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Stability Study of Model Predictive Control in Presence of Undesirable Factors

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Abstract: In this study, the stability behavior of Generalized Predictive Controllers (GPC) as one of model predictive control algorithm is studied and effects of noise, error in delay estimation, input disturbances, unstable system and non-minimum phase system is analyzed. The results showed that GP-controller can be achieved stability and resist against wrong parameterization. This stability studies is completed by means of a numerical example. The results show that the GP algorithm can guarantee the stability of this system.

Key words: Model based control, receding horizon control, dynamic control, non-minimum phase system, robustness of MPC

INTRODUCTION

Model predictive control technique belongs to a class of computer control procedures in which a dynamic model of the plant is employed to forecast and optimize the future behavior of the process. Various MPC algorithms are just differing in using model of the plants, noise and cost functions, for deriving optimal control law. The most applicable and representative algorithm of model predictive control are as follows: Dynamic Matrix Control (Camacho and Bordons, 2004), Matrix Algorithm Control (Alowger and Zheng, 2000), Generalized Predictive Control (Camacho and Bordons, 2004; Camacho, 1993), Predictive Functional Control (Alowger and Zheng, 2000), Extended prediction Self Adaptive Control and Extended Horizon Self Adaptive Control. Due to importance, popularity and significant practical importance of Generalized Predictive Control (GPC) in industry and academia, this algorithm is used to complete this study. Generalized predictive control, which is very effective for the self tuning control of linear and nonlinear plant, is proposed by Clarke *et al*. (1987b). It is based on the minimization of a long-range cost function. It is also able of stable control of processes with variable parameters, variable dead-time and with a model order which changes instantaneously. This method is effective for plant which is simultaneously non-minimum-phase and open-loop unstable and whose model is over-parameterized. It is also suitable for high-performance applications such as the control of flexible systems. Although among the control theory, cruise control and self tuning control have good background but none of them can be used as a suitable and robust control for many of real world control applications. For this purpose a common algorithm is needed to be suitable for below usages (Camacho and Bordons, 2004; Clarke *et al*., 1987a):

- A non-minimum-phase plant: most continues time transfer functions when descretized by a fast sample time will provide at least one discrete time zero out of unit circle.
- An open loop unstable plant with badly-damped poles
- A plant with unknown or changeable dead time
- A plant with un-known degree

GPC algorithm can solve all the above problems in a multi input multi output systems. The algorithm is already able to design stable and robust controller for changable parameter, changeable dead time and also for those processes that the degree of plant is changing randomly. One of the employed models for designing GPC controller is called CARMA (Control Auto-Regressive Integrated Moving Average) model that is one of the time series model.

Basic principles for model predictive control: In fact expression of predictive control based on model is not associated with any specific strategy. It is referred to a large domain of control method in which an explicit model of process is used to derive a control law by minimizing an objective function. Main idea in all predictive control family is based on using below items (Camacho and Bordons, 2004):

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- Using an accurate model of process for predicting future output
- Computing a control sequence by minimizing an objective function
- Sending the first element of computed control signal u(t/t) to the process in time t and using the strategy of rejecting other elements of control signal

Because one time step later the controller is updated with actual measurements from the system and updated disturbance predictions. Then, the optimization is done all over again.

Thus it can be said that MPC controller prophesies the plant behavior over a prediction horizon while using the dynamic model and measurements, then determines a manipulated variable sequence that optimizes some open-loop performance objective over the control horizon. This manipulated variable sequence is implemented until the next measurement becomes available. Then the optimization problem is solved again (Bemporad *et al.*, 2002). The most significant property about the MPC is the distinct use of a model. This feature can be the advantage and the disadvantage of MPC at the same time. The study of the process and controller behavior, the ability to do the simulations and fault detection of the plant are the most important advantages of using the model in the design. As well the detailed study of the plant behavior is a disadvantage since it has to be done before the actual MPC design can be started (Morari and Lee, 1999; Chen *et al.*, 2000).

Advantages of MPC

- It can be used in most real world industry to do its ability in solving problems of multi-input multi-output (MIMO) system (Qin and Badgwellb, 2003)
- The concepts of MPC aroused from industry
- It is able to control a large number of processes, consist of those with non-minimum phase, long time delay, input/output constraints or open-loop unstable characteristics
- Able to handle the process constraints
- It can be applied to batch processes where the future reference signals are known

Limitation of model predictive control: Some of the main disadvantages and limitation of MPC which has been concentrated by researchers and academic studies to expand the applicability of this kind of model based controllers are as follows (Tyagunov, 2004):

- **Stability/Robustness:** Theoretical aspects associated with stability and performance properties of MPC have proven to be a complicated and difficult issue. Stability tends to be problematic because of the following facts that presence of constraints in the optimization problem results in a nonlinear closed-loop system; even if the model and plant dynamics are linear.

- **Optimization:** Due to results of nonlinear optimization problems which rarely have exploitable convexity properties, an essential issue, both theoretical and practical, is whether the optimization can be successfully employed in MPC. Hence in most of the cases, the optimal solution is unknown and un-computable.

- **Noise handling:** The feedback strategies have an advantage that the effect of noise and disturbances can be considered. Since most of the MPC techniques are open-loop strategies, the design of such feedback strategies is more difficult than open-loop MPC control.

- **Performance assessment:** Generally, the optimization problem for nonlinear plant is non-convex and leads to many difficulties. These difficulties are related to feasibility, optimality, stability issues and computation time.

Since MPC is a natural method it can generate feedback control actions for linear nonlinear and hybrid system and piecewise affine plants subject to point-wise-in-time input and/or state-related constraints. In the Fig. 1 the basic MPC block and their connection has been shown (Camacho and Bordons, 2004; Oliveira and Morari, 2000).

From an empirical point of view, what makes MPC very useful in a variety of applications is an attractive feature of MPC and its ability to naturally

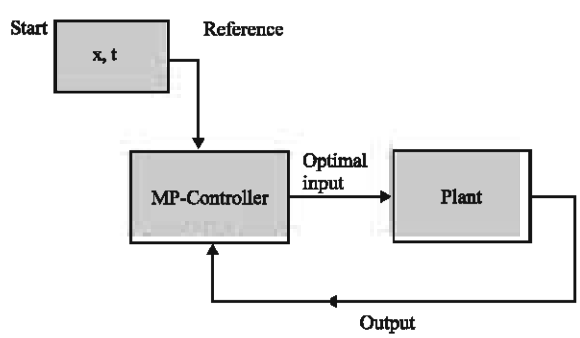

Fig. 1: Basic MPC block

and explicitly handle MIMO plants, time delay and constraints by direct incorporation into the on-line optimization. Particularly, the continuing optimization performed at each time step is needed to fulfill the forced constraints. The most important and also the most easily handled, are hard constraints or saturation of the control input. For more information on finite horizon MPC (Jadbabaie *et al*., 1999).

MATERIALS AND METHODS

Here, the GPC algorithm and its necessary knowledge for employing into this algorithm have described. This method is described for a general continues and nonlinear system and it has the ability to apply to a linear plant. First of all, since GPC is a model based control technique and what increases the process performance in such methods is an exact model of the system, consequently it is suffice to say that how the accurate model of the plant would help to increase the control system's performance. Therefore the first step to design model predictive controller is to come up with a mathematical differential or difference equation of the system. Due to limitation of physical elements such as control valves, sensors, tanks and actuators the constraints is defined. The initial step has just done once. When the model is derived if the system is nonlinear, the equation of the system should be linearized. Therefore, by using MATLAB software programming the nonlinear system is changed to linear to avoid loss of convexity and to keep away the troublesome setup of the NMPC. At this time which the transfer function of the plant is linearized and considering that the real dynamic model of a plant is a continuous model and regarding to utilize a computer control method, the dynamic equation of the system should convert to discrete form by sampling time of T_S. Now all the essential matrices are ready for deriving the CARMA model of the plant. Then it is time for computing step response (the first column of step response matrix (G) can be calculated as a step response of a plant, when a unit step is applied to the manipulated variables, then for composing G matrix, these coefficients are arranged in the following form):

$$G = \begin{bmatrix} g_0 & 0 & 0 & 0 \\ g_1 & g_0 & 0 & 0 \\ . & . & & \\ . & . & g_0 & 0 \\ . & . & & \\ g_{N-1} & g_{N-2} & \cdots & g_0 \end{bmatrix} \tag{1}$$

In which g_0 to g_{N-1} are step response coefficient when a unit step is applied to a plant under control. After derivation of matrix G it is turn for computing free response. Finally in order to start GPC design, the algorithm has to find the prediction horizon (N_p), control horizon (N_c) and lambda (λ_r and λ_q).

In MPC design the number of samples one looks ahead is called the prediction horizon N_p. Control horizon N_c or it may call N_u is the number of sample that the optimal input is calculated for that. For the simple control system design, with a shorter control horizon than prediction horizon the complexity of the problem can be reduced. Choosing a value for Lambda is a trade off between a smooth signal and a fast system performance. If a smooth signal is desirable then the fraction ($\lambda_q|\lambda_r$) should be low and if a fast system is needed then this fraction should be greater. The entire necessary algorithms for finding these essential arrays or matrices have been fully described by Camacho and Bordons (2004). Next step is to calculate weighting matrices R and Q (these matrices are tuned till the desire performance is achieved).

$$R = \lambda_r \times \text{eye}(2\times N_c) \quad \text{and} \quad Q = \lambda_q \times \text{eye}(2\times N_c) \tag{2}$$

In which N_c is the control horizon and eye ($2\times N_c$) returns the ($2\times N_c$)-by-($2\times N_c$) identity matrix.

Regarding to use the algorithm of reference tracking, the references signals are designing. Afterward it is turn for solving an optimization problem with a quadratic objective function and linear constraints which is called a quadratic programming. QP solvers gave good performance results, where the reference trajectories were followed quite closely.

Therefore, the last step to complete the GPC algorithm is to solve optimization problem and applying the three existing constraints (input, output and terminal) and the reference trajectory. To make this algorithm more obvious, it has been applied to a system (9) describing in the next section. Before that the CARMA model and basic formulation of GPC is described by Camacho and Bordons (2004).

Consider a CARMA model of a SISO plant.

$$A(z^{-1}).y(t) = B(z^{-1}).z^{-d}.u(t-1) + C(z^{-1}).\frac{e(t)}{\Delta} \tag{3}$$

With $\Delta = 1 - z^{-1}$ and consider d as a dead time of the system. Where y(t), u(t) and e(t) are an output sequence, manipulated sequence and zero mean white noise respectively and A, B and C are polynomial in backward shift operator z^{-1}.

$$A(z^{-1}) = 1\text{-}a_1.z^{-1}+a_2.z^{-2}+...+a_{na}.z^{-na} \tag{4}$$

$$B(z^{-1}) = b_0\text{-}b_1.z^{-1}+b_2.z^{-2}+...+b_{nb}.z^{-nb} \tag{5}$$

$$C(z^{-1}) = 1\text{-}\,c_1.z^{-1}+c_2.z^{-2}+...+c_{nc}.z^{-nc} \tag{6}$$

The GPC algorithm is to apply a control sequence which can minimize a bellow multi stage cost function:

$$J\left(N_{P1}, N_{P2}, N_u\right) = \sum_{j=N_{p1}}^{N_{p2}} R.\left[\hat{y}(t+j|t) - w(t+j)\right]^2 + \sum_{j=1}^{N_u} Q.\left[\Delta u(t+j-1)\right]^2 \tag{7}$$

$\hat{y}(t+j|t)$ is the j-step ahead prediction of the system on data up to time t, w(t+j) is the future reference trajectory. N_{p1} is the minimum value for prediction horizon and N_{p2} is the maximum value for prediction horizon and N_u is the control horizon. Q and R are weighting matrices.

Therefore the optimal input is derived by the following formula:

$$\Delta u = K\ (w - f) \tag{8}$$

where, K is the first row of matrix $(G^TG+\lambda I)^{-1}G^T$ and G is step response, Q and R are weighting matrices, f is free response and w is the future reference trajectory.

RESULTS AND DISCUSSION

Control problem: Consider a state-space model of the system.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{9}$$

$$y(t) = Cx(t) \tag{10}$$

$$\left.\begin{array}{ll} A = \begin{bmatrix} 1.2 & -0.35 \\ 1 & 0 \end{bmatrix} & B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ C = \begin{bmatrix} 1 & -0.6 \end{bmatrix} & D = [0] \end{array}\right\} \tag{11}$$

The purpose of this study is to analyze the stability, convexity and robustness behavior of GPC. This is done by applying different undesirable factors occurring in real practices. Therefore the control objective is to illustrate the effect of noise, wrong delay estimation, unstable and non-minimum phase system on a performance of generalized predictive controller (9).

Here the design of GPC is carried out for system (11). As described in the previous section, the control objective is to analyze the effect of noise, wrong delay estimation, unstable and non-minimum phase on stability behavior of generalized predictive controller.

For this purpose the generalized predictive controller is designed for the system (11) and then the effects of five different parameters on control performance are examined.

Calculating matrices A and B.

$$A(q^{-1}) = 1 - 1.2q^{-1} + 0.35q^{-2} \tag{12}$$

$$B(q^{-1}) = q^{-1}(1 - 0.6q^{-1}) \tag{13}$$

In this case first the bellow Diophantine equation for $N_{P1} \le j \le N_{P2}$ is solved.
Matrix G is introduced as follow:

$$G = \begin{bmatrix} g_0 & 0 & 0 & 0 \\ g_1 & g_0 & 0 & 0 \\ . & . & & \\ . & . & g_0 & 0 \\ . & . & & \\ g_{N-1} & g_{N-2} & ... & g_0 \end{bmatrix} \tag{14}$$

$$G = \begin{bmatrix} 1.0000 & 0 & ... & & & & & & & 0 \\ 1.6000 & 1.0000 & 0 & ... & & & & & & \\ 1.9700 & 1.6000 & 1.0000 & 0 & ... & & & & & \\ 2.2040 & 1.9700 & 1.6000 & 1.0000 & 0 & ... & & & & \\ 2.3553 & 2.2040 & 1.9700 & 1.6000 & 1.0000 & 0 & ... & & & \\ 2.4550 & 2.3553 & 2.2040 & 1.9700 & 1.6000 & 1.0000 & 0 & ... & & \\ 2.5216 & 2.4550 & 2.3553 & 2.2040 & 1.9700 & 1.6000 & 1.0000 & 0 & ... & \\ 2.5667 & 2.5216 & 2.4550 & 2.3553 & 2.2040 & 1.9700 & 1.6000 & 1.0000 & 0 & ... \\ 2.5975 & 2.5667 & 2.5216 & 2.4550 & 2.3553 & 2.2040 & 1.9700 & 1.6000 & 1.0000 & 0 \\ 2.6186 & 2.5975 & 2.5667 & 2.5216 & 2.4550 & 2.3553 & 2.2040 & 1.9700 & 1.6000 & 1.0000 \end{bmatrix} \tag{15}$$

Therefore the control signal is obtained (Eliasi *et al*., 2007).

Generalized predictive control design: Here the generalized predictive controller is design for the system (10).

During the design the control horizon is considered to be 5. As it is appeared in the Fig. 2, the controller provides stable and acceptable performance during reference tracking. To show the effect of control horizon on error, we consider a constant value of 1 for lambda fraction and increase control horizon.

From Fig. 3 it is obvious that by increasing the control horizon (horizontal axis), the rate of change of error (vertical axis) is increased but it reaches a constant value when control horizon is more than 10. This output can also track the variable reference showing in Fig. 4.

Noise sensitivity: In this section the effect of colored-noise case on controller performance is analyzed. Therefore the $C(z^{-1})$ polynomial is not 1 anymore and the optimal prediction is derived as bellow.

$$u(t) = (G^T G + \lambda I)^{-1} G^T (w - f) \tag{16}$$

$$C(q^{-1}) = E_j(q^{-1})^2 A(q^{-1}) + q^{-1} F_j(q^{-1}) \tag{17}$$

The controller is designed when colored-noise case with bellow dynamic equation is considered.

$$e(t) + 0.5e(t-1) \tag{18}$$

The simulation results in Fig. 5 have shown that this controller delivers both good response and stability for reference tracking and noise rejection.

An unstable system: According to the system's state space, this system has two poles at -0.5 and -0.7. In this case if a pole was placed in -1.5 instead of -0.5 the system would be unstable system.

As can be seen this GP-controller can control the unstable system and the output track the reference with a good performance. Notice that for closed-loop stability of this plant there is no condition on open-loop system.

Influence of error in delay estimation: We considered that the designer has made a mistake in delay estimation. For this system the real delay d = 1 but to investigate the effect of delay on controller performance we choose d = 5.

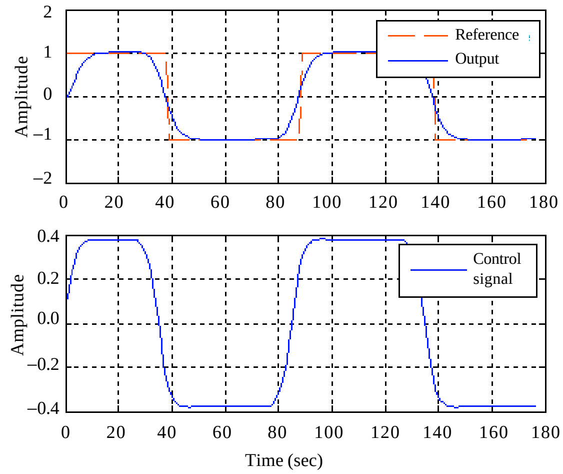

Fig. 2: GP-controller design

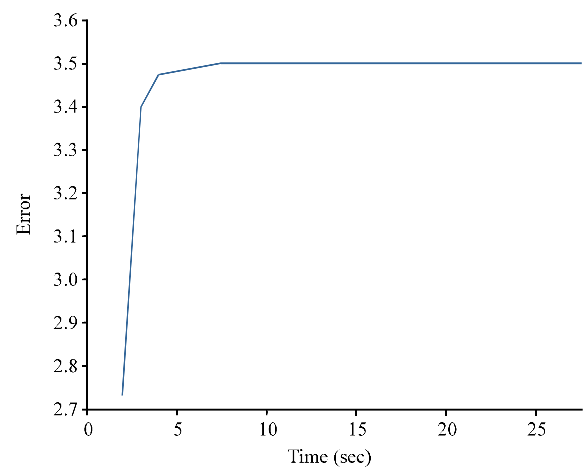

Fig. 3: Error value for different control horizon

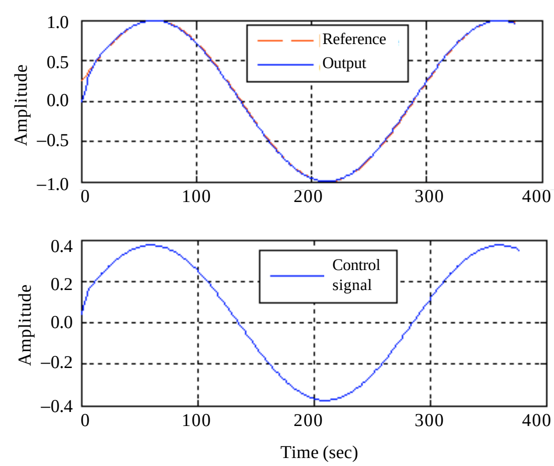

Fig. 4: GP-controller design with sine input

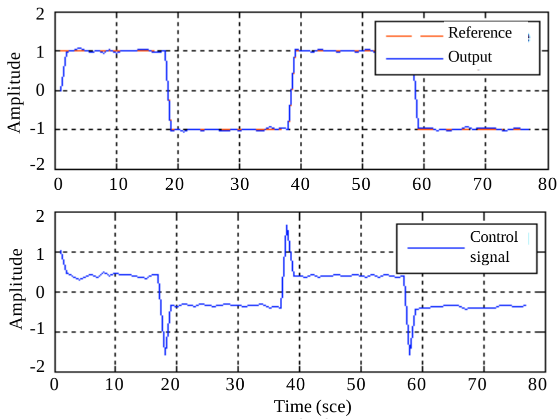

Fig. 5: Effect of colored noise on GP-controller

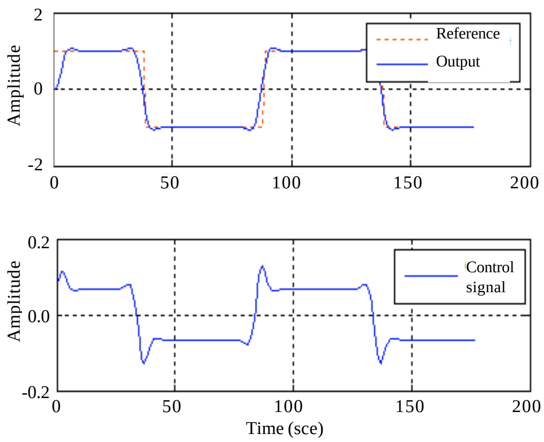

Fig. 6: GP-controller design in unstable case

Although Fig. 7 is shown the initial overshoot but this does not influence the reference tracking and the controller can resist the wrong delay estimation.

Generalize predictive control for a non-minimum phase system: According to the system's state space model, this system has one zero at -0.6. In this case if a zero was placed in 1.2 instead of -0.6 the system would be a non-minimum phase system.

Figure 8 shown the non-minimum-phase behavior. This system is identical except for the sign of the gain due to having one zero in the RHP. Thus, the step response goes in the wrong direction first, then in the correct direction, ending with its appropriate final value.

Input disturbances: Suppose that after 70 samples from beginning of the simulation, disturbance occurs, in this case the controller explicitly considers the measurable disturbances so it is able to reject it.

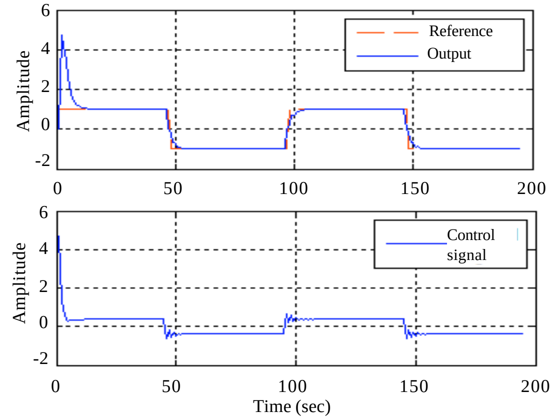

Fig. 7: GP-controller design with error in delay estimation

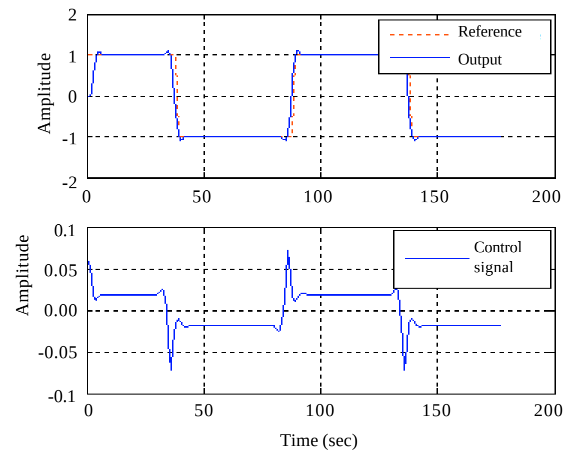

Fig. 8: GP-controller design for non-minimum phase case

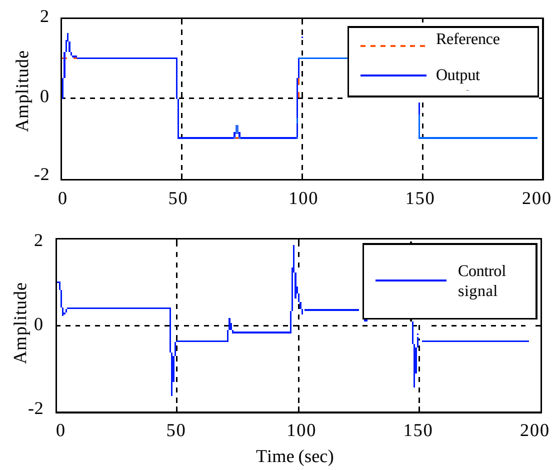

Fig. 9: Disturbance rejection

As it is appear in the Fig. 9, the system is able to measure the disturbances and the controller can reject the disturbance effects before it appears in the output.

CONCLUSION

In this study stability study of GPC as one of the MPC algorithm was carried out. In particular, the stability behavior of GPC was studied and the effect of noise, error in delay estimation, unstable system, non-minimum phase system and input disturbance are analyzed. The results showed that GP-Controller can achieve stability and resist against wrong parameterization.

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