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A Control Theoretic Approach for Congestion Control of Multi-Hop Wireless Computer Networks

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Abstract: Congestion control in different network architectures implies different formulations and requirements. In this study, a control theory based approach has been employed to formulate and solve the congestion control problem in multi-hop wireless networks. Both wireless networks with ring and tree topologies will be considered in this paper. Such networks will be formulated in general form while the wireless interference in the system is considered. For the tree topology a multi-level wireless access network in its general form is considered. Closed form model equations will be derived and next, by linearizing the resulting nonlinear model equations, controller designing problem will be dealt with. Simulation results are representative of the controller designing procedure and its performance evaluation for some typical multi-hop wireless networks.

Key words: Wireless multi-hop networks, wireless interference, IEEE 802.11, tree topology, active queue management

INTRODUCTION

Congestion in computer networks occurs when there are excessive needs for a limited network resource such as routers out-link bandwidth and leads to aggressive losses of data, resource under-utilization and even congestion collapse. Therefore, congestion control algorithms are employed to manage the entire network in order for achieving two main goals: (1) to prevent the occurrence of congestion or overcome it, if it occurs; (2) to maximize the throughput of the network. To meet these purposes, congestion control has been implemented as a distributed control strategy, in which the Transmission Control Protocol (TCP) variants in the end users control their sending rates based on the feedback signals from the network congestion status and the link algorithms in the routers supply the end users with some explicit/implicit congestion notifications through marking/dropping of packets.

Over the past few years, the problem of congestion control has received wide-spread attention, both in the Internet context (Hollot *et al.*, 2002; Kelly *et al.*, 1998; Low, 2003; Low *et al.*, 2002) as well as in an wireless and ad-hoc network context (Yi and Shakkottai, 2007; Raghunathan and Kumar, 2007). For wired networks, there are lots of control-theory based methods developed for congestion control problem (Bigdeli and Haeri, 2005, 2007; Fengyuan *et al.*, 2002; Hollot *et al.*, 2002; Jain *et al.*, 2004;

Ryu *et al.*, 2003; Sun *et al.*, 2003; Yan *et al.*, 2005), which try to solve this problem as an Active Queue Management (AQM) method. AQM methods are those link algorithms that determine their packet marking/dropping probabilities based on the queue length in the router buffer. Design of proper algorithms for marking/dropping of packets in a router even in wired networks is an open problem yet. Besides, some optimization methods for distributed resource allocation in such networks have been proposed. These methods are based on some pricing algorithms which try to maximize network utilization as well as fairness in the network (Kunniyur and Srikant, 2003; Kelly *et al.*, 1998; Low, 2003).

For wireless and especially ad-hoc networks, the related researches are mostly directed towards the adaptation of the end-to-end control schemes such as TCP to ad-hoc networks. Some other works try to extend the optimization problem of (Kelly *et al.*, 1998) to wireless/ad-hoc networks (Raghunathan and Kumar, 2007; Yi and Shakkottai, 2007).

In this study, we attempt to formulate the hop-by-hop congestion control of wireless multi-hop ad-hoc networks from a control theoretic view point. Nodes in such networks are radio-equipped and communicate by broadcasting over wireless links. Communication paths between nodes which are not in radio range of each other are established by intermediate nodes acting as relays to forward data toward the destination. The diverse

applications of such networks range from community based roof-top networks to large-scale ad-hoc networks (Yi and Shakkottai, 2007).

Here, we consider multi-hop wireless networks with both ring and tree topologies and try to develop a frame for AQM designing for such networks. To do so, the main characteristic of such networks should be considered. In a wireless context, an important constraint that arises is that the wireless channel is a broadcast medium and transmissions by nearby nodes cannot proceed simultaneously. These constraints introduce various forms of interference i.e.,

- **Self-interference:** This occurs when a flow's transmission on a hop interferes with its transmission on the previous and next hops
- **Inter-flow interference:** This occurs when flows with no common link or node still interfere with each other

These interference constraints result in correlations between the instantaneous capacities of wireless links and introduce a spatial nature to congestion in wireless networks. In contrast, wired networks can be accurately modeled by a graph model with independent capacity constraints on the links. The absence of link interference and the independent nature of the capacity constraints in wired networks ensure that:

- A flow can obtain the needed congestion feedback information from just links along its own path (and no other links)
- Conversely, the links along a flow's path are the only links whose congestion is affected by the flow's traffic. No other links are affected by a flow's traffic

It turns out that these two facts are crucial to the stability of wire-line Internet congestion control mechanisms (Raghunathan and Kumar, 2007). Such a wired graph model with link capacity constraints is however not an accurate model of wireless networks, where the congestion in a spatial neighborhood interferes with the directed edges along a flow's path. More precisely, we observe that:

- As in wired networks, with common TCP, a flow can only obtain congestion feedback from every directed link along its path and no other
- However, nearby links not directly on the path can interfere with this flow, since congestion in wireless networks is of a spatial nature. Thus, traffic at nearby links affects the congestion at directed links along a flow's path. Conversely, a flow's traffic can affect links which are near a flow, but not on the flow's route

In this study, the model of wireless networks with ring and tree topologies is derived in such a way that considers the interferences in the network IEEE 802.11 physical carrier sense model (Raghunathan and Kumar, 2007) is used for representing the interferences in the networks. Afterwards, the general AQM controller will be formulated for these types of networks, separately. For the tree topology a multi-level wireless access network in its general form is considered. The resulting models are sets of nonlinear equations. By linearizing the resulting equations, small signal models will be obtained for these networks which may be used for controller designing purposes. Controller designing procedure and its performance evaluation via simulations are also considered in this study. As it will be seen, simulation results are representative of good performance of the developed method for both ring and tree topologies.

MATERIALS AND METHODS

TCP/AQM model of wireless networks: In order for describing the wireless networks the modified Kelly's *et al.* (1998) model that includes interference constraints (Raghunathan and Kumar, 2007) will be used in this study. Consider a network with a set L of links and a set V of vertices. For this network, there exists a set S containing all flows in the network and a route matrix A which is defined in following manner: $A_{sj} = 1$ if flow s passes through link j and $A_{sj} = 0$ otherwise. Then the modified Kelly's model describing the dynamics of rate x_s allocated to user s at time t can be represented as:

$$\frac{d}{dt}x_s(t) = K(\omega_s - x_s(t) \sum_j A_{sj} p_j(y_j(t))) \quad (1)$$

where, K is a constant, ω_s is the desired window size of TCP or the willingness to pay for user s and $p_j(y_j(t))$ is the packet marking (with enabled ECN) probability/price function of link j . The aggregate input rate $y_j(t)$ is defined as:

$$y_j = \sum_s B_{js} x_s \quad (2)$$

where, $B = TA$ is the flow-interference matrix. The adjacency matrix T is determined by IEEE 802.11 physical carrier sense (Raghunathan and Kumar, 2007) and is a $V \times V$ matrix that captures the interference constraints. This matrix is dependent to the structure of the network i.e., its topology and the relative location of nodes with respect to each other. This the main difference of wireless and wired network, as in wired networks, the matrix B in Eq. 2 is replaced with A . In this context, two important

topologies that are to be considered are the ring and tree topologies which are representative of ad-hoc and access wireless networks, respectively.

In analysis of network behavior, there are two approaches about dealing with the marking function $p(\cdot)$. In the first approach known as the Kelly's approach, $p(\cdot)$ is considered as a non-decreasing price function which is an indicator of time. The goal in era is to find this function such that the best overall fairness is achieved. A good study about the performance of common price functions in wireless networks may be found in (Ragunathan and Kumar, 2007). In the second approach, the marking function $p(\cdot)$ is considered as a congestion notification signal feeding back to the senders to prevent/overcome congestion in the network. This function represents the probability of marking packets. This approach has been widely used for wired networks (Bigdeli and Haeri, 2005; Bigdeli and Haeri, 2007; Fengyuan *et al.*, 2002; Holot *et al.*, 2002; Jain *et al.*, 2004; Ryu *et al.*, 2003; Sun *et al.*, 2003; Yan *et al.*, 2005), but for wireless networks it has been used in some extent in (Yi and Shakkottai, 2007) for wireless access multi-hop networks. In this study, we will generalize this idea for both ring and tree topologies and represent a framework for designing congestion controllers based on control theory.

For the congestion control problem, $p(\cdot)$ in a node should be defined as a function of congestion status in that node. That is, in a congested node, the rate of input data is more than its out-link capacity. Therefore, in the entrance of link it forms a queue whose length is indicator of the congestion status. Therefore, for congestion control problem, Eq. 1 can be represented as:

$$\frac{d}{dt}x_s(t) = \kappa(\omega_s - x_s(t) \sum A_{js} p_j(q_j(t))) \quad (3)$$

where, the queue length $q_j(t)$ follows the dynamics (Yi and Shakkottai, 2007):

$$\dot{q}_j = \begin{cases} -C_{i,j} + \sum_s B_{js} x_s & \text{if } q_j > 0 \\ \max(0, -C_j + \sum_s B_{js} x_s) & \text{if } q_j = 0 \end{cases} \quad (4)$$

where, $C_{i,j}$ stands for the out-link capacity of link j in packets sec^{-1} . Note that in Eq. 3 and 4, for simplicity the effect of fading and channel loss has been ignored. It however, does not dramatically affect congestion control because of employing ECN which considers packet marking instead of packet dropping.

In the following sections, at first we use the general Eq. 3 and 4 for formulating congestion control problem for the ring and tree topologies. Linearizing the derived

equations, results in transfer functions which are then used for designing congestion controllers based on control theory approaches.

Ring topology: Here, the Eq. 3 and 4 will be expanded for a network of N_d nodes forming a ring topology as in Fig. 1. Suppose each node of the network generate a forward flow with N_i hops. Besides, let all the links be identical with the same capacity and drop rate. Then using IEEE 802.11 physical carrier sense for modeling the interference in the network is represented as:

IEEE 802.11 physical carrier sense (Ragunathan and Kumar, 2007): IEEE 802.11 physical carrier sense prevents a node from transmitting if the received energy from any other transmission in its spatial neighborhood is greater than a certain threshold, called the carrier-sensing threshold. The typical value of this threshold results in a

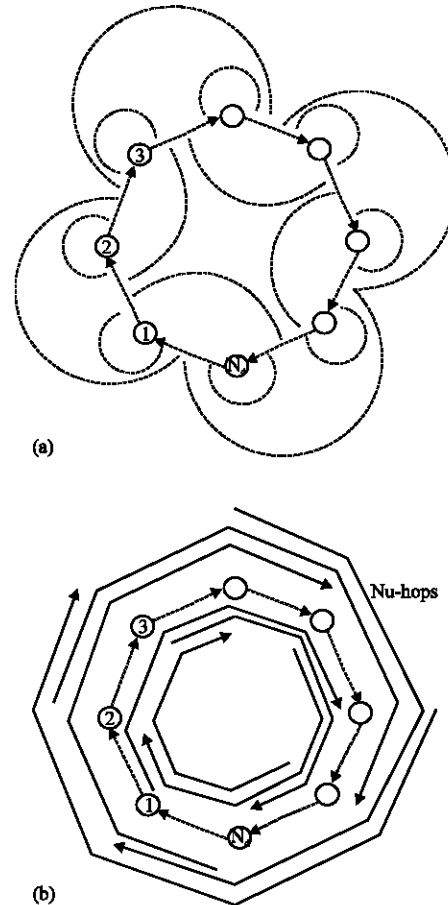


Fig. 1: Wireless network and flow pattern for ring network. (a) The dashed lines indicate interference from the two-hop neighborhood using the IEEE 802.11 physical carrier sense model and (b) The solid lines indicate the flows

IEEE 802.11 carrier sensing range of two hops. In other words, $T_{ij} = 1$, if the respective transmitters at the head of directed edges i and j are within two hops of each other.

Therefore, the $N_n \times N_n$ matrices A, T and B will be derived as:

$$\begin{aligned} T &= \text{Toeplitz}(\underline{u}, \underline{u}^T) \\ A &= \text{Toeplitz}(\underline{v}, \underline{w}) \\ B &= TA \end{aligned} \quad (5)$$

where, Toeplitz $(\underline{m}, \underline{n})$ stands for a Toeplitz matrix with the first row and the first column of vectors \underline{m} and \underline{n} respectively. Beside, in Eq. 5, the vectors $\underline{u}, \underline{v}$ and \underline{w} are defined as:

$$\begin{aligned} \underline{u} &= [1 \ 1 \ 1 \ 0 \ \dots \ 0 \ 1 \ 1] \quad \text{and,} \\ \underline{v} &= \left[\underbrace{1 \ 1 \ \dots \ 1}_{N_1} \ \underbrace{0 \ 0 \ \dots \ 0}_{N_d - N_1} \right], \quad \underline{w} = \left[\underbrace{1 \ 0 \ 0 \ \dots \ 0}_{N_d - N_1} \ \underbrace{1 \ 1 \ \dots \ 1}_{N_1 - 1} \right]^T \end{aligned} \quad (6)$$

Considering Eq. 5 and 6, it is observed that each column of matrix A is a permutation of vector \underline{v} and so contains N_1 ones. Therefore, the dynamics of homogeneous flow of Eq. 3 will be reduced to:

$$\frac{d}{dt} x_s(t) = \kappa(\omega_s - N_1 x_s(t) p(q_s(t))) \quad (7)$$

where, $p(q_s(t))$ stands for the marking probability of queue s . In order to simplify Eq. 4, consider Theorem 1 as follows:

Theorem 1: Consider a network with topology with N_d nodes and forward flow of N_1 hops. If the matrices A, T and B are defined as in Eq. 5 and 6, then we have:

$$F(j, N_1, N_d) = F(N_1, N_d) = \sum_s B_{js} = 5N_1, \quad \{ \forall j, \forall N_d, N_1 | N_d \geq N_1 \geq 5 \} \quad (8)$$

Proof: Let \underline{t}_i and \underline{a}_i be the i th rows of matrices A and T , correspondingly and $T_{i,j}$ and $A_{i,j}$ be the corresponding elements of these matrices. Considering the definition of matrix B , $F(j, N_1, N_d)$ can be written as:

$$F(j, N_1, N_d) = \sum_s B_{js} = \sum_s \underline{t}_j \underline{a}_s^T = \sum_s \left(\sum_k T_{j,k} A_{k,s} \right) = \sum_k \left(T_{j,k} \times \sum_s A_{k,s} \right) \quad (9)$$

But, from Eq. 5 each row of matrix A comes from a permutation of vector \underline{v} . Therefore, from the definition of vector \underline{v} in Eq. 6, we have:

$$\sum_s A_{k,s} = N_1 \quad (10a)$$

Which is independent of N_d . Therefore, from Eq. 10a and the definition of matrix T (Eq. 5, 6), we have:

$$F(j, N_1, N_d) = \sum_k \left(T_{j,k} \times \sum_s A_{k,s} \right) = N_1 \sum_k T_{j,k} = 5N_1 \quad (10b)$$

And the proof is completed.

Considering Theorem 1, Eq. 4 will be simplified as:

$$\dot{q}_s = \begin{cases} -C_{1,s} + 5N_1 x_s & \text{if } q_s > 0 \\ \max(0, -C_{1,s} + 5N_1 x_s) & \text{if } q_s = 0 \end{cases} \quad (11)$$

Eq. 7 and 11 are a set of nonlinear equations which describe the network dynamics. In order to design a congestion controller, one should design the function $p(q_s(t))$ to obtain desired behavior. Assuming the data nodes always have enough data to send, the desired behavior may be defined as regulating the queue length about a desired value. This method is the well-known Active Queue Management Method (AQM) for congestion management. In this way, not only the link is fully utilized, but also a controllable queuing delay as well as small jitter is achieved. For this purpose, we linearize the system dynamics about its operating point to derive input/output transfer function.

Linearization: The equilibrium point (x_e, p_e, q_e) of (7) and (11) is defined by:

$$\begin{aligned} \dot{x}_s &= 0 \rightarrow w = N_1 x_s \times p_s(q_s) \\ \dot{q}_s &= 0 \rightarrow x_s = \frac{C_{1,s}}{5N_1} \end{aligned} \quad (12)$$

Linearizing the nonlinear model about this equilibrium point results in:

$$\begin{aligned} \delta \dot{x}_s &= \frac{\partial x_s}{\partial x_s} \bigg|_Q \delta x_s + \frac{\partial x_s}{\partial p} \bigg|_Q \delta p = -N_1 p_e(q_e) \delta x_s - N_1 x_e \delta p \\ \delta \dot{q}_s &= \frac{\partial q_s}{\partial x_s} \bigg|_Q \delta x_s = 5N_1 \delta x_s \end{aligned} \quad (13)$$

where, δx_s , δq_s , and δp are the corresponding small signal perturbations as:

$$\delta x_s(t) = x_s(t) - x_e, \quad \delta q_s(t) = q_s(t) - q_e \quad \text{and} \quad \delta p(t) = P(t) - p_e$$

The rate and queue transfer functions therefore are:

$$P_{\text{rate}}(s) = \frac{\delta x_s(s)}{\delta p(s)} = \frac{-N_1 x_e}{s + \frac{w}{x_e}} = \frac{-N_1 x_e}{s + N_1 p_e}, \quad P_{\text{que}}(s) = \frac{\delta q_s(s)}{\delta x_s(s)} = \frac{5N_1}{s} \quad (14)$$

Therefore, small signal transfer function of the system can be represented by:

$$P_{oi}(s) = \frac{\delta q_s(s)}{\delta p(s)} = -\frac{5N_i^2 x_s}{s(s + N_i p_s)} \quad (15)$$

Tree topology: Here, we deal with tree topologies with wireless access structure. Consider Fig. 2, in which there are a number of wireless access points which are connected to each other through a wired network. Connected to each access point, there are N_s nodes which are similarly connected to N_s wireless. This hierarchical structure with the same number of branches in each intermediate node has been assumed in order to keep some types of homogeneity in the network.

Here, our problem is to formulate the congestion control problem in each access point. For this purpose, consider Fig. 3 in which a wireless point with the above-mentioned branch structure and numbered nodes has been shown. Considering each flow starting from identical nodes in Level 0, passing through the corresponding Level 1 node as shown in Fig. 3 towards the access node, then the matrix A can be written as:

$$A_s = \begin{cases} \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} & \text{for } s \leq N_s^2 \\ \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} & \text{for } N_s^2 < s \leq N_s^2 + N_s \\ \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix} & \text{for } s = N_s^2 + N_s + 1 \end{cases} \quad (16)$$

where, in Eq. 16, A_s is the row of matrix and $l_s = \text{int}(s/N_s)$ stands for the integer value s of over N_s . Correspondingly, considering the IEEE 802.11 carrier sense method, the matrix T can be written as:

$$T_s = \begin{cases} \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & 1 \end{bmatrix} & \text{for } s \leq N_s^2 \\ \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 \end{bmatrix} & \text{for } N_s^2 < s \leq N_s^2 + N_s \\ \begin{bmatrix} 1 & 1 & \dots & 1 \end{bmatrix} & \text{for } s = N_s^2 + N_s + 1 \end{cases} \quad (17)$$

And $B = TA$ can be written as:

$$B_s = \begin{cases} \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & N_s + 1 & 0 & 0 & \dots & 0 & N_s + 2 \end{bmatrix} & \text{for } s \leq N_s^2 \\ \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & N_s + 1 & 0 & 0 & \dots & 0 & 2N_s + 1 \end{bmatrix} & \text{for } N_s^2 < s \leq N_s^2 + N_s \\ \begin{bmatrix} 1 & 1 & \dots & 1 & N_s + 1 & N_s + 1 & \dots & N_s + 1 & N_s^2 + N_s + 1 \end{bmatrix} & \text{for } s = N_s^2 + N_s + 1 \end{cases} \quad (18)$$

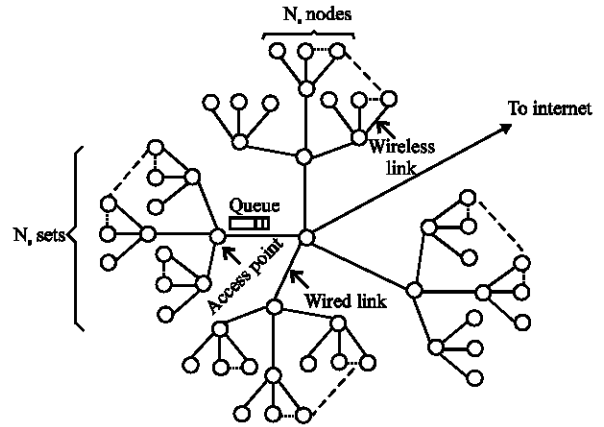


Fig. 2: Wireless access network structure; dark lines: wired links and light lines: wireless links

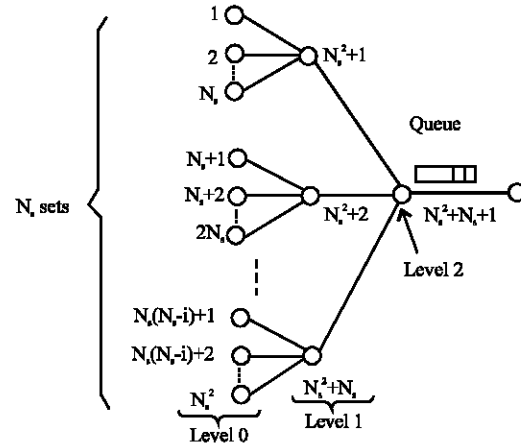


Fig. 3: A wireless access point with numbered nodes

Considering symmetry in the network, we have:

$$\begin{cases} x_1 = x_2 = \dots = x_{N_s^2} \\ x_{N_s^2+1} = x_{N_s^2+2} = \dots = x_{N_s^2+N_s} \end{cases} \quad (19)$$

From Eq. 16-19, Eq. 1 for level 0 nodes can be represented as:

$$\frac{d}{dt} x_1(t) = \kappa(\omega_1 - x_1(t)P_1(t)) \quad (20)$$

Where:

$$P_1(t) = P_1(q_1(t)) \triangleq P(N_s x_1(t) + (N_s + 1)x_{N_s^2+1}(t) + (N_s + 2)x_{N_s^2+N_s+1}(t))$$

Next, Eq. 1 for level 1 nodes can be written as:

$$\frac{d}{dt} x_{N_s^2+1}(t) = \kappa[\omega_{N_s^2+1} - x_{N_s^2+1}(t)(N_s P_1(t) + P_2(t))] \quad (21)$$

Where:

$$P_2(t) = P_2(q_2(t)) \triangleq p(N_s x_1(t) + 2N_s x_{N_s^2+1}(t) + (2N_s + 1)x_{N_s^2+N_s+1}(t))$$

Similarly, for the Level 2 access node, the sending rate Equation is:

$$\frac{d}{dt} x_{N_s^2+N_s+1}(t) = \kappa[\omega_{N_s^2+N_s+1} - x_{N_s^2+N_s+1}(t)(N_s^2 P_1(t) + N_s P_2(t) + P_3(t))] \quad (22)$$

Where:

$$P_3(t) = P_3(q_3(t)) \triangleq p(N_s^2 x_1(t) + (N_s^2 + N_s)x_{N_s^2+1}(t) + (N_s^2 + N_s + 1)x_{N_s^2+N_s+1}(t))$$

The queue lengths in different nodes follow the dynamics of Eq. 4 (Yi and Shakkottai, 2007). Let us denote the queue length in levels 0, 1 and 2, as q_0 , q_1 and q_2 , respectively, then we have:

$$\begin{aligned} \dot{q}_0 &= (-C_{l1} + \sum_s B_{l,s} x_s)^+ \\ &= (-C_{l1} + [N_s x_1 + (N_s + 1)x_{N_s^2+1} + (N_s + 2)x_{N_s^2+N_s+1}])^+ \\ \dot{q}_1 &= (-C_{l,N_s^2+1} + \sum_s B_{N_s^2+1,s} x_s)^+ \\ &= (-C_{l,N_s^2+1} + [N_s x_1 + 2N_s x_{N_s^2+1} + (2N_s + 1)x_{N_s^2+N_s+1}])^+ \\ \dot{q}_2 &= (-C_{l,N_s^2+N_s+1} + \sum_s B_{N_s^2+N_s+1,s} x_s)^+ \\ &= (-C_{l,N_s^2+N_s+1} + [N_s^2 x_1 + (N_s^2 + N_s)x_{N_s^2+1} + (N_s^2 + N_s + 1)x_{N_s^2+N_s+1}])^+ \end{aligned} \quad (23)$$

where, $(a)^+ = \max(0, a)$ and $C_{i,j}$ stands for the out-link capacity of j th link. Due to the hierarchical and symmetrical structure of considered tree network and in order to simplify the above equations, one can assume P_1 , P_2 and P_3 , are linear functions with the following relationships:

$$P_1 = \frac{1}{N_s} P_2 = \frac{1}{N_s^2} P_3 \quad (24)$$

Then, congestion can be controlled via proper feedback from the access node of number $N_s^2 + N_s + 1$.

Linearization: In order for deriving the linear model of system around its operating point, at first, the equilibrium point of system dynamics should be derived. For this purpose, let us consider the network is in the fair operating regime, where:

$$x_1 = x_2 = \dots = x_{N_s^2+N_s} = x_{N_s^2+N_s+1} = x_e \quad (25)$$

Therefore, if the bottleneck link be the access link of capacity $C_{l,N_s^2+N_s+1}$, then the equilibrium point of the network will be derived as:

$$q_2 = 0 \rightarrow -C_{l,N_s^2+N_s+1} + [N_s^2 + (N_s^2 + N_s) + (N_s^2 + N_s + 1)]x_e = 0$$

Therefore,

$$x_e = \frac{C_{l,N_s^2+N_s+1}}{3N_s^2 + 3N_s + 1} \quad (26)$$

And

$$\begin{aligned} \dot{x}_1 = 0 &\rightarrow \omega_1 = x_e P_{1e} = x_e \frac{P_{3e}}{N_s^2} \\ \dot{x}_{N_s^2+1} = 0 &\rightarrow \omega_{N_s^2+1} = x_e [N_s P_{1e} + P_{2e}] = 2x_e \frac{P_{3e}}{N_s} \\ \dot{x}_{N_s^2+N_s+1} = 0 &\rightarrow \omega_{N_s^2+N_s+1} = x_e [N_s^2 P_{1e} + N_s P_{2e} + P_{3e}] = 3x_e P_{3e} \end{aligned} \quad (27)$$

where, P_{1e} , P_{2e} and P_{3e} are the equilibrium values of $P_1(t)$, $P_2(t)$ and $P_3(t)$. Then linearizing the nonlinear model about this equilibrium point results in:

$$\delta \dot{x}_1(t) = \frac{\partial \dot{x}_1}{\partial x_1} \bigg|_Q \delta x_1 + \frac{\partial \dot{x}_1}{\partial P_1} \bigg|_Q \delta p_1 = -P_{1e} \delta x_1 - x_e \delta p_1 = -\frac{P_{3e}}{N_s^2} \delta x_1 - \frac{x_e}{N_s^2} \delta p_3 \quad (28)$$

$$\begin{aligned} \delta \dot{x}_{N_s^2+1}(t) &= \frac{\partial \dot{x}_{N_s^2+1}}{\partial x_{N_s^2+1}} \bigg|_Q \delta x_{N_s^2+1} + \frac{\partial \dot{x}_{N_s^2+1}}{\partial P_1} \bigg|_Q \delta p_1 + \frac{\partial \dot{x}_{N_s^2+1}}{\partial P_2} \bigg|_Q \delta p_2 \\ &= -(N_s P_{1e} + P_{2e}) \delta x_{N_s^2+1} - x_e (N_s \delta p_1 + \delta p_2) \\ &= -\frac{2P_{3e}}{N_s} \delta x_{N_s^2+1} - \frac{2x_e}{N_s} \delta p_3 \end{aligned} \quad (29)$$

(30)

Analogously, for the bottleneck queue we have:

(31)

Where:

$$\begin{aligned} \delta x_1(t) &= x_1(t) - x_e, \quad \delta x_{N_1^2+1}(t) = x_{N_1^2+1}(t) - x_e, \quad \delta x_{N_1^2+1}(t) = x_{N_1^2+1}(t) - x_e, \\ \delta p_1(t) &= P_1(t) - p_{1e}, \quad \delta p_2(t) = P_2(t) - p_{2e}, \quad \delta p_3(t) = P_3(t) - p_{3e} \quad \text{and} \\ \delta q_s(t) &= q_s(t) - q_e \end{aligned}$$

These are the corresponding small signal perturbations. Taking Laplace transform we have:

(32)

(33)

(34)

And so the open loop transfer function will be:

(35)

Considering Eq. 35, the dominant non-zero pole of this system can be considered at $s = -P_{3e}/N_g^2$.

Controller designing: The block diagram of the above-mentioned ring and tree wireless topologies in addition to AQM controller is depicted in Fig. 4. In Fig. 4, q stands for the desired queue to be controlled and P is the corresponding marking probability. From this figure, one should design proper AQM controllers for the linearized system models of Eq. 15 or 35. The performance of

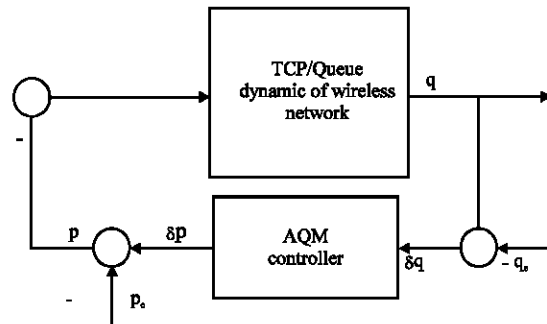


Fig. 4: Block diagram of wireless network in conjunction with an AQM controller

designed controllers then can be evaluated via simulations of the nonlinear system in addition to the linear small signal controller. In continue the results of designing such a controller and its performance evaluation will be presented via some examples.

RESULTS AND DISCUSSION

Here, the above-described method would be applied to two sample networks to investigate its applicability and performance. For this purpose, the considered networks have the following properties:

- For ring topology, a circle network with $N_d = 8$ nodes has been assumed that each node can generate a flow with $N_l = 5$ hops. For this network matrices A, T and B will be:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 4 & 5 & 4 & 3 & 2 & 2 & 2 \\ 2 & 3 & 4 & 5 & 4 & 3 & 2 & 2 \\ 2 & 2 & 3 & 4 & 5 & 4 & 2 & 2 \\ 2 & 2 & 2 & 3 & 4 & 5 & 4 & 3 \\ 3 & 2 & 2 & 2 & 3 & 4 & 5 & 4 \\ 4 & 3 & 2 & 2 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 2 & 2 & 3 & 4 \\ 4 & 5 & 4 & 3 & 2 & 2 & 2 & 3 \end{bmatrix}$$

For $p_e = 0.1$, $C_{1s} = 300 \text{ packets sec}^{-1}$ and $q_e = 50$ packets we have:

$$x_e = \frac{C_{1s}}{5N_1} = 12, w = N_1 x_e \times p_e(q_e) = 6$$

So, the open loop small signal transfer function is:

$$P_d(s) = \frac{\delta q_e(s)}{\delta p(s)} = -\frac{5N_1^2 x_e}{s(s + N_1 p_e)} = -\frac{1500}{s(s + 0.5)} \quad (36)$$

- For tree topology, a tree structure with $N_s = 3$ has been considered. For this network matrices A, T and B will be:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 5 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 5 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 4 & 0 & 5 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 4 & 0 & 5 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 4 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 4 & 5 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 1 & 1 & 7 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 4 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 4 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 4 & 4 & 4 & 13 \end{bmatrix}$$

For $p_{3c} = 0.13$, $C_{113} = 500 \text{ packets sec}^{-1}$ and $q_c = 50$ packets we have

$$x_e = \frac{C_{113}}{3N_s^2 + 3N_s + 1} = 13.5$$

And

$$\begin{aligned} \alpha_1 &= x_e \frac{P_{3e}}{N_s^2} = 0.2 \\ \alpha_0 &= 2x_e \frac{P_{3e}}{N_s} = 1.18 \\ \alpha_3 &= 3x_e P_{3e} = 5.3 \end{aligned}$$

So, the open loop small signal transfer function is:

$$\begin{aligned} P_d(s) &= \frac{\delta q_2(s)}{\delta p_3(s)} = -\frac{N_s^2 x_e}{s(N_s^2 s + P_{3e})} - \frac{2(N_s^2 + N_s)x_e}{s(N_s s + 2P_{3e})} - \frac{(N_s^2 + N_s + 1)3x_e}{s(s + 3P_{3e})} \\ &= -\frac{13.5}{s(s + 0.0144)} - \frac{108}{s(s + 0.87)} - \frac{526.5}{s(s + 0.39)} \end{aligned} \quad (37)$$

For the above-mentioned networks, Proportional (P) and Proportional Plus Integral (PI) controllers has been designed by the authors using well-known frequency response method. However, simulation results show that in the case of system parameter variations, P controller does not perform properly. But, PI performs robustly in spite of such variations. Therefore, the results for PI controller will be presented in following subsections.

Simulation results for ring topology: For the model of Eq. 36, PI controller has been designed and its performance has been evaluated via MATLAB simulations. For this purpose, the nonlinear equations of system model have been considered as the plan model

and the small signal controller has been designed based on the small signal transfer function of Eq. 36. Figure 5 shows the closed loop behavior of the network with its nominal parameters. As seen, the underlying parameters have converged to their desired values with a reasonable overshoot, i.e., 15% for the queue length.

In continue, the performance of the closed loop system, under network parameter variations will be evaluated. For this purpose, in Fig. 6, the bottleneck out-link capacity i.e., C_b has been changed at time 100 sec once to 250 packets sec^{-1} (dotted line) and once to 350 packets (dash line). As seen in this figure, in both case,

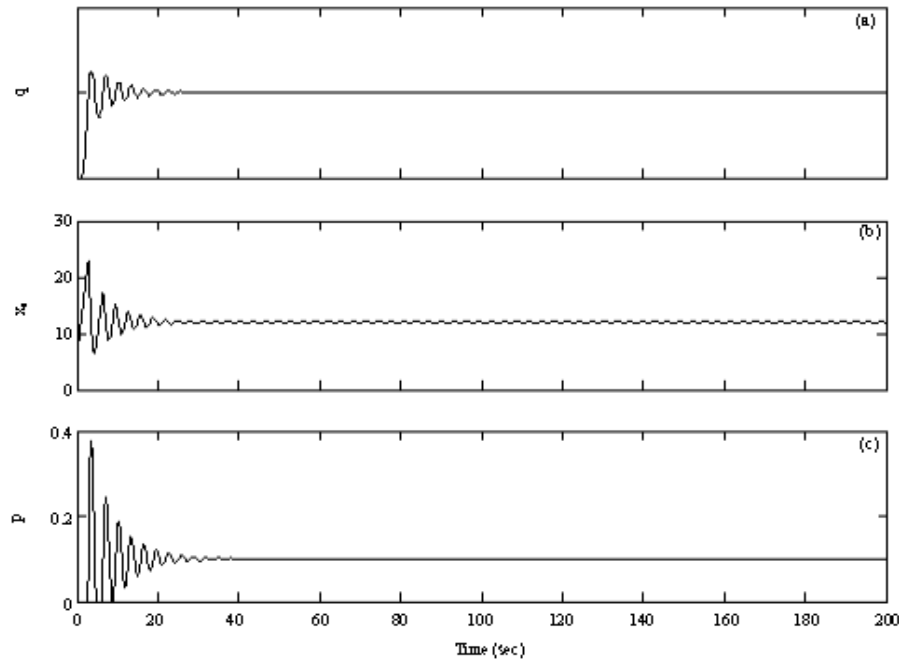


Fig. 5: Nominal response of ring network with PI controller (a) queue length (b) sending rate and (c) marking probability

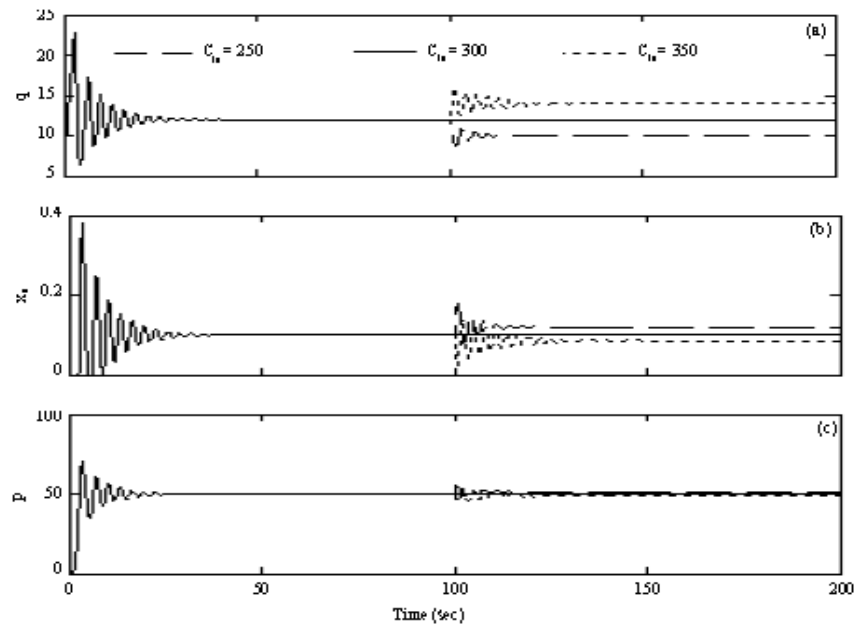


Fig. 6: Closed-loop ring network response for out-link capacity variations (a) queue length (b) sending rate and (c) marking probability

the AQM controller has been able to compensate the network dynamic variation as the queue length show very small offset (about 1.2%). However, the new sending rate has been matched truly with the new condition, while the marking probability has not been increased noticeably. Similarly, in Fig. 7, the parameter N_1 has been change at time 100 sec to values 3 and 7 for the dotted and dash curves, respectively. From Fig. 7, once again good ability of the designed controller for compensating network dynamics is obvious.

As our final experiment in this part, consider the graphs in Fig. 8, where the network parameters i.e., C_{in} and N_1 has been changed randomly over time in a range of $\pm 25\%$ around their nominal values. As seen in this figure, by employing the AQM strategy the average values of the queue length as well as the sending rate has been kept almost constant. However, the marking probability as the control command shows more fluctuations with respect to previous cases.

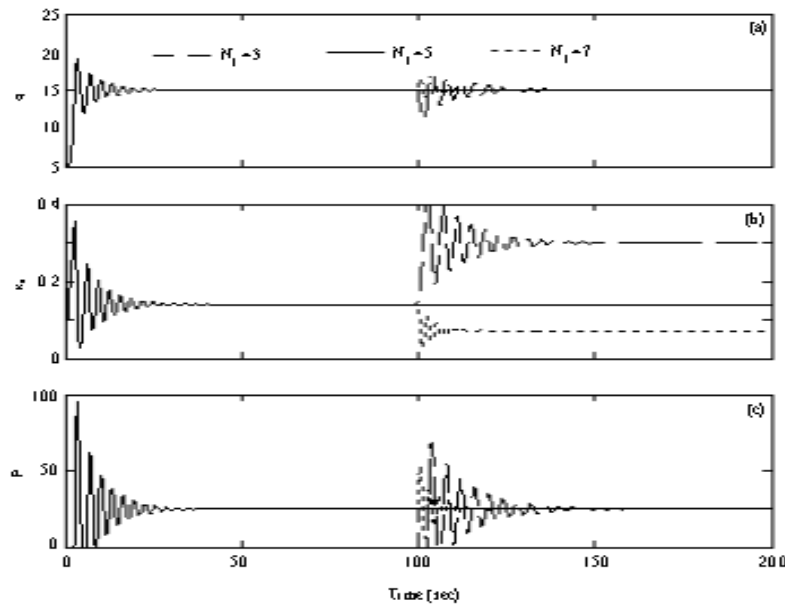


Fig. 7: Closed-loop ring network response for N_1 variations (a) queue length (b) sending rate and (c) marking probability

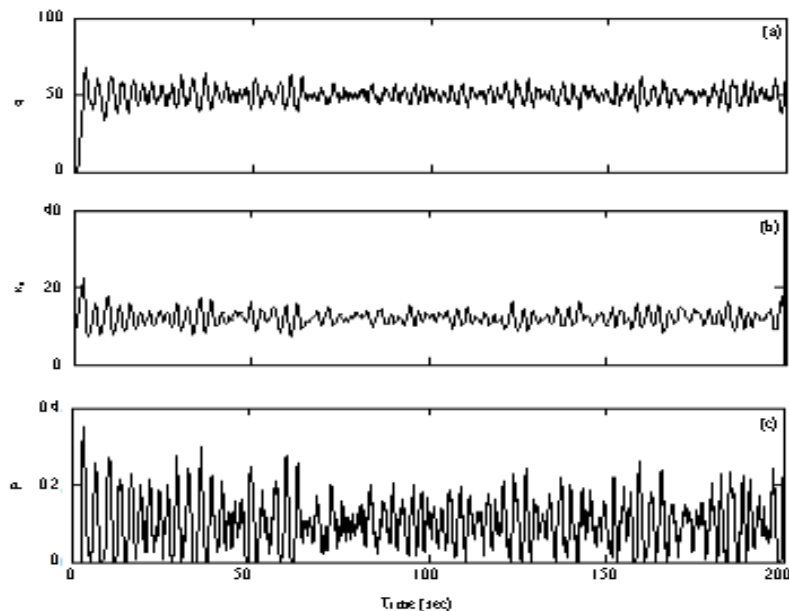


Fig. 8: Closed-loop ring network response for random network parameter variations (a) queue length (b) sending rate and (c) marking probability

Simulation results for tree topology: Now, let us switch to the above-described tree topology. For this network, PI controller has been also designed by the frequency

method. The step response of the closed loop system has been shown in Fig. 9. As seen in this figure, the PI controller has been able to properly regulate the queue

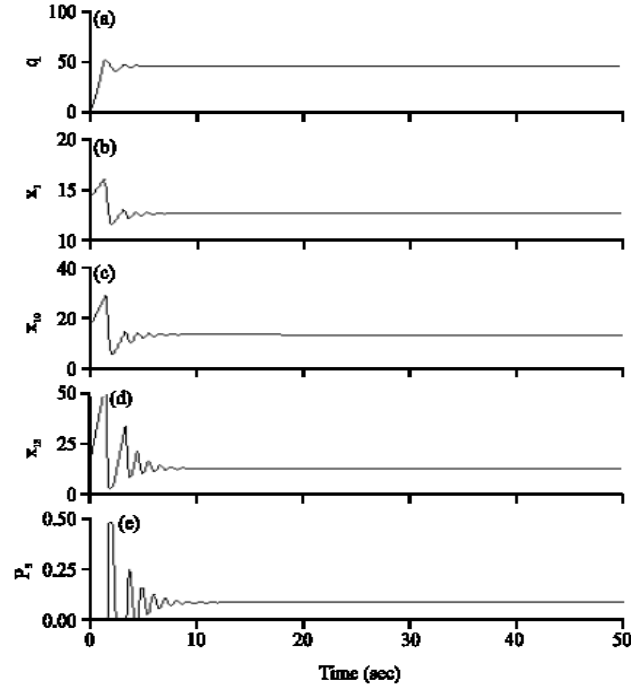


Fig. 9: Nominal response of tree network with PI controller (a) queue length (b, c and d) sending rates and (e) marking

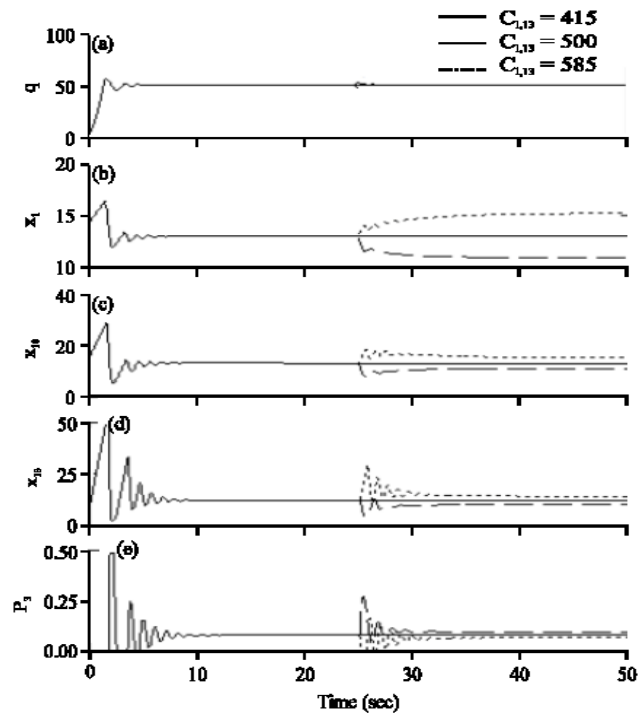


Fig. 10: Closed-loop tree network response for out-link capacity variations (a) queue length (b, c and d) sending rates and (e) marking

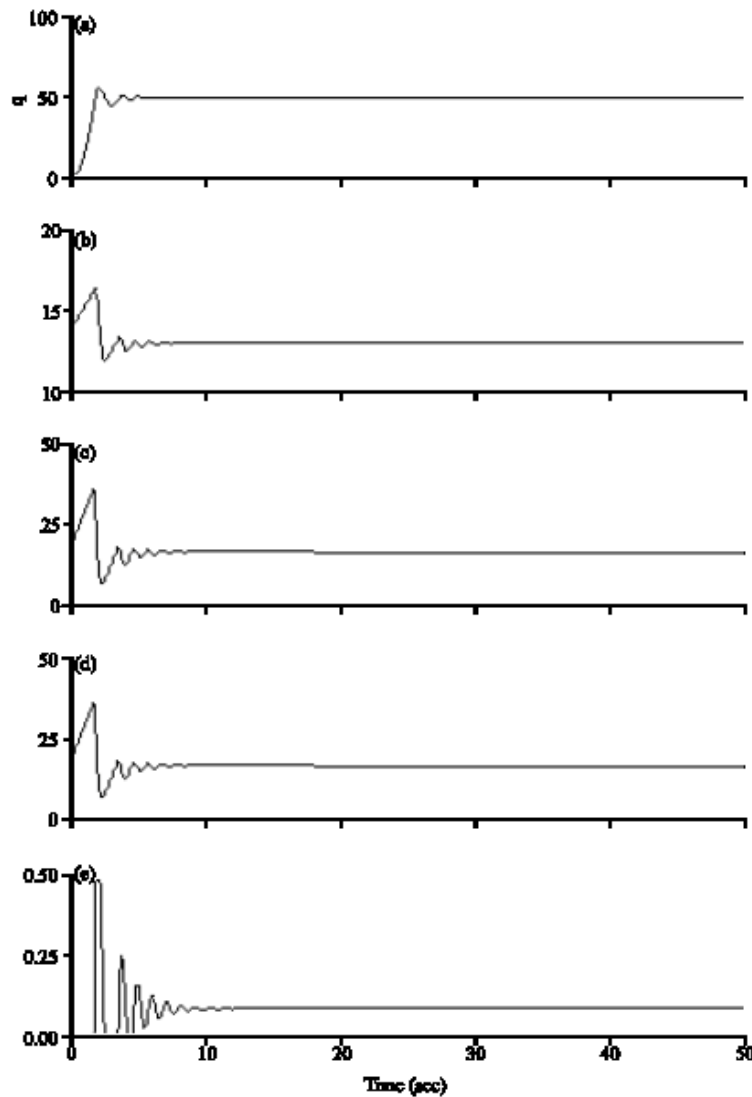


Fig.11: Closed-loop tree network response for random out-link capacity variations (a) queue length (b, c and d) sending rates and (e) marking

length about its desired value. Also note that sending rates in different levels have been converged to the same values which mean fairness in the algorithm.

Additionally, in Fig. 10 and 11, at time 25 sec, the out-link capacity C_{113} has been changed in step and random manner, respectively. In Fig. 10, C_{113} has been changed about $\pm 17\%$ around its nominal value, i.e., it has been changed to 415 (dotted line) and 585 (dashed line) packets sec^{-1} , respectively. In Fig. 11, random variations in a range of about $\pm 25\%$ around the nominal value of C_{113} has been applied to the network. As seen in both cases, the closed loop system has been compensate for the dynamic variations properly. Besides, the fair resource allocation has been also preserved in both cases.

CONCLUSION

In this study, a framework for designing AQM controllers for congestion control of wireless networks has been proposed. For this purpose, the model of wireless networks with ring and tree topologies has been derived in such a way that considers the interferences in the network. IEEE 802.11 model has been used for representing the interferences in the networks. Then, the general AQM controller has been formulated for these types of networks, separately. For the tree topology a multi-level wireless access network in its general form has been considered. In order for controller designing, the resulting models which are sets of nonlinear equations

have been linearized. Finally PI controller has been designed as an AQM controller and its performance has been evaluated via simulations. Simulation results are representative of good performance of the developed method for both ring and tree topologies.

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