central Iran, where Jiroft dam is located within the basin. Rainfall data used were the records of five stations of the area named Soltani, Baft, Henjan, Cheshm Aroos and Meydan; Konarooey station data was used for discharging. Halil Rud River watershed and location of rainfall and hydrometry stations are shown in Fig. 2. Data base of the present study comprises of 34 floods data. The unit of all data is  $m^3 sec^{-1}$ , cms. In the following modeling steps, rainfall data were used as input and runoff data as output.

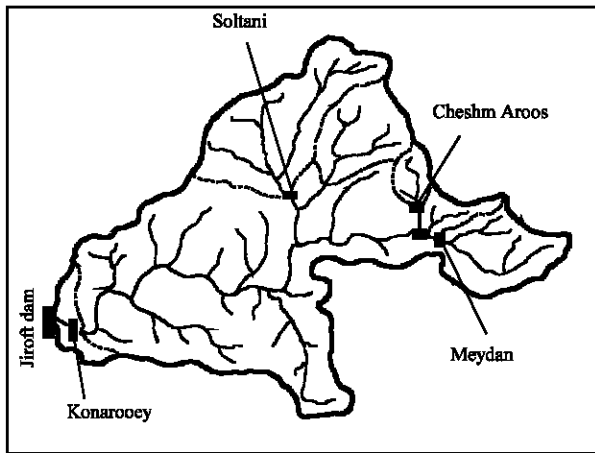


Fig. 2: Halil Rud River watershed and location of rainfall and hydrometry stations

**THE FUZZY RULE-BASED MODEL**

Organization of an RR simulation system can be delineated and evaluated through rules derived from expert's point of view corresponding to a specified structure based on the measurement data. Compared to routine modeling, fuzzy rule-based structures provide a robust tool which is directly able to handle the semantic models of human interpretation of the system of interest. Accordingly, some appropriate fuzzy expressions such as very low (VL), low (L), medium low (ML), medium (M), medium high (MH), high (H) and very high (VH), should firstly be appointed for RR modeling. The modeling practice proceeds towards the mathematical presentation of those fuzzy sets with some standard membership functions. As shown in Fig. 3 and 4, Gaussian shapes were chosen for the input and output variables and then adjusted to fine-tune the modeling. The standard deviation values for rainfall and runoff variables were fixed on 9.48 and 15.1, respectively.

A generalized deductive reasoning scheme comprised of fuzzy if-then rules lends itself to model a highly nonlinear relationship that exists between rainfall and runoff values. Every fuzzy rule is responsible for mapping some part of the input space to suitable part of the output space. A fuzzy rule base consists of fuzzy rules and accumulates what knowledge a system expert has acquired through experience, experimental data and common sense. The fuzzy rule base for the present model is formed by the following statements:

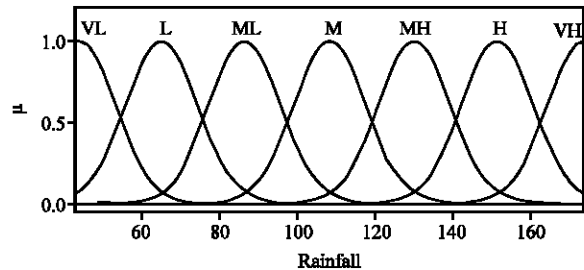


Fig. 3: Membership functions for rainfall variable

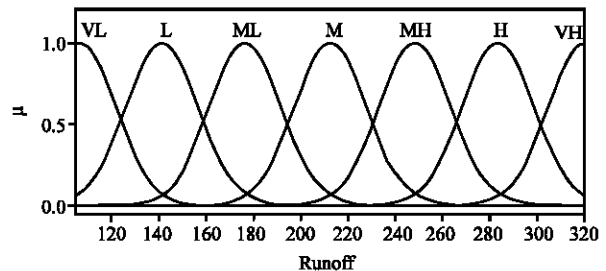


Fig. 4: Membership functions for runoff variable

- Rule 1:** If rainfall is VL, then runoff is VL
- Rule 2:** If rainfall is L, then runoff is L
- Rule 3:** If rainfall is ML, then runoff is ML
- Rule 4:** If rainfall is ML, then runoff is L
- Rule 5:** If rainfall is M, then runoff is L
- Rule 6:** If rainfall is M, then runoff is M
- Rule 7:** If rainfall is MH, then runoff is L
- Rule 8:** If rainfall is MH, then runoff is ML
- Rule 9:** If rainfall is H, then runoff is L
- Rule 10:** If rainfall is H, then runoff is ML
- Rule 11:** If rainfall is VH, then runoff is M
- Rule 12:** If rainfall is VH, then runoff is ML
- Rule 13:** If rainfall is VH, then runoff is L

As can be seen from the above rule base, this model contains some contradictory rules showing the same antecedent and different consequence parts. This conflict arises when data have largely scattered over their measured range. These rules can also be included in the fuzzy model to make the simulation action more effective and precise.

If-then rules can mathematically be interpreted by means of the so-called Mamdani synthesis. The input variable is evaluated for each rainfall and a truth value matched to the grade of membership of the input variable in each predefined antecedent fuzzy set is computed. For every Mamdani rule, minimum truth value of the antecedent then propagates through and truncates the membership function for the consequent graphically. Since the rules are disjunctive, every fired rule produces its own truncated membership function in the consequent fuzzy sets. To make a single decision, however, the rules must be integrated in some suitable style. Aggregation is referred to the process by which the fuzzy sets that portray the outputs of each rule are combined into a single fuzzy set. An aggregated membership function comprised of the outer envelope of the individual truncated membership shapes from each rule is created by taking maximum truth values of the output results. Combination of mentioned logical connectors minimum and maximum employed in the present fuzzy rule-based model is called max-min inference method. Generally, these connectors can also be replaced with other standard functions such as product, probabilistic sum and other agents in a similar way to the membership functions in order to improve the performance of the model.

Completing the modeling procedure necessitates some mechanism to be capable of converting the aggregated fuzzy set, which encompasses a range of output values, to a single value. To do this, an appropriate defuzzification method must be used. The adopted defuzzifier for the present model is the bisector

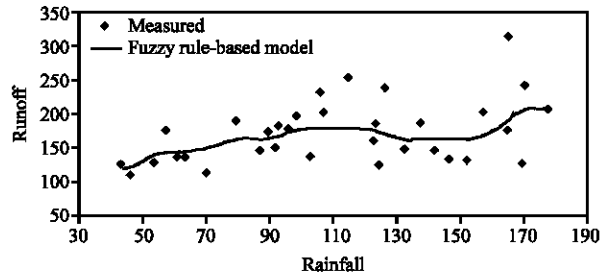


Fig. 5: Comparison of results of fuzzy rule-based model with experimental data

agent. In this method a non-fuzzy value that divides the area bounded by the final membership function curve into two equal segments is yielded as the most typical crisp value of the union of all output fuzzy sets. The results were acquired by making use of Fuzzy Logic Toolbox integrated in MATLAB software. The membership functions, rules and mathematical operators can be chosen step by step in the software. As can be shown from Fig. 5, the predicted results and real data are in good agreement with each other.

### THE FUZZY REGRESSION MODELS

Fuzzy regression techniques are considered to be useful when it is known that a causal relation exists, but only few data points are available (Bardossy *et al.*, 1990; Ozelkan and Duckstein, 2000). The correlated equations for a fuzzy regression program can be written in the usual forms:

$$y = a+bx \tag{2}$$

It should be pointed out that four parameters of the Eq. 2 can generally be fuzzy. When it is so, ~ symbol over the parameter will hereafter represent the fuzzy number. In the context of modeling, it is expected that neither the model and its parameters nor the data are certain. Because the oversimplification of a modeling system has always been considered as a great risk in the evaluation process the uncertainties arisen from incomplete or imprecise information must be reflected in some appropriate manner. The uncertainties in the model parameters, model inputs, etc., can then be taken into account by fuzzy numbers with their shape derived from experimental data or expert knowledge (Hanss, 1999). The text continues on considering two different states, where in phase (1) a, b and y and in phase (2) x (rainfall) and y (runoff), are taken as fuzzy numbers. It is worthy of special mentioning that the developed fuzzy regression models do not correspond

to an ad hoc implementation and can be applied for any other hydrologic modeling purposes, the only difference, however, will appear in the context of measured data.

**Possibility regression:** Here, those models with fuzzy coefficients of the regression are considered. The mathematical details and background are well documented in Tanaka *et al.* (1982) and Yen *et al.* (1999). Let the model be of the form:

$$\tilde{Y} = \tilde{A}_0 + \tilde{A}_1 X_1 \tag{3}$$

where  $\tilde{A}_0$  and  $\tilde{A}_1$  correspond to the fuzzy numbers of the following form:

$$\tilde{A} = \begin{cases} 1 - \frac{x-a}{S^L} & a - S^L \leq x \leq a \\ 1 - \frac{a-x}{S^R} & a < x \leq a + S^R \end{cases} \tag{4}$$

Note that the subscripts 0 and 1 were ignored for the sake of simplicity and the triangular fuzzy number can then be denoted by.  $\tilde{A} = T(a - S^L, a, a + S^R)$ .  $S^L$  and  $S^R$  are called the left and right spreads of the fuzzy number, respectively.  $\tilde{A}_0$  and  $\tilde{A}_1$  must be determined in such a way that the fuzziness in the output of fuzzy  $\tilde{y}$  of all the observed responds becomes minimum. In other words, it is desired that we could find the regression parameters that have the smallest spreads around their central value. Mathematically, it corresponds to the minimization action of the following target function:

$$Z = m(S_0^L + S_0^R) + \sum_{j=1}^n \left[ (S_j^L + S_j^R) \sum_{i=1}^m |x_{ij}| \right] \tag{5}$$

which is subjected to the following constraints:

$$(1-H)S_0^L + (1-H) \sum_{j=1}^n S_j^L |x_{ij}| - a_0^c - \sum_{j=1}^n a_j^c |x_{ij}| \geq -Y_i \tag{6}$$

$$(1-H)S_0^R + (1-H) \sum_{j=1}^n S_j^R |x_{ij}| + a_0^c + \sum_{j=1}^n a_j^c |x_{ij}| \geq Y_i \quad i=1,2,\dots,m \tag{7}$$

It is supposed that the regression parameters are symmetric fuzzy numbers, that is  $S_j^R = S_j^L = S_j$ . Also, note that there exists only one input of  $X_1 = \begin{bmatrix} 1 \dots x_{11} \\ 1 \dots x_{21} \\ \vdots \\ 1 \dots x_{m1} \end{bmatrix}$  in the model. Thus the problem is:

$$\min Z = mS_0 + S_1 \sum_{i=1}^m |x_{ij}| \tag{8}$$

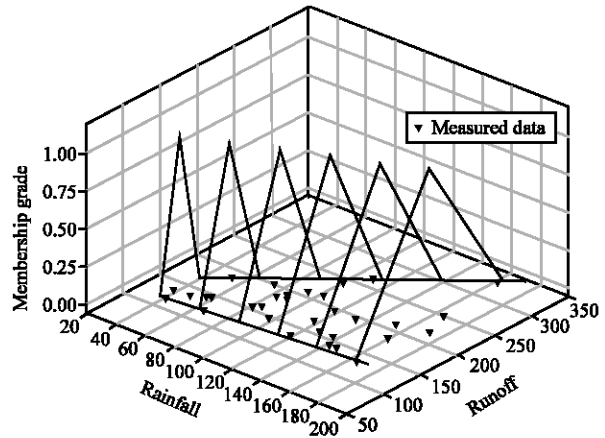


Fig. 6: Possibility regression modeling of rainfall and runoff variables

which is subjected to the following refined constrains:

$$a_0 + a_1 x_{ij} + (1-H)(S_0 + S_1 x_{ij}) \geq Y_i, \tag{9}$$

$$-a_0 - a_1 x_{ij} + (1-H)(S_0 + S_1 x_{ij}) \geq -Y_i \tag{10}$$

In the present study, QSB software was used and the desired parameters viz.  $a_0$ ,  $a_1$ ,  $S_0$  and  $S_1$  obtained.

As can be shown from Fig. 6, the response of the possibility regression modeling corresponds to a special region where nearly all data are dispersed within. The lower and upper limits of this region depict a fuzzy box i.e., the generalized concept of intervals for a two-dimensional problem. These limits possess the membership grade of zero. The line placed in rainfall-runoff plane that possess the highest membership grade will be given by this method. The projection of this line in rainfall-runoff plane yields the average expected runoff values.

**Least square regression with ordinary coefficients, but fuzzy data:** In this case, the regression model is written as the following from Diamond (1998) and Ming *et al.* (1997):

$$\tilde{y} = a + b\tilde{x} \tag{11}$$

where, the rainfall and runoff values are essentially considered as symmetric triangular fuzzy numbers for simplicity and can then be represented by  $x_i = T(a_{x_i}, b_{x_i}, c_{x_i}) = (b_{x_i}, S_{x_i})_T$  and  $y_i = T(a_{y_i}, b_{y_i}, c_{y_i}) = (b_{y_i}, S_{y_i})_T$ , respectively. For short, the triangular fuzzy numbers are now denoted by the pair of  $(b, S)_T$  where  $b$  and  $S$  are mean

value and spread of the membership function. Therefore, all  $S_{x_i}$  and  $S_{y_i}$  are taken the same and equal to  $S = 2.5$ . The values of  $a$  and  $b$  can be obtained by minimizing,

$$\min_{a,b} \sum_i [\bar{y} - (a + b\bar{x})]^2 \quad (12)$$

Minimization action of the sum of square of distances between inputs and estimated outputs according to the above equation leads to appearance of the following equations:

$$\left\{ \begin{aligned} 2na^+ + b^+ \sum_{i=1}^n \left\{ \int_0^1 (\underline{x}_i(\mu) + \bar{x}_i(\mu)) \cdot d\mu \right\} &= \quad (13a) \\ \sum_{i=1}^n \left\{ \int_0^1 (\underline{y}_i(\mu) + \bar{y}_i(\mu)) \cdot d\mu \right\} \\ a^+ \sum_{i=1}^n \left\{ \int_0^1 (\underline{x}_i(\mu) + \bar{x}_i(\mu)) \cdot d\mu \right\} + b^+ \sum_{i=1}^n \left\{ (\underline{x}_i^2(\mu) + \bar{x}_i^2(\mu)) \cdot d\mu \right\} &= \\ \sum_{i=1}^n \left\{ \int_0^1 (\underline{x}_i(\mu)\underline{y}_i(\mu) + \bar{x}_i(\mu)\bar{y}_i(\mu)) \cdot d\mu \right\} & \quad (13b) \end{aligned} \right.$$

$$\left\{ \begin{aligned} 2na^- + b^- \sum_{i=1}^n \left\{ \int_0^1 (\underline{x}_i(\mu) + \bar{x}_i(\mu)) \cdot d\mu \right\} &= \quad (13c) \\ \sum_{i=1}^n \left\{ \int_0^1 (\underline{y}_i(\mu) + \bar{y}_i(\mu)) \cdot d\mu \right\} \\ a^- \sum_{i=1}^n \left\{ \int_0^1 (\underline{x}_i(\mu) + \bar{x}_i(\mu)) \cdot d\mu \right\} + b^- \sum_{i=1}^n \left\{ (\underline{x}_i^2(\mu) + \bar{x}_i^2(\mu)) \cdot d\mu \right\} &= \\ \sum_{i=1}^n \left\{ \int_0^1 (\underline{x}_i(\mu)\bar{y}_i(\mu) + \bar{x}_i(\mu)\underline{y}_i(\mu)) \cdot d\mu \right\} & \quad (13d) \end{aligned} \right.$$

Where:

$$\underline{x}_i(\mu) = a_{x_i} + \mu(b_{x_i} - a_{x_i}) = a_{x_i} + 2.5\mu \quad (14a)$$

$$\bar{x}_i(\mu) = c_{x_i} + \mu(b_{x_i} - c_{x_i}) = c_{x_i} - 2.5\mu \quad (14b)$$

$$\underline{y}_i(\mu) = a_{y_i} + \mu(b_{y_i} - a_{y_i}) = a_{y_i} + 2.5\mu \quad (14c)$$

$$\bar{y}_i(\mu) = c_{y_i} + \mu(b_{y_i} - c_{y_i}) = c_{y_i} - 2.5\mu \quad (14d)$$

and  $\mu$  stands for membership grades of  $x_i$  and  $y_i$ . Indeed, the inverse of triangular membership functions for left-hand and right-hand sides of rainfall and runoff variables are computed through Eq. 14a-d. It is possible to find  $a^+$  and  $b^+$  from Eq. 13a, b and similarly  $a^-$  and  $b^-$  from Eq. 13c, d). It is obvious that  $b^+ \geq b^-$  and to find the feasible solutions, one has to check whether  $0 \leq b^- \leq b^+$  or  $b^- \leq b^+ \leq 0$ . If  $b^- \leq b^+ \leq 0$  then the data are positively tight and thus the unique solution for the problem of finding regression parameters in Eq.11 is the pair ( $a^+$ ,  $b^+$ ), otherwise the unique solution is ( $a^-$ ,  $b^-$ ).

The system of equations that appeared as Eq. 13a-d was solved and answers were obtained by making use of MAPLE package.

### RESULTS AND DISCUSSION

In order to compare the obtained results from the different models considered under the different assumptions, conditions and considerations, the Mean of Relative Errors (MRE) for each model were computed. The predicted results of fuzzy rule-base model show an MRE of 16.29%. This amount of error seems to be acceptable because data has widely scattered. As mentioned before, two fuzzy regression models were also developed and the results have been expressed in the form of triangular numbers. Obtained fuzzy results for the Possibility Regression (PR) and Fuzzy Least Squares Regression (FLSR) are shown in Table 1.

In case of fuzzy regression models, different comparison criteria could be considered according to operator's point of view because those models predict an interval of possible runoff values whose importance or

Table 1: Predicted results with fuzzy regression models

| Rainfall | PR runoff |       |       | FLSR runoff |       |       |
|----------|-----------|-------|-------|-------------|-------|-------|
|          | Min.      | Mean  | Max.  | Min.        | Mean  | Max.  |
| 43       | 106.4     | 131.7 | 157.1 | 135.2       | 136.4 | 137.6 |
| 46       | 106.8     | 133.9 | 161.0 | 136.7       | 137.9 | 139.1 |
| 53       | 107.8     | 139.1 | 170.3 | 140.0       | 141.2 | 142.4 |
| 57       | 108.4     | 142.0 | 175.6 | 141.9       | 143.1 | 144.3 |
| 60.5     | 109.0     | 144.6 | 180.3 | 143.6       | 144.8 | 146.0 |
| 63       | 109.3     | 146.5 | 183.6 | 144.8       | 146.0 | 147.2 |
| 69.5     | 110.3     | 151.3 | 192.2 | 147.9       | 149.1 | 150.3 |
| 79       | 111.7     | 158.3 | 204.9 | 152.5       | 153.7 | 154.9 |
| 86.5     | 112.8     | 163.8 | 214.8 | 156.1       | 157.3 | 158.5 |
| 89       | 113.2     | 165.7 | 218.1 | 157.3       | 158.5 | 159.7 |
| 91       | 113.5     | 167.1 | 220.8 | 158.3       | 159.5 | 160.7 |
| 92       | 113.6     | 167.9 | 222.1 | 158.8       | 160.0 | 161.2 |
| 95.5     | 114.2     | 170.5 | 226.8 | 160.4       | 161.6 | 162.8 |
| 98       | 114.5     | 172.3 | 230.1 | 161.6       | 162.8 | 164.0 |
| 102      | 115.1     | 175.3 | 235.4 | 163.6       | 164.8 | 166.0 |
| 105.5    | 115.6     | 177.8 | 240.0 | 165.2       | 166.4 | 167.6 |
| 106.5    | 115.8     | 178.6 | 241.4 | 165.7       | 166.9 | 168.1 |
| 114.5    | 117.0     | 184.5 | 252.0 | 169.6       | 170.8 | 172.0 |
| 122      | 118.1     | 190.0 | 262.0 | 173.2       | 174.4 | 175.6 |
| 123      | 118.2     | 190.8 | 263.3 | 173.6       | 174.8 | 176.0 |
| 124      | 118.4     | 191.5 | 264.6 | 174.1       | 175.3 | 176.5 |
| 126      | 118.7     | 193.0 | 267.3 | 175.1       | 176.3 | 177.5 |
| 132      | 119.6     | 197.4 | 275.2 | 178.0       | 179.2 | 180.4 |
| 137      | 120.3     | 201.1 | 281.9 | 180.4       | 181.6 | 182.8 |
| 141.5    | 121.0     | 204.4 | 287.9 | 182.5       | 183.7 | 184.9 |
| 146      | 121.7     | 207.7 | 293.8 | 184.7       | 185.9 | 187.1 |
| 151.6    | 122.5     | 211.9 | 301.3 | 187.4       | 188.6 | 189.8 |
| 156.8    | 123.3     | 215.7 | 308.2 | 189.9       | 191.1 | 192.3 |
| 164.5    | 124.4     | 221.4 | 318.4 | 193.6       | 194.8 | 196.0 |
| 166      | 124.6     | 222.5 | 320.4 | 194.3       | 195.5 | 196.7 |
| 169      | 125.1     | 224.7 | 324.4 | 195.7       | 196.9 | 198.1 |
| 170      | 125.2     | 225.5 | 325.7 | 196.2       | 197.4 | 198.6 |
| 177      | 126.3     | 230.6 | 335.0 | 199.6       | 200.8 | 202.0 |

degree of believeth in their occurrence would be determined by their corresponding membership grades. That is, fuzzy regression models provide more informative results since they predict minimum, mean and maximum expected runoff values and the operator can therefore obtain a wider perspective over the situation. To have a consistent criterion, the average of predicted runoff values was considered for computing MRE. The mean of relative errors for PR and FLSR were 21.76 and 19.38%, respectively. This means that the results obtained by the use of fuzzy rule-base model show better correlation with observed data.

### CONCLUSIONS

With respect to importance of flood modeling in water resources management, several models were developed by means of advanced fuzzy methodologies. These novel models employ fuzzy logic and fuzzy regression techniques. On the basis of data measured at the hydrometry stations, the models were customized for Halil Rud River watershed. From the numerical results predicted by the developed models, it can be pointed out that the adopted mathematical approaches can effectively be used in flood simulation. While the fuzzy rule-based model shows the lowest mean of relative errors, fuzzy regression models are capable of predicting whole anticipated runoff values accompanied by their respective grade of membership or possibility of occurrence on the basis of observed data. The fuzzy rule-based model has the advantage of flexibility and simplicity because the ill-defined relation between rainfall and runoff variables can be described in a semantic form. The model is also considered as robust, that is, the performance do not depend upon training phase and probable new input variables and rules can be easily added. Fuzzy regression models are regarded more informative because they can forecast the spectrum of possible results. The fuzzy methodology succeeds in situations where data has broadly scattered. In such situations, other global estimators like artificial neural networks seem to give results of limited accuracy and precision.

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