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Explicit Solution of Nonlinear ZK-BBM Wave Equation Using Exp-Function Method

[1]J. Mahmoudi, [2]N. Tolou, [2]I. Khatami, [3]A. Barari and [2]D.D. Ganji
[1]Department of Mechanical Engineering, K.N. Toosi University of Technology, Tehran, Iran
[2]Department of Mechanical Engineering, Babol University of Technology, Mazandaran University, Babol, Iran
[3]Department of Civil Engineering, Babol University of Technology, Mazandaran University, Babol, Iran

Abstract: This study is devoted to studying the (2+1)-dimensional ZK-BBM (Zakharov-Kuznetsov-Benjamin-Bona-Mahony) wave equation in an analytical solution. The analysis is based on the implementation a new method, called Exp-function method. The obtained results from the proposed approximate solution have been verified with those obtained by the extended tanh method. It shows that the obtained results of these methods are the same; while Exp-function method, with the help of symbolic computation, provides a powerful mathematical tool for solving nonlinear partial differential equations of engineering problems in the terms of accuracy and efficiency.

Key words: Benjamin-Bona-Mahony-Burgers (BBMB) equations, Exp-function method

INTRODUCTION

In the recent decade, the study of nonlinear partial differential equations (NLEEs) modeling physical phenomena, has become an important toll. Seeking exact solutions for (NLEEs) has long been one of the central themes of perpetual interest in Mathematics and Physics. These solutions may well describe various phenomena in physics and other fields and thus may give more insight into the physical aspects of the problems. In this aspect, nonlinear wave equations in mathematical physics play a major role in various fields, such as plasma physics, fluid mechanics, optical fibers, solid state physics, optical fibers, chemical kinetics and geochemistry (Benjamin *et al*., 1972; Ablowitz and Clarkson, 1991; Rosenau and Hyman, 1993; Hereman *et al*., 1985; Kadomtsev and Petviashvili, 1970; Zakharov and Kuznestov, 1974; Li *et al*., 2003; Malfliet, 1992; Malfliet and Hereman, 1996; Monro and Parkes, 1999). But it is well known that except a limited number most of them do not have any precise analytical solutions. So, to solve these nonlinear equations other methods are needed, however, in recent decades numerical methods have well used to analysis the nonlinear partial equations such as (2+1)-dimensional ZK-BBM (Benjamin-Bona-Mahony) equation. Long with the numerical methods, the semi-exact analytical methods have been improved; for instance inverse scattering method (Ablowitz and Clarkson, 1991), Hirota's bilinear method (Hirota, 1971), homogenous balance method (Wang, 1996), homotopy perturbation method (He, 2005; Ganji and Rafei, 2006; Tolou *et al*., 2007), variational iteration method (He, 1999, 2000, 2004; Ganji and Rafei, 2006), asymptotic methods (He, 2006), non-perturbative methods (He, 2006), tanh-function method (Zayed *et al*., 2004; Wazwaz, 2007; Zhang and Xia, 2006a; Zhang, 2007b), algebraic method (Hu, 2005; Zhang and Xia, 2006b), Jacobi elliptic function expansion method (Liu *et al*., 2001; Zhao *et al*., 2006), F-expansion method (Zhang, 2006, 2007a) and so on. Recently, He and Wu (2006) proposed a straightforward and concise method, called Exp-function method, to obtain generalized solitonary solutions and periodic solutions of NLEEs. The solution procedure of this method, by the help of Matlab or Mathematica, is of utter simplicity and this method can be easily extended to all kinds of NLEEs.

In our previous study we implemented the HPM and VIM to BBMs equations (Fakhari *et al*., 2007; Tari and Ganji, 2007). Those show the applicability, accuracy and finally efficiency of both VIM and HPM. The Benjamin-Bona-Mahony equation (Benjamin *et al*., 1972) described by the following:

$$u_t+u_x+uu_x-u_{xxt} = 0 \tag{1}$$

This equation has been proposed as a model for propagation of long waves where nonlinear dispersion is incorporated.

Corresponding Author: N. Tolou, Department of Mechanical Engineering, Babol University of Technology, Mazandaran University, Babol, Iran

Also we implemented HPM to the fifth-order Korteweg de Vries (KdV) and generalized Hirota-Satsuma coupled KdV equations (Rafei and Ganji, 2006) which again approve the accuracy and efficiency of proposed method to this type of equation. The spatially one-dimensional KdV equation;

$$u_t + auu_x - u_{xxx} = 0 \quad (2)$$

is a model that governs the one-dimensional propagation of small amplitude, weakly dispersive waves and plays a major role in the solitons concepts. The term soliton coined by Zabusky and Kruskal (He, 2006) who found particle like waves which retained their shapes and velocities after collisions. The balance between the nonlinear convection term uux and the dispersion effect term u_{xxx} in the KdV Eq. 2 gives rise to solitons. The K (n, n) equation (Rosenau and Hyman, 1993):

$$u_t + a\,(u^n)_x - u^n_{xxx} = 0 \quad (3)$$

Where, in addition to the nonlinear convection term $(u^n)_x$, the dispersion effect term $(u^n)_{xxx}$ is genuinely nonlinear as well. The delicate interaction between the convection with the genuine nonlinear dispersion generates solitary waves with exact compact support that are termed compactons. Compactons are defined as solitons with finite wavelengths. Compactons are compact solutions that are usually expressed by powers of trigonometric functions sine and cosine. Unlike soliton that narrows as the amplitude increases, the compacton's width is independent of the amplitude.

In modern physics, a suffix-on is used to indicate the particle property (Wang, 1996), for example phonon, photon, peakon, soliton and compacton. One of the well-known two-dimensional generalizations of the KdV eqtiations is developed, namely the Zakharov-Kuznetsov (ZK) (1974) given by:

$$u_t + auu_x + (\nabla^2 u)_x = 0 \quad (4)$$

Where, $\nabla^2 = \partial^2 x + \partial^2 y + \partial^2 z$ is the isotropic Laplacian (Zakharov and Kuznetsov, 1974; Li *et al*., 2003).

The present letter is motivated by the desire to extend these works via implementation the Exp-function method to a modified form of BBMs equations formulated in the ZK sense by examination (2+1) dimensional ZK-BBM problem.

Consider the model describe by the generalized of the ZK-BBM equation (Wazwaz, 2007).

$$u_t + u_x + a\,(u^n)_x + b\,(u_{xt} + u_{yy})_x = 0 \quad (5)$$

To clarify the validity of proposed method, the obtained solutions are compared with their corresponding tanh methods.

MATERIALS AND METHODS

Implementation the Exp-function method: In this study, we apply the Exp-function method (He and Wu, 2006) for the solution of the ZK-BBM equations (Eq. 5) (Wazwaz, 2007).

Using the transformation

$$u = u(\eta), \qquad \eta = kx + qy + \omega t, \quad (6)$$

Where, k and ω are constants. Eq. 5 becomes:

$$\omega u' + ku' + 3aku^2u' + b\,(\omega k^2 u^m + kq^2 u^m) = 0 \quad (7)$$

Where, prime denotes the differential with respect to η. The Exp-function method is based on the assumption that traveling wave solution can be expressed in the following form (He and Wu, 2006):

$$u(\eta) = \frac{\sum_{n=-c}^{d} a_n \exp(n\eta)}{\sum_{m=-p}^{q} b_m \exp(m\eta)} \quad (8)$$

Where, c, d, p and q are positive integers which are unknown to be further determined, a_n and b_m are unknown constants. Equation 7 can be re-written in an alternative form (He and Wu, 2006) as follows:

$$u(\eta) = \frac{a_c \exp(c\eta) + \ldots + a_{-d} \exp(-d\eta)}{a_p \exp(p\eta) + \ldots + a_{-q} \exp(-q\eta)} \quad (9)$$

In order to determine the values of c and p, we balance the linear term of highest order in Eq. 7 with the highest order nonlinear term (He and Wu, 2006). By calculation, obtained:

$$u''' = \frac{c_1 \exp[(7p + c)\eta] + \ldots}{c_2 \exp[8p\eta] + \ldots} \quad (10)$$

and

$$u^2u' = \frac{c_3 \exp[(p + 3c)\eta] + \ldots}{c_4 \exp[4p\eta] + \ldots} = \frac{c_3 \exp[(5p + 3c)\eta] + \ldots}{c_4 \exp[8p\eta] + \ldots} \quad (11)$$

Where, c_i are determined coefficients only for simplicity. Balancing highest order of Exp-Function in Eq. 10 and 11 we have:

$$7p + c = 2\,(3p + c) \quad (12)$$

This leads to the following result:

$$p = c \tag{13}$$

Similarly, to determine the values of d and q, we balance the linear term of lowest order in Eq. 7 with the lowest order nonlinear term.

$$u''' = \frac{\dots + d_1 \exp[-(7q + d)\eta]}{\dots + d_2 \exp[-8q\eta]} \tag{14}$$

and

$$u^2 u' = \frac{\dots + d_3 \exp[-(q + 3d)\eta]}{\dots + d_4 \exp[(-4q)\eta]} = \frac{\dots + d_3 \exp[-(5q + d)\eta]}{\dots + d_4 \exp[(-8q)\eta]} \tag{15}$$

Where, d_i are determined coefficients only for simplicity. Balancing lowest order of Exp-Function in Eq. 14 and 15:

$$-(7q+d) = -2\,(3q+d) \tag{16}$$

which leads

$$q = d \tag{17}$$

Case 1: p = c = 1, d = 1: We can freely choose the values of c and d, but we illustrate that the final solution does not strongly depend upon the choice of values c and d (He and Wu, 2006). By setting p = c = 1 and q = d = 1 the trial function Eq. 9 becomes:

$$u(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{\exp(\eta) + b_0 + b_{-1} \exp(-\eta)} \tag{18}$$

Substituting Eq. 18 into 7 we have:

$$\frac{1}{A}\left[C_{-3}e^{-3\eta} + C_{-2}e^{-2\eta} + C_{-1}e^{-\eta} + C_0 + C_1 e^{\eta} + C_2 e^{2\eta} + C_3 e^{3\eta}\right] = 0, \tag{19}$$

Where:
$A = \left(e^{\eta} + b_0 + b_{-1}e^{-\eta}\right)^4$ and C_i are as below:

$$\begin{aligned}
&2ka_1b_{-1} + 2ka_1b_0^2 + 8b\omega k^2a_1b_{-1} - 4b\omega k^2a_1b_0^2 - 2\omega a_{-1} - 2ka_0b_0 \\
&-2\omega a_0b_0 + 6aka_1^3b_{-1} - 4bkq^2a_1b_0^2 + 4bkq^2a_0b_0 - 2ka_{-1} \\
&+4b\omega k^2a_0b_0 - 6aka_1a_0^2 + 6aka_1^2a_0b_0 + 8bkq^2a_1b_{-1} - 8b\omega k^2a_{-1} \\
&-8bkq^2a_{-1} - 6aka_1^2a_{-1} + 2\omega a_1b_0^2 + 2\omega a_1b_{-1} = 0
\end{aligned}$$

$$\begin{aligned}
&-b\omega k^2a_0 + 3aka_1^3b_0 + ka_1b_0 + \omega a_1b_0 - \omega a_0 - bkq^2a_0 \\
&+b\omega k^2a_1b_0 + bkq^2a_1b_0 - 3aka_1^2a_0 - ka_0 = 0
\end{aligned}$$

$$\begin{aligned}
&-ka_{-1}b_0^3 + k^2a_0b_{-1}b_0^2 + \omega a_0b_{-1}b_0^2 + 3aka_0^3b_{-1} - 15aka_{-1}^2a_0 \\
&-6ka_{-1}b_0b_{-1} + 5ka_1b_{-1}^2b_0 + 5\omega a_1b_0b_{-1}^2 - 6\omega a_{-1}b_0b_{-1} + \omega a_0b_{-1}^2 \\
&+ka_1b_{-1}^2 - \omega a_{-1}b_0^3 - b\omega k^2a_{-1}b_0^3 + b\omega k^2a_0b_{-1}b_0^2 + bkq^2a_0b_{-1}b_0^2 \\
&-bkq^2a_{-1}b_0^3 - 23b\omega k^2a_0b_{-1}^2 - 23bkq^2a_0b_{-1}^2 + 18b\omega k^2a_{-1}b_{-1}b_0 \\
&+5b\omega k^2a_1b_0b_{-1}^2 + 5bkq^2a_1b_0b_{-1}^2 + 18bkq^2a_{-1}a_1b_0 - 3aka_{-1}^2a_1b_0 \\
&+18aka_1a_0a_{-1}b_{-1} - 3aka_0^2a_{-1}b_0 = 0
\end{aligned}$$

$$\begin{aligned}
&-4bkq^2a_0b_{-1}^2b_0 + 8bkq^2a_1b_{-1}^3 + 2\omega a_0b_{-1}^2b_0 + 2ka_0b_{-1}^2b_0 \\
&-4b\omega k^2a_0b_{-1}^2b_0 + 2ka_1b_{-1}^3 - 8bkq^2a_{-1}b_{-1}^2 - 2\omega a_{-1}b_0^2b_{-1} \\
&+4bkq^2a_{-1}b_{-1}b_0^2 - 2ka_{-1}b_0^2b_{-1} - 2ka_{-1}b_{-1}^2 + 6aka_{-1}^2a_1b_{-1} \\
&-6aka_{-1}^2a_0b_0 + 8b\omega k^2a_1b_{-1}^3 - 6aka_{-1}^3 - 2\omega a_{-1}b_{-1}^2 \\
&-8b\omega k^2a_{-1}b_{-1}^2 + 6aka_{-1}a_0^2b_{-1} + 2\omega a_1b_{-1}^3 + 4b\omega k^2a_{-1}b_{-1}b_0^2 = 0
\end{aligned}$$

$$\begin{aligned}
&-bkq^2a_{-1}b_{-1}^2b_0 + b\omega k^2a_0b_{-1}^3 - \omega a_{-1}b_0b_{-1}^2 - b\omega k^2a_{-1}b_{-1}^2b_0 \\
&+3aka_{-1}^2b_0b_{-1} - 3aka_{-1}^3b_0 + bkq^2a_0b_{-1}^3 - ka_{-1}b_0b_{-1}^2 + \omega a_0b_{-1}^3 \\
&+ka_0b_{-1}^3 = 0
\end{aligned}$$

$$\begin{aligned}
&6\omega a_1b_0b_{-1} + 6ka_1b_{-1}b_0 + ka_1b_0^3 - \omega a_0b_{-1} + \omega a_1b_0^3 - ka_0b_0^2 \\
&-\omega a_0b_0^2b\omega k^2a_1b_0^3 + bkq^2a_1b_0^3 - bkq^2a_0b_0^2 - 5\omega a_{-1}b_0 \\
&-b\omega k^2a_0b_0^2 - 3aka_0^3 - 5ka_{-1}b_0 + 23bkq^2a_0b_{-1} + 23b\omega k^2a_0b_{-1} \\
&+3aka_0^2a_1b_0 - 5bkq^2a_{-1}b_0 - 5b\omega k^2a_{-1}b_0 - 18b\omega k^2a_1b_0b_{-1} \\
&+15aka_1^2a_0b_{-1} - 18aka_1a_0a_{-1} - 18bkq^2a_1b_0b_{-1} + 3aka_1^2a_{-1}b_0 = 0
\end{aligned}$$

$$\begin{aligned}
&-4\omega a_{-1}b_{-1} - 4ka_{-1}b_{-1} - 32b\omega k^2a_1b_{-1}^2 + 32bkq^2a_{-1}b_{-1} \\
&+32b\omega k^2a_{-1}b_{-1} - 12aka_{-1}a_0^2 - 4ka_{-1}b_0^2 - 4\omega a_{-1}b_0^2 \\
&+4\omega a_1b_0^2b_{-1} - 32bkq^2a_1b_{-1}^2 + 4b\omega k^2a_1b_0^2b_{-1} + 4bkq^2a_1b_{-1}b_0^2 \\
&-4b\omega k^2a_{-1}b_0^2 - 4bkq^2a_{-1}b_0^2 + 4ka_1b_{-1}^2 + 4\omega a_1b_{-1}^2 + 12aka_1^2a_{-1}b_{-1} \\
&+12aka_1a_0^2b_{-1} + 4ka_1b_0^2b_{-1} - 12aka_{-1}^2a_1 = 0
\end{aligned}$$

Solving the system of algebraic equations, we obtain the following results:

$$\begin{aligned}
&a_{-1} = -\frac{1}{8}a_1b_0^2 \qquad a_0 = 0 \qquad b_0 = b_0 \qquad b_{-1} = \frac{1}{8}b_0^2 \\
&a = -\frac{1}{2}\frac{k+\omega}{ka_1^2} \qquad b = \frac{1}{2}\frac{k+\omega}{k(\omega k + q^2)}
\end{aligned} \tag{20}$$

Where, a_1, b_0 and k are free parameters.

Substituting these results into Eq. 18, we obtain the following exact solution.

$$\begin{aligned}
u &= \frac{a_1 e^{(kx+qy+\omega t)} + \frac{1}{8}a_1b_0^2 e^{-(kx+qy+\omega t)}}{e^{(kx+qy+\omega t)} + b_0 + \frac{1}{8}b_0^2 e^{-(kx+qy+\omega t)}} \\
&= a_1 - \frac{8a_1b_0}{8e^{(kx+qy+\omega t)} + 8b_0 + b_0^2 e^{-(kx+qy+\omega t)}}
\end{aligned} \tag{21}$$

When k is an imaginary number, the obtained solitary solution can be converted in to periodic solution (He and Wu, 2006). We write k = ik, q = iq and ω = iω using the transformation

$$e^{i(kx+qy+\omega t)} = \cos[kx+qy+\omega t] + i\sin[kx+qy+\omega t] \qquad (22)$$

and

$$e^{-i(kx+qy+\omega t)} = \cos[kx+qy+\omega t] - i\sin[kx+qy+\omega t] \qquad (23)$$

Then Eq. 21 becomes

$$u = a_1 - \frac{8a_1 b_0}{(8+b_0^2)\cos(kx+qy+\omega t) + 8b_0 + (8-b_0^2)\sin(kx+qy+\omega t)} \qquad (24)$$

If we search for a periodic solution or compact solution, the imaginary part in Eq. (24) must be zero, that requires:

$$8 - b_0^2 = 0 \qquad (25)$$

From Eq. 25 we obtain

$$b_0 = \pm 2\sqrt{2} \qquad (26)$$

Substituting Eq. 26 in Eq. 24 yields two periodic solutions:

$$u = a_1 - \frac{16\sqrt{2}a_1}{16\cos(kx+qy+\omega t) + 16\sqrt{2}} \qquad (27)$$

and

$$u = a_1 + \frac{16\sqrt{2}a_1}{16\cos(kx+qy+\omega t) - 16\sqrt{2}} \qquad (28)$$

It can be seen good approximation with the extended tanh method (Wazwaz, 2007).

Case 2: p = c = 2, q = d = 2: As mentioned above the values of c and d can be freely chosen, we set p = c = 2 and q = d = 2 then the trial function, Eq. 9 becomes:

$$u(\eta) = \frac{a_2\exp(2\eta) + a_1\exp(\eta) + a_0 + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{b_2\exp(-2\eta) + b_1\exp(\eta) + b_0 + b_{-1}\exp(-\eta) + b_{-2}\exp(-2\eta)} \qquad (29)$$

There are some free parameters in Eq. 29, we set $b_2 = 1$, $b_1 = 0$ and $b_{-1} = 0$ for simplicity, then the trial function, Eq. (29) is simplified as follows:

$$u(\eta) = \frac{a_2\exp(2\eta) + a_1\exp(\eta) + a_0 + a_{-1}\exp(-\eta) + a_{-2}\exp(-2\eta)}{\exp(-2\eta) + b_0 + b_{-2}\exp(-2\eta)} \qquad (30)$$

By the manipulation as illustrated above, we obtain

$$a_{-2} = -\frac{1}{4}\frac{a_2^2 b_0^2 - a_0^2}{a_2} \qquad a_0 = a_0 \qquad b_0 = b_0 \qquad b_{-2} = \frac{1}{4}\frac{a_2^2 b_0^2 - a_0^2}{a_2^2}$$
$$a = -\frac{k+\omega}{ka_2^2} \qquad b = \frac{1}{2}\frac{k+\omega}{k(\omega k + q^2)} \qquad a_1 = 0 \qquad a_{-1} = 0 \qquad (31)$$

Substituting Eq. 31 in Eq. 30 yields to the following solution

$$u = \frac{a_2 e^{2(kx+qy+\omega t)} + a_0 - \frac{1}{4}\frac{(a_2^2 b_0^2 - a_0^2)}{a_2} e^{-2(kx+qy+\omega t)}}{e^{2(kx+qy+\omega t)} + b_0 + \frac{1}{4}\frac{(a_2^2 b_0^2 - a_0^2)}{a_2^2} e^{-2(kx+qy+\omega t)}} \qquad (32)$$

It can be proved that the obtained solution (Eq. 29) is equivalent to the solution obtained in case 1.

Case 3: p = c = 2, q = d = 1: We consider the case p = c = 2 and q = d = 1, Eq. 9 can be expressed as:

$$u(\eta) = \frac{a_2\exp(2\eta) + a_1\exp(\eta) + a_0 + a_{-1}\exp(-\eta)}{b_2\exp(-2\eta) + b_1\exp(\eta) + b_0 + b_{-1}\exp(-\eta)} \qquad (33)$$

There are some free parameters in Eq. 33 we set $b_2 = 1$ for simplicity, by the same manipulation as illustrated above we obtained:

$$a_0 = a_0 \qquad a_{-1} = -\frac{1}{8}\frac{\gamma_1}{a_2^2} \qquad a_1 = a_1$$
$$b_0 = \frac{1}{2}\frac{\gamma_2}{a_2^2} \qquad b_{-1} = \frac{1}{8}\frac{\gamma_1}{a_2^3} \qquad b_1 = b_1 \qquad (34)$$
$$a = -\frac{\omega+k}{ka_2^2} \qquad b = \frac{2(\omega+k)}{k(\omega k + q^2)} \qquad \gamma_i = \text{const} \quad (i = 1, 2)$$

By Substituting Eq. 34 in Eq. 33 we obtained a solution that easily proved that this equation is same with that obtained in case 1.

CONCLUSION

In this survey, our objective has been to show that exact solution of the (2+1)-dimensional ZK-BBM

(Benjamin-Bona-Mahony) equation can be obtained by Exp-Function method. The method is used to finding the traveling wave solutions of ZK-BBM nonlinear partial differential equation. We also found new exact solution that is not obtained by other existed method. Furthermore, the method leads to both the generalized solitary solutions and periodic solutions. The results obtained from proposed method have been compared and verified with those obtained by the extended tanh method. The results revealed that Exp-function method is powerful mathematical tool for solutions of nonlinear partial differential equations in the terms of accuracy and efficiency while systems of nonlinear partial differential equations having wide applications in engineering.

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