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Determining the Order Penetration Point in Auto Export Supply Chain by the Use of Dynamic Programming

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Abstract: In this study, after studying the characteristics and concepts relating to the Order Penetration Point (OPP), a dynamic programming model which determines this point in auto export supply chain was proposed. One of the most important characteristics of this supply chain was that, the product was packaged in different modules and after various stockings and passing long routs, was assembled in the target country. This modularized characteristic of the product was encouraging to explore the OPP of the chain from one point to several points in which the OPP of each module was located. Our proposed model tried to put the OPP of expensive modules (that have higher inventory holding cost) in the upstream section of the chain and puts the OPP of cheaper ones which created delay, in the downstream section of the chain. And finally, a numerical example was provided and solved to illustrate the application of our proposed model.

Key words: Supply chain management, logistics, order penetration point, customer order decoupling point, dynamic programming

INTRODUCTION

The positioning of the Order Penetration Point (OPP) is successively becoming a topic of strategic interest (Olhager, 2003). In the existing literature, this point is also called: Customer Order Decoupling Point (CODP), Decoupling Point (DP) and Customer Order Point (COP). This point defines the stage in the manufacturing value chain, where a particular product is linked to a specific customer order. Sometimes the OPP is called the customer order decoupling point to highlight the involvement of a customer order (Olhager, 2003).

Upstream from the OPP the supply chain is initially forecast driven. However, with the advent of Kanban driven supply this has become more than simply a push system. Downstream from the OPP all products are pulled by the end-user, that is, they are market driven. The OPP separates the part of the supply chain that responds directly to the customer from the part of the supply chain that uses forward planning and a strategic stock to buffer against the variability in the demand of the supply chain (Naylor *et al.*, 1999).

Associated with the positioning of the OPP is the cognate issue of postponement. The aim of postponement is to increase the efficiency of the supply chain by moving product differentiation (at the Decoupling Point) closer to the end user (Naylor *et al.*, 1999). Postponement centers around delaying activities in the supply chain until real information about the markets is available

(Yang and Burns, 2003). HP is very famous in the implementation of this strategy, because HP has designed its modules in a way that by different combinations of the modules, it can produce different products. Yet, if this company would supply its products as finalized and assembled products, the amount of its inventories would be increased dramatically. So, HP prepares the modules in its supply chain and wait for the customer to order. After the customer order is specified, modules are assembled to each other and the requested product would be created. Summing up, HP has reduced its inventories in its supply chain by using the postponement strategy.

A 2×2 matrix shown in Fig. 1 identifies four generic supply chain Postponement/Speculation strategies, by combining manufacturing and logistics postponement and speculation. The matrix will be referred to as the P/S matrix. The four strategies are (Pagh and Cooper, 1998):

		Logistics	
		Speculation decentralized inventories	Postponement centralized inventories and direct distribution
Manufacturing	Speculation Make To Stock (MTS)	The full speculation strategy	The logistics postponement strategy
	Postponement Make To Order (MTO)	The manufacturing postponement strategy	The full postponement strategy

Fig. 1: The P/S matrix and generic supply chain P/S strategies. Source: (Pagh and Cooper, 1998)

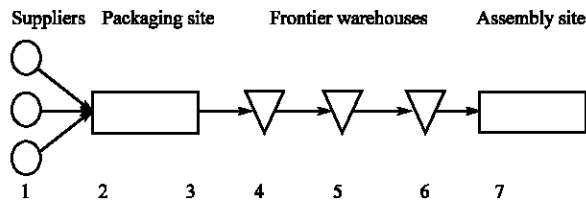


Fig. 2: Possible points for positioning the OPP of modules

- The full speculation strategy
- The logistics postponement strategy
- The manufacturing postponement strategy
- The full postponement strategy

The subject of this study was more relative to the logistics postponement strategy, because the extended model was for a supply chain where its manufacturing process, is not based on the customer order. In other words, there was no postponement in the manufacturing part of the chain and postponement can only be made in the logistics part (including the packaging of modules and shipping). Figure 2 shows the overall structure of this chain and the possible points for positioning the OPPs.

Some of the existing researchers in this field as Adan and Wal (1998), Arreola-Risa and Decroix (1998), Donk (2001), Federgruen and Katalan (1999), Wikner and Rudberg (2005b) and Youssef *et al.* (2004) only discuss about strategies that are related to postponements in production. Adan and Wal (1998) believe that it is not necessary for a system to be completely MTS or completely MTO. But, it could be as a combination of these two and by this reasoning he has proposed models for compound models. Arreola-Risa and Decroix (1998) have studied the MTS and MTO strategies for different products that are produced by one machine and has finally determined that production processes in which the products must be either as MTS or MTO. Being MTS or MTO in the food industry is studied as a case study by Donk (2001). Effects of adding a MTO product to a MTS system is studied by Federgruen and Katalan (1999). Determining the position of OPP, Wikner and Rudberg (2005b) in addition to production range, have considered the issues relating to product engineering. Youssef *et al.* (2004) have proposed several efficient rules for a combinational system of MTS and MTO.

The other study that have not limited themselves to postponement in production and have discussed about positioning the OPP in the total supply chain, have studied the subject in different points of view. The following paragraphs give a proper overview about these studies.

Positioning the OPP is called a strategic decision by Olhager (2003). In the mentioned study, the major factors that are effecting the positioning of the OPP were propounded and were classified in three categories: market specifications, product specifications and production specifications.

The impact of the OPP positioning, on productivity of the supply chain is studied by Hull (2005). Olhager and Ostlund (1990) discuss relations between push-pull systems and the positioning of the OPP point. The mentioned study believed that supply chain acts as a push system in the upstream of the OPP and in the downstream section of the OPP acts as a pull system.

In two interesting studies (Mason-Jones *et al.*, 2000; Naylor *et al.*, 1999), the relation between lean and agile concepts with the positioning of the OPP in the supply chain is propounded. These studies have pointed to this important issue that the supply chain in the upstream section of the OPP must be lean (low costs) and in the downstream section of the OPP that the customer demand is specified must be agile. These studies believe that instead of being completely lean or being completely agile, it is better to implement a combination of these concepts in the supply chain strategy. Also, the OPP acts as a divider between the lean and agile section of the supply chain. Wikner and Rudberg (2005a) have extended the concept of Customer Order Decoupling Point (CODP) to customer order decoupling zone (CODZ).

Garg and Tang (1997), Mikkola and Skjott-Larsen (2004), Pagh and Cooper (1998), Yang *et al.* (2004) and Yang and Burns (2003) have studied the issue of postponement in the supply chain. Garg and Tang (1997) propound this subject that, it is not necessary that all products have a common OPP and a family of products could have several OPP points. The considered study presented two models for products that have more than two OPP points. Mass customization and modularization concepts and their relation with postponement strategies are propounded by Mikkola and Skjott-Larsen (2004). Four strategies of postponement/speculation in supply chain and specifications of each strategy in terms of production, inventory and distribution costs and the status of service levels in each strategy are presented by Pagh and Cooper (1998). Yang *et al.* (2004) have written a review study in postponement. Yang and Burns (2003) have viewed the supply chain from the point of view of postponement and has studied the postponement requirements in relation to integration of supply chain and issues relating to capacity planning and control in supply chain and order decoupling point.

Rudberg and Wikner (2004) have studied the problem of determining the position of OPP in supply chains which act as mass customization. Wanke and Zinn (2004) have

defined the strategic decisions of logistics as: being MTS or MTO, being pulled or pushed and centralizing inventories or decentralizing inventories. And as propounded as before, each of these three decisions are related to the positioning of OPP. Fisher (1997) believes that supply chain of new and competitive products must have high responsiveness and supply chain of common and non-competitive products must have a high efficiency, where this subject was also related to the positioning of the OPP in supply chain.

MATERIALS AND METHODS

Consider the supply chain of auto export to a specific country. The assembling company in the target country is our only customer and batch sized orders are specified. But the time of order placement is a stochastic variable that can give values between α and β .

Suppose that after the placement of order by customer, the required time for moving the modules between each two points, has two elements where one is fixed and another is a variable. The fixed element is not dependent on the number of moving modules between points j and $j+1$, because in any case it occurs and we show this element by l_j . But the variable element shows the amount of delay in delivery that occurs by shifting the OPP of module i from point $j+1$ to point j and we show it by r_{ij} . Also suppose that after the placement of order by customer, for every unit of time (for example, day) delay in delivery, a penalty cost is paid that we show it by p .

These autos must be exported as separated modules (final assembly must take place in the target country) and different modules differ in inventory cost and the amount of delay that occurs in their shipment. So, the position of their OPP can be at different points. Consequently, we let the OPP of each module to be located at any point of the chain.

Transportation cost has two elements, fixed and variable, but the variable cost of transportation has no effect on the solution of problem, because in any case it would be paid. Therefore we only consider the fixed transportation costs. As a whole, the following notations are required:

- Q : Customer order size
- t : Time of order placement by customer
- $f(t)$: Probability distribution function for the time of order placement
- $F(t)$: Cumulative distribution function of order placement by customer
- t^* : Optimal time of being prepared for modules in OPPs (decision variable)
- h_{ij} : Inventory holding cost for the unit of module i in point j

- K_j : Fixed transportation cost from point $j-1$ to point j
- k_{ij} : Variable transportation cost for module i from point $j-1$ to point j
- T_j : Sum of fixed transportation costs from the chain beginning to point j ($T_j = \sum_{p=1}^j K_p$)
- l_j : Time distance (transportation time) between points $j-1$ and j
- n : No. of modules
- N : No. of points of chain
- r_{ij} : Amount of delay in delivery that occurs by shifting the module i from point $j+1$ to point j
- R_{ij} : Amount of delay in delivery that occurs by positioning the module i in point j ($R_{ij} = \sum_{k=j}^{N-1} r_{ik}$)
- p : Penalty cost for delay in delivery of order
- x_{ij} : If the OPP of module i is located at point j , equals to 1 and otherwise equals to 0 (decision variable)
- t_i : Amount of time where module i is in its OPP before t^*
- tp_j : Amount of time where modules that their OPP is in point j are in their OPP before t^*
- y_j : If there is a transshipment to destination point of j , equals to 1 and otherwise equals to 0 (decision variable)

Modeling the problem by dynamic programming: We decompose the problem into two phases. In phase (1), we determine the OPP of each module without considering the other modules. In this phase we do not consider the transportation costs, because they have no impact on the answer of our problem, i.e. the position of the OPP for that module is not affected by the transportation costs. The output of this phase is the primary OPP for each module and the sum of inventory holding and penalty costs (for delay in delivery of order) for positioning the OPP of each module in each point. We show these costs by c_{ij} and use them in phase (2) as some of the input data.

Phase (1): determining t^* and primary OPP for each module: In this phase, because the OPP of a module is placed at any point, all of its fixed transportation costs must be paid, we do not consider these costs in this section. On the other hand, it is not necessary that the considered module arrives before t^* , because if it arrives before t^* the time duration until t^* , inventory holding cost (type II) is included. Consequently, in this phase we should just consider the inventory holding cost (type I) and the delay in delivery cost. If module i placed in point j , sum of these costs (c_{ij}), is computed as follows:

$$c_{ij} = Q \times h_{ij} \times \int_{t^*}^{\beta} (t - t^*) f(t) dt + p \times \left[R_{ij} + \int_{\alpha}^{t^*} (t^* - t) f(t) dt \right] \quad (1)$$

The Eq. 1 shows that, if the time of order placement by customer (t) is after t* the type I of inventory holding costs increases. And if the time of order placement by customer is before t*, delay costs will be increased. We know that in any case, we have a delay for each module equal to R_{ij} and the penalty cost for it must be paid. So, for computing the c_{ij} for each module and each point, t* must be specified. In the other hand, value of t* is determined by differentiating Eq. 1 with respect to t*, equating to zero and solving for the t*. Differentiating with respect to t* gives:

$$\begin{aligned} & \frac{\partial}{\partial t^*} \left[\sum_i \sum_j \left(Q \times h_{ij} \times \int_{t^*}^{\beta} (t - t^*) f(t) dt + p \times \left[R_{ij} + \int_{\alpha}^{t^*} (t^* - t) f(t) dt \right] \right) \right] \\ &= \sum_i \sum_j \left(-Q \times h_{ij} \int_{t^*}^{\beta} f(t) dt + p \int_{\alpha}^{t^*} f(t) dt \right) \\ &= \sum_i \sum_j \left(-Q \times h_{ij} \times (1 - F(t^*)) + p \times F(t^*) \right) = 0 \end{aligned} \tag{2}$$

By simplifying the Eq. 2, we have:

$$t^* = F^{-1} \left(\frac{\sum_i \sum_j Q \times h_{ij}}{\sum_i \sum_j (Q \times h_{ij} + p)} \right) \tag{3}$$

So, there is a mutual relationship between t* and c_{ij}, where for computing c_{ij}, t* should be determined and for computing t*, c_{ij} for each i and j should be determined, because in Eq. 3 for each i there is only one j and that is where c_{ij} is minimum. So, for computing t* and c_{ij}, we propose the following algorithm:

- Step 0:** Solve Eq. 3 for t* and consider all i's and all j's
- Step 1:** Consider the current t* and Solve Eq. 1 for c_{ij} at all i's and all j's
- Step 2:** Solve Eq. 3 for t* but for each i only consider the j where c_{ij} = min{c_{ij}}
- Step 3:** while t* in the two consecutive stages is not fixed, repeat steps 1 and 2

The following example shows the method of using this algorithm.

Suppose that Q = 96 and time of order placement by the customer has a uniform distribution between 0 and 30 and also p = 20 units of money, n = 8 and N = 7. And inventory holding costs for each unit module, at each point is shown in Table 1.

Also, R_{ij} for all modules and all points are shown in Table 2.

Iterations of the algorithm for obtaining t* and c_{ij} are shown in Table 3. In Table 3, for each i only the minimum of c_{ij} and the relative j are shown.

Values of c_{ij} for all modules and all points in the final iteration of the algorithm are shown in Table 4.

Now, we obtain the primary OPP of each module as:

$$OPP_i = \{j | c_{ij} = \min\{c_{ij}\}\} \quad i = 1, 2, \dots, n \tag{4}$$

So, in the mentioned example we have: OPP₁ = 6, OPP₂ = 3, OPP₃ = 6, OPP₄ = 4, OPP₅ = 3, OPP₆ = 7, OPP₇ = 5, OPP₈ = 2. And the range that the OPP of each module can be located in it (i.e., [a, b]) is determined as:

$$a = \min\{OPP_i\}, \quad b = \max\{OPP_i\} \tag{5}$$

Table 1: Inventory holding cost of modules at points (h_{ij})

Modules	Points						
	1	2	3	4	5	6	7
1	0.15	0.20	0.25	0.30	0.35	0.40	0.50
2	0.10	0.15	0.20	0.25	0.30	0.45	0.60
3	0.20	0.26	0.35	0.50	0.60	0.70	0.80
4	0.07	0.12	0.20	0.24	0.36	0.50	0.58
5	0.10	0.16	0.25	0.40	0.55	0.80	1.00
6	0.15	0.30	0.50	0.70	0.76	0.80	0.90
7	0.10	0.16	0.21	0.25	0.30	0.50	0.60
8	0.20	0.28	0.31	0.34	0.40	0.44	0.50

Table 2: Amount of delay in delivery that occurs by positioning the module i in point j (R_{ij})

Modules	Points						
	1	2	3	4	5	6	7
1	26	22	15	12	10	9	8
2	25	21	19	24	20	16	13
3	23	21	18	14	8	6	5
4	26	22	15	12	10	9	8
5	25	21	19	24	20	16	13
6	28	26	24	21	18	13	10
7	14	11	8	5	3	2	1
8	18	14	19	14	20	14	21

Table 3: Iterations of the algorithm for obtaining t* and c_{ij}

Iteration	Minimum c _{ij} for each i								
	i=1	2	3	4	5	6	7	8	t*
0	-	-	-	-	-	-	-	-	19.5
1	223.9(j=6)	288.7(3)	246.9(6)	225.7(4)	297.5(3)	322.1(7)	146.3(5)	252.8(2)	19.9
2	221.3(6)	288.6(3)	240.3(6)	224.8(5)	296.8(3)	312.9(7)	145.0(5)	251.7(2)	20.1
3	220.1(6)	288.7(3)	237.1(6)	223.8(5)	296.5(3)	308.5(7)	144.4(5)	251.2(2)	20.1

So, for the above example we have: $a = 2$ and $b = 7$. At the end of phase (2), the OPP of all modules locates in range $[a, b]$ because considering the fixed transportation costs (economies of scale) causes the OPP of some modules to shift toward the OPP of some other modules but there is no factor that would cause the OPP of a module to come out from this range.

Phase (2): determining t_i and final OPP for each module by the use of dynamic programming: Dynamic programming is an effective method to find a global optimum in some optimization problems. We now briefly describe the formulation of problem of determining t_i and final OPP for each module and the solution by dynamic programming.

- Stage (n) : Points a to b
- State (s) : Set of modules that are not already assigned to any nodes
- Decision variable : Modules that are assigned in each stage and the amount of time that they should arrive in their OPP earlier than t^*

The recursive formulation of the model in dynamic programming is as follows:

$$f_n^*(s) = \min_{(x_n, t_n)} \{P_n(x_n, t_n) + f_{n+1}^*(s - x_n)\} \quad (6)$$

where, $(s - x_n)$ shows the set of s after the assignment of set x_n of modules in stage n . The decision vector is (x_n, t_n) which contributes to the objective function by $p_n(x_n, t_n)$ that is stated by the following formulation:

$$P_n(x_n, t_n) = \begin{cases} \sum_{i \in x_n, j=n} c_{ij} + T_j & \text{if } t_n = 0 \\ \sum_{i \in x_n, j=n} c_{ij} + Q \times \left(\sum_{i \in x_n, j=n} h_{ij} \right) \times t_n & \text{if } t_n > 0 \end{cases} \quad (7)$$

As is stated by the Eq. 7 the decision (x_n, t_n) has two conditions. In both conditions the sum of c_{ij} for assigned

Table 4: Values of c_{ij} for all modules and all points in the final iteration of the algorithm

c _{ij} modules	Points						
	1	2	3	4	5	6	7
1	350.9	318.7	256.5	234.4	222.2	220.1	225.7
2	333.0	300.9	288.7	346.5	314.4	297.9	291.4
3	328.7	318.1	302.2	285.7	241.4	237.1	242.8
4	338.3	306.2	248.7	225.0	223.8	235.7	238.3
5	333.0	302.4	296.5	370.1	353.6	352.8	354.2
6	370.9	374.4	385.7	387.1	366.5	322.8	308.5
7	223.0	202.4	180.3	156.5	144.4	165.7	171.4
8	278.7	251.2	305.9	260.7	330.1	276.3	355.7

modules must be calculated. But between fixed transportation costs and inventory holding cost of type II, only one will occur. In the case of $t_n = 0$, modules that are assigned to point n , must arrive there exactly at t^* . In another words, there is a consignment to this point which involves the fixed transportation cost represented by T_j . But if these modules are transported by a consignment that its destination is one of the points after n , there would be no fixed transportation costs but instead the inventory holding cost of type II is involved. the proposed dynamic programming is explained more clearly in an example problem, with two modules and three points.

Stages of dynamic programming in a backward method are shown at Table 5-8:

So, $(x_1, t_1) = (\{1\}, 4)$, $x_2 = \emptyset$ and $x_3 = 2$. It means that only one consignment consisting of both modules must go to point 3. It delivers the module 1 to point 1, 4 days before t^* and delivers the module 2 to point 3 at t^* . The cost of the optimal solution is 35 which consist of three parts. Fixed transportation cost is equal to 12 units, inventory holding cost (type II) is 4 units and finally $8+11 = 19$ units for c_{ij} .

Numerical example: Here, it will solve the mentioned example that had 8 modules and 7 points, by using the model that was presented in phase (2). The required data are: inventory holding cost, that are shown in Table 1, c_{ij} that obtained in phase (1) and are shown in Table 4. Also values of a and b that are 2 and 7, respectively. Also

Table 5: Values of I_j and T_j for point a to b and c_{ij} for modules and points in dynamic programming of the example problem

j	1	2	3
1	8	18	21
2	26	19	11
T_j	10	11	12
I_j	-	2	3

Table 6: Stage 3 of dynamic programming of the example problem (n = 3)

s	$f_n^*(s)$	(x_n^*, t_n^*)
\emptyset	0	\emptyset
{1}	21+12 = 33	{1}, 0
{2}	11+12 = 23	{2}, 0
{1,2}	32+12 = 44	{1, 2}, 0

Table 7: Stage 2 of dynamic programming of the example problem (n = 2)

x _n s	$f_n(s) = P_n(x_n, t_n) + f_{n+1}^*(s - x_n)$						$f_n^*(s)$	(x_n^*, t_n^*)
	t _n = 0			t _n = 2				
\emptyset	0						0	\emptyset
{1}	33	18+11	33				29	({1}, 0)
{2}	23		19+11				23	\emptyset
{1,2}	44	11+23	11+23	19+11	2+23	2+23	43	(({1}, 2)
		+18	+19	+18	+18	+19		

Table 8: Stage 1 of dynamic programming of the example problem (n = 1)

$$f_n(s) = P_n(x_n, t_n) + f_{n+1}^{*}(s - x_n)$$

x_n	$t_n = 0$				$t_n = 2$		$t_n = 4$		$f_n^*(s)$	(x_n^*, t_n^*)
	\emptyset	{1}	{2}	{1,2}	{1}	{2}	{1}	{2}		
{1,2}	43	10+23+8	10+29+26	8+26+10	8+2+23	26+2+29	8+4+23	26+4+33	35	{1},4

Table 9: Values of l_j and T_j for point a to b

j	2	3	4	5	6	7
l_j	0	2	2	3	4	2
T_j	30	40	55	60	70	80

Table 10: Values of decision variables (x_{ij} and t_i and y_j for all modules and all points)

x_{ij}	j							t_i
	2	3	4	5	6	7		
1	0	0	0	0	1	0	0	
2	0	0	0	0	1	0	0	
3	0	0	0	0	1	0	0	
4	0	0	0	0	1	0	0	
5	1	0	0	0	0	0	0	
6	0	0	0	0	1	0	0	
7	0	0	0	0	1	0	0	
8	1	0	0	0	0	0	0	
y_j	1	0	0	0	1	0	Z = 2132.9	

Table 11: Iterations of the algorithm for obtaining the final t^* and other decision variables (x_{ij} and t_i and y_j) i and (j) that $x_{ij} = 1$

Iteration	i = 1								t^*
	j = 6	2	3	4	5	6	7	8	
0	6	6	6	2	6	6	2	19.7	
1	5	2	5	5	2	5	5	17.7	
2	5	2	5	5	2	2	5	15.7	
3	5	2	5	5	2	2	5	15.7	

suppose that time distance between points and summation of fixed transportation costs until different points being as shown in Table 9. And note that l_j for first point always equals to zero.

Now, we solve the presented model for the above inputs and obtain the values for decision variables that are shown in Table 10.

Obtained OPPs in Table 6 are not equal to primary OPPs. So, we must compute the t^* from equation 3 for modules and points in Table 6 where, x_{ij} is equal to 1. Finally we propose the following algorithm:

- Step 1:** Solve equation (3) for t^* and consider the modules and points that x_{ij} for them is equal to 1.
- Step 2:** Consider the current t^* and solve Eq. 1 for c_{ij} and all modules and all points
- Step 3:** Solve the mathematical model by considering the current c_{ij} (all i and all j) and go to step 1.
- Step 4:** while t^* in two consecutive stages is not fixed, repeat steps 1 to 3.

Iterations of the above algorithm for the mentioned example are shown in Table 11.

So, the final values of decision variables are:

$$t^* = 15.7$$

$$y_1 = y_3 = y_4 = y_6 = y_7 = 0 \text{ and } y_2 = y_5 = 1$$

$$t_1 = t_2 = t_3 = t_4 = t_5 = t_6 = t_7 = t_8 = 0$$

$$x_{15} = x_{35} = x_{45} = x_{75} = 1 \text{ and } x_{22} = x_{52} = x_{62} = x_{82} = 1$$

$$z = 2366.6$$

CONCLUSION

Summary: we decomposed the problem of determining the OPP points for modules into two different phases. In phase (1), we only considered the delay in delivery costs and type I of inventory holding costs. And then in phase (2), we used the output of phase (1) as some of inputs and also in this phase we considered the transportation costs and the type II of inventory holding costs that may occur for saving in transportation costs. We modeled this problem that had three cost elements as a dynamic programming model and solved a numerical example and presented its results.

Future research: This research has the ability to be extended in two directions. First is the elimination of some assumptions that causes the creation of new problems 4 issues of these problems are:

- In this problem, we assumed that the demand is deterministic, whereas in real problems it may be stochastic.
- We modeled and solved this problem for export to one country. If we want to consider multi countries, the supply chain becomes a divergent chain and the complexity of the problem would be increased. In other words, we supposed that there is only one customer (target country) and as a result, the chain is linear, whereas it may be multiple customers and the chain as divergent.
- The case that there are multiple customers and also their demand being stochastic is a complex problem that study on it could be very attractive.
- If the time between two sequential customer orders is short, it is better that the amount of held inventory in OPP point to be as a multiple of Q, i.e., batch sizes in points before OPP in the chain must be larger than batch sizes in points after the OPP. This case is very close to real supply chains.

Another direction that we can extend this research is related to the solving method. We modeled this problem by mathematical programming, whereas we can model this problem by dynamic programming. But the efficiency of dynamic programming for this problem is very low, because by increasing the number of modules, the number of states and decision variables in each stage would be increased exponentially. If we could limit the number of states or decision variables, this method could be an appropriate technique for this problem.

If the dimensions of problem become large enough, we must use the metaheuristic algorithms such as GA, SA, TS and also a combination of these algorithms can be used. Modeling and solving the problem by each of these methods and then a comparison of them in the light of obtained solutions and time solution by computer could be a good research.

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