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Numerical Simulation of the Swelling Pressure Test on Compacted Boom Clay

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Abstract: This study presents an investigation into the numerical simulation of the behavior of a compacted Boom clay under various loading paths. It is studied in particular the swelling pressure during the wetting path and the peak observed in the test performed at a low density as a consequence of the plastification of the sample at constant volume. A comparison of blind prediction and experimental results is made by using a highly swelling elastoplastic numerical model. This comparison has showed the ability of the model to correctly predict the complex trend of behavior during the swelling pressure test.

Key words: Numerical model, suction, expansive clays, elastoplasticity

INTRODUCTION

The mechanical behavior of compacted expansive clays is a critical factor in the design of seals for repositories to store hazardous wastes. An adequate constitutive model for this type of material is therefore required when realistic numerical simulations of sealing systems are to be performed. The clays are generally compacted and are consequently in unsaturated state.

A number of constitutive relationships have been proposed to describe the stress-strain behaviour of the material. Alonso *et al*. (1990) presented the Barcelona Basic Model (BBM) describing the stress-strain behaviour of partially saturated soils of low activity. This model is able to reproduce most responses of partially saturated soils under simple loading-unloading and wetting-drying paths.

The BBM model presents, however, limitations in modelling the behaviour of more expansive soils. Pousada (1984) and Dif and Bluemel (1991) reported test results from wetting and drying cycles performed on highly expansive clay which showed a large irreversible component of swelling on first wetting. Subsequently, Alonso *et al*. (1995a) reported cyclic wetting-drying test results for another highly expansive clay, which showed an irreversible component of shrinkage during each drying stage. Neither of these patterns of behaviour could be represented by the existing elastoplastic models which assume that both wetting- induced swelling and drying-induced shrinkage are elastic processes.

Again, suction controlled experiments (Cui and Delage, 1996; Bernier *et al*., 1997; Alonso *et al*., 1999) have revealed that wetting under a given confining stress may induce an irreversible volumetric deformation in unsaturated soils.

It is believed that such behaviour features are mainly related to the existence of coupled chemo-hydro-mechanical phenomena between distinct levels of structure within the material. Gens and Alonso (1992) presented a conceptual basis for a model for expansive soils. Two distinct levels are distinguished: the microstructural level at which swelling of active minerals takes place and the macrostructural level responsible for major structural rearrangement. Following this concept, Alonso *et al*. (1999) proposed an extended form of elastoplastic model for highly expansive unsaturated clays which they called the Barcelona Expansive Model (BExM). This model was based on an assumption of two levels of soil fabric (an unsaturated macrofabric and a saturated microfabric within individual clay packets) with coupling between the mechanical behaviour of the two levels.

In this study, it is presented a simulation of one of the responses of expansive soils observed in the experimental study (Romero, 1999) carried out on compacted Boom clay in which a peak is observed during a swelling pressure test performed at low density as a consequence of the plastification of the sample at constant volume (Fig. 1).

A numerical simulation of the behaviour of compacted Boom clay under various loading paths is presented. An elastoplastic model based on the constitutive model BExM of Alonso *et al*. (1999) is used to perform a blind prediction of the swelling pressure test.

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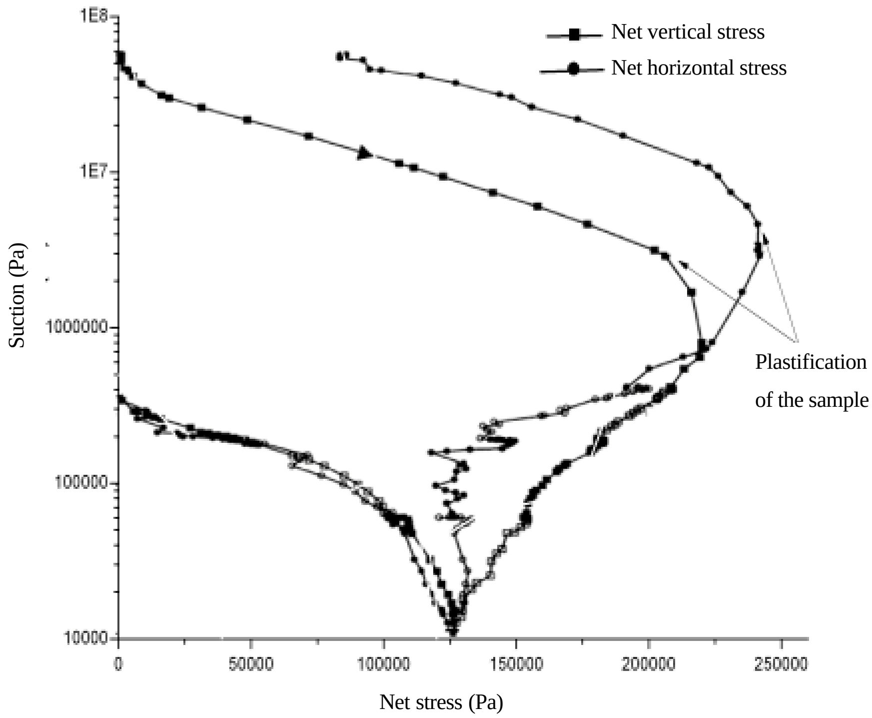

Fig. 1: Swelling pressure test on Boom clay (after Romero, 1999)

A full details of the numerical algorithm are given in Bendani *et al*. (2008). A comparison between the predicted and the experimental results is then made.

The objective of the study is to check the ability of model to correctly predict the complex trend.

MATERIALS AND METHODS

Theory and formulation: The deformation of unsaturated soil is coupled to both stress variation and moisture movement. An extended critical state model proposed by Alonso *et al*. (1999) is adopted here to relate the deformation in the variations of stress and suction in the soil.

Stress-strain constitutive relationship: A double structure elastoplastic model based on the framework for expansive soils presented by Gens and Alonso (1992) is described here. This framework assumes that a double structure model can describe the behaviour of expansive soils. The double structure is defined as consisting of a macro- and a microstructure. A slightly swelling model is utilised to describe the macrostructure of the soil, defined as the level at which the major structural arrangements take place (Alonso *et al*., 1990). The section of the model describing the microstructure, defined as the level at which basic swelling of active minerals occurs, assumes a saturated state and an effective stress law may be applied. Only reversible strains are allowed to develop in the microstructure whilst irreversible strains can develop in the macro level. Coupling of the two levels allows only the microstructure to influence the macrostructure. This coupling provides a mechanism that allows the development of strains in the microstructure to lead to softening/hardening of the macrostructure by the development of irreversible strains. This is accomplished by assuming that the irreversible strains will occur when the effective stress goes beyond either a maximum or a minimum level. The authors propose that these maximum and minimum levels of effective stress be defined by a pair of yield surfaces between which the stress state of the soil lies. These are known as the suction increase SI and the suction decrease SD yield surfaces, respectively. Furthermore, it is assumed that these two lines are coupled, leading to the movement of either when a load path move to the other.

Following the approach presented by Gens and Alonso (1992), the increment of total strain can be defined as:

$$d\varepsilon = d\varepsilon^{e}_{Mp} + d\varepsilon^{e}_{Ms} + d\varepsilon^{e}_{m} + d\varepsilon^{p} \qquad (1)$$

where, the subscripts M and m represent the macro and micro contributions, respectively and the superscripts e and p represent elastic and plastic contributions.

The expression for the macrostructural elastic volumetric strain component due to stress changes is given by Alonso *et al.* (1990) as:

$$d\varepsilon^{e}_{Mp} = -\frac{\kappa}{\nu}\frac{dp}{p} \tag{2}$$

where, κ is the stiffness parameter for changes in net mean stress p in elastic region and ν is the specific volume.

Alonso *et al.* (1990) also defined the macrostructural elastic volumetric strain component due to suction changes, at a constant stress, as:

$$d\varepsilon^{e}_{Ms} = -\frac{\kappa_s}{\nu}\frac{ds}{(s+p_{atm})} = A_s ds \tag{3}$$

where, κ_s is the stiffness parameter for changes in suction in elastic region, s is suction and p_{atm} is the atmospheric pressure.

As discussed earlier, the framework is based on the assumption that the microstructure remains saturated even when the macrostructure has a negative pore water pressure and that the principle of effective stress will therefore be valid within the aggregates. In term of net mean stress and suction, effective mean stress is defined as (Alonso *et al.*, 1994). The microstructural elastic volumetric strain component $d\varepsilon^{e}_{m}$ is therefore expressed as a function of, that is (Alonso *et al.*, 1994):

$$d\varepsilon^{e}_{m} = \beta_m e^{-\alpha_m(p+s)} d(p+s) = A_{(p+s)} d(p+s) \tag{4}$$

where, α_m and β_m are material parameters and $A_{(p+s)}$ is the microstructural stiffness.

The yield function related to the preconsolidation stress, for isotropic stress states is identical to the yield function presented by Alonso *et al.* (1990) for the slightly swelling model, i.e.:

$$\text{LC} \qquad F_1(p,q,s,p_0) = q^2 - M^2(p-p_s)(p_0-p) = 0 \tag{5}$$

where, p is the net mean stress, q is the deviatoric stress and p_0 is the preconsolidation stress, p_s is the parameter controlling suction effect on cohesion and M is the slope of the critical state line.

The preconsolidation stress p_0, was defined as:

$$\left(\frac{p_0}{p_c}\right) = \left(\frac{p_0^*}{p_c}\right)^{\left(\frac{\lambda(0)-\kappa}{\lambda(s)-\kappa}\right)} \tag{6}$$

$$\lambda(s) = \lambda(0)\left[(1-r)\exp(-\beta s) + r\right] \tag{7}$$

where, p_0^* is the preconsolidation stress of saturated soil, p_c is the reference stress, $\lambda(0)$, $\lambda(s)$, β and r are the parameters controlling the stiffness of the soil.

In addition to the Load Collapse (LC) yield surface, two further equations are required to define SI and SD yield surfaces. Based on the above assumptions of the definition of these surfaces, Alonso *et al.* (1994) presented them as:

$$s_I \qquad F_2(p, s, s_I) = (p+s) - s_I = 0 \tag{8}$$

$$s_D \qquad F_3(p, s, s_D) = s_D - (p+s) = 0 \tag{9}$$

where, s_I and s_D are stress parameters that can physically be defined as the level of suction at which, under zero mean confining stress, irreversible shrinkage or swelling strains are produced on drying or wetting paths. The authors also present two yield loci to define the amount of plastic strains produced when the surfaces yield. From these, two hardening laws for the SI and SD yield surfaces were presented (Alonso *et al.*, 1994) as:

$$ds_I = \frac{1}{h_I(p,p_0,s_I)} d\varepsilon^{p}_{m} \tag{10}$$

$$ds_D = \frac{1}{h_D(p,p_0,s_D)} d\varepsilon^{p}_{m} \tag{11}$$

where, h_I and h_D are functions defining the relationship between microstructural elastic and plastic strains. With suitable substitution of Eq. 1 the stress-strain relationship can be expressed as:

$$d\sigma'' = D\left(d\varepsilon - d\varepsilon^{e}_{Ms} - d\varepsilon^{e}_{m} - d\varepsilon^{p}\right) \tag{12}$$

which can be rewritten as:

$$d\sigma'' = D\left[d\varepsilon - \left(A_s + A_{(p+s)}\right)ds - A_{(p+s)}dp - d\varepsilon^{p}\right] \tag{13}$$

where, σ'' is the net stress.

The above constitutive relationship has been incorporated in a more general numerical model to achieve a solution satisfying the stress equilibrium equation:

$$\frac{\partial(\sigma_{ij} - \delta_{ij}u_a)}{\partial x_j} + \frac{\partial u_a}{\partial x_i} + b_i = 0 \tag{14}$$

where, δ_{ij} is the Kronecker delta function and b_i is the body force.

Moisture transfer: The velocity of liquid is based on the generalised Darcy's law and is then expressed as:

$$V_w = -K_w\left[\nabla\left(\frac{u_w}{\gamma_w}\right) + \nabla z\right] \quad (15)$$

where, V_w is the velocity of the liquid, K_w is the non saturated hydraulic conductivity, u_w is the pore water pressure, γ_w is the unit weight of water et z is the reference position and ∇ is the gradient operator.

An expression of the degree of saturation is obtained from the experimental results performed on Boom clay by Centro International de Metodos Numericos en Ingeniera, UPC, Spain, which gives:

$$S_w = \frac{\log(s) - 8.5}{-4.6} \qquad 10\text{ kPa} < s < 23600\text{ kPa} \quad (16)$$

where, s is the suction in Pa.

The hydraulic conductivity relationship for Boom clay is expressed by Alonso *et al.* (1995b) as:

$$K_w = \frac{1.3\times10^{-12}}{1 + 1.35\times10^{-10}(s)^{1.692}}\ \text{m sec}^{-1} \quad (17)$$

The governing equation for moisture transfer in an unsaturated soil can be expressed as:

$$\frac{\partial(\rho_w n S_w)}{\partial t} + \nabla.(\rho_w V_w) = 0 \quad (18)$$

where, ∇ is the divergence operator.

Dry air transfer: Air in unsaturated soil is considered to exist in two forms: bulk air and dissolved air (Fredlund and Rahardjo, 1993). In this approach, the proportion of dry air contained in the pore liquid is defined using Henry's law.

The pore air velocity is assumed to satisfy the Darcy's law, i.e.:

$$V_a = -K_a\nabla\left(\frac{u_a}{\gamma_a}\right) \quad (19)$$

where, K_a is the air conductivity, u_a is the pore air pressure and γ_a is the unit weight of air.

Alonso *et al.* (1995b) have proposed a relationship for air conductivity for Boom clay as:

$$K_a = 1.31\times10^{-19}\frac{\gamma_a}{\mu_a}\left[e(1-S_r)\right]^{2.47}\quad \text{m sec}^{-1} \quad (20)$$

where, μ_a is the air viscosity.

To take into account the dissolved air in water, Henry's law is used and the governing equation of the air flow can be expressed as (Thomas and Sansom, 1995):

$$\frac{\partial(S_a + H_s S_w)}{\partial t} = -\nabla.\left[\rho_a(V_a + H_s V_w)\right] \quad (21)$$

where, H_s is Henry's volumetric coefficient of solubility and ρ_a is the density of air.

Numerical algorithm: The complexity and coupled nature of the governing differential equations presented in previous sections prevents a direct analytical solution. A numerical approach is then required and the formulation of this solution is based on the finite element method for the spatial discretisation and on a backward difference mid-interval time stepping scheme for temporal discretisation. Before the elastoplastic relationship can be developed, the plastic contribution needs to be addressed. A viscoplastic algorithm has been found to give efficient solutions to elastoplastic problems (Thomas and Harb, 1990).

In this study the Galerkin weighted residual approach (Zienkiewicz and Taylor, 2000) is employed. Two dimensional eight noded isoparametric elements are used. The system of the three Eq. 14, 18 and 21 can be discretised spatially and converted as:

$$\begin{bmatrix} K_{ww} & K_{wa} & 0 \\ K_{aw} & K_{aa} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} u_w \\ u_a \\ \mathbf{u} \end{Bmatrix} + \begin{bmatrix} C_{ww} & C_{wa} & C_{wu} \\ C_{aw} & C_{aa} & C_{au} \\ C_{uw} & C_{ua} & C_{uu} \end{bmatrix}\begin{Bmatrix} \dot{u}_w \\ \dot{u}_a \\ \dot{\mathbf{u}} \end{Bmatrix} + \begin{Bmatrix} f_w \\ f_a \\ f_u \end{Bmatrix} = \{0\} \quad (22)$$

u_w, u_a and **u** are pore water pressure, pore air pressure and displacement respectively. The dot above variables refers to the derivative with respect to time. K_{ij} and C_{ij} represent the corresponding matrices of the governing equations with (i, j = w, a, **u**).

For simplicity, it is convenient to rewrite Eq. 22 in the following form:

$$K\{\varphi\} + C\left\{\frac{\partial\varphi}{\partial t}\right\} + \{f\} = \{0\} \quad (23)$$

where, $\{\varphi\}$ represents the global unknown vector, i.e., $\{u_w\ u_a\ \mathbf{u}\}^T$.

To solve Eq. 23, a general form of fully implicit mid-interval backward difference time stepping algorithm is used to discretise the governing equation temporally. The solution algorithm is given by Bendani *et al.* (2008).

The number of corrector iterations required for convergence is related to the size of the time step used. This property has been used to modify the time step size used as a simulation processes. If the number of iterations exceeds a specified value, the time step size is reduced by a factor. Likewise, if the number of iterations required falls below a specific value, the time step is then increased by a factor. This variable time stepping scheme helps to provide an efficient solution algorithm.

Suction controlled swelling pressure test: The test was carried out in a suction controlled oedometer capable of measuring lateral stress which had been modified to ensure zero horizontal deformation. The vertical tress was measured by varying the oil pressure in the confining stress. This was achieved by monitoring the vertical strain via strain gauge and allowing any vertical strain to be cancelled by a change in the oil pressure. This oil pressure was therefore a measure of the vertical stress of the oil.

The samples were prepared from a Boom clay obtained from the HADES underground laboratory in Mol, Belgium. The soil was compacted to a water content of 3% and a dry density of 2000 kg m^{-3}. This material was then crushed and sieved to isolate aggregates of a size of 2 to 2.2 mm. These aggregates was then compacted to a dry density of 1400 kg m^{-3} to produce the testing samples.

The sample was compressed with the moisture content remaining constant within a suction controlled oedometer apparatus capable of monitoring the variation of vertical and horizontal stress. The loading path is shown in Fig. 2. The applied loading was removed resulting in a reduction of vertical displacement. This process was completed when a zero net vertical stress had been achieved. This unloading path is also shown in Fig. 2. The sample now at the required dry density and moisture content was then placed in the suction controlled swelling pressure apparatus.

During the test, deformation of the sample was prevented. A constant air pressure of 500 kPa was applied to the sample through the porous stone at the top of the sample. The suction of the sample was controlled by raising or lowering the pore water pressure, applied through the porous stone at the base of the sample. The flow of water both into and out of the sample was allowed to occur freely through the porous stone at the base of the sample. The planned load path for suction of the sample was to reduce the suction to a value of 10 kPa from the initial value of 79420 kPa and then raised it to a value of 450 kPa. The test was halted during the redrying stage at a suction of 373 kPa, as the sample was reported to have begun shrinking, thereby breaking the requirement of zero deformation. No further details of the test were

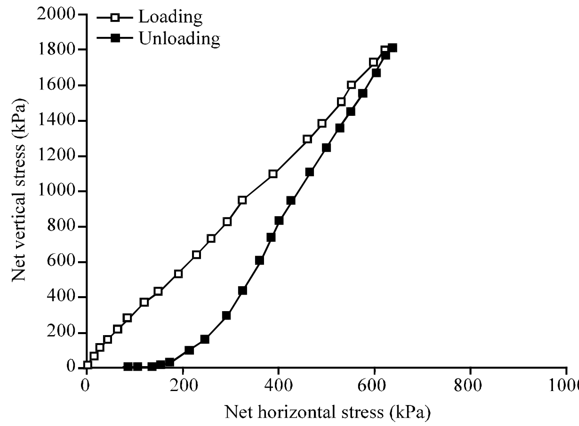

Fig. 2: Stress path during sample preparation

Table 1: Material parameters used in the numerical example

Parameters	Values
$\lambda(0)$	0.25
r	0.86
β (MPa^{-1})	1.25
β_m	2.3×10^{-7}
α_m	5.0×10^{-7}
ρ_w (kg m^{-3})	1000
γ_a (kN m^{-3})	12.75×10^{-3}
μ_a (Ns m^{-2})	1.1×10^{-5}
κ	0.06
p_c (kPa)	10.0
κ_s	0.0085
h_I	$92\times10^{-7}\exp\left(-5\times10^{-7}s_D\right)\left(1-{p}/{p_0}\right)^{1.1}$
h_D	$46\times10^{-7}\exp\left(-5\times10^{-7}s_I\right)\left(1-{p}/{p_0}\right)^{1.1}$
M	1
h_s	0.1

available until the blind prediction of the development of swelling pressure due to the suction variations had been made.

Numerical simulation of swelling pressure test: Here, a blind prediction of the swelling pressure test is presented.

The initial void ratio is given a value of e = 0.94. The initial suction of the sample was set at the initial reported value of 79420 kPa. The initial stress of the sample was taken as the measured values of stress at the completion of the sample preparation stage with zero net vertical stress and a net horizontal stress of 85 kPa.

The simulation reduced the suction of the sample from its initial value to the value of 10 kPa in a series of steps so the predicted increase in net stress could be recorded. The model and parameters used for the blind prediction are shown in Table 1.

The position of the yield surfaces also needs to be defined. This is achieved by defining the values of the hardening parameters p_0^*, s_I and s_D. The SD and SI yield surfaces are assumed to be just above the initial point in (p, s) space. The values assigned of the relevant hardening parameters are:

s_I = 79477.0 kPa
s_D = 79470.0 kPa

As no further information is available to define the initial position of the LC yield surface the hardening parameter was initially set at the value of:

$p_0^* = 90$ kPa

The simulation was then performed and the value of suction on the rewetting path at which a negative stress in the soil was predicted was compared to the value reported for shrinkage of the sample experimentally measured. The value of saturated preconsolidation stress p_0^* was then revised until the two values of suction corresponded to each other. The value of p_0^* which was used for the blind prediction was $p_0^* = 120$ kPa.

The predicted development of stress within the soil is shown in Fig. 3. It can be shown that a peak value of horizontal stress of 195 kPa and a peak value of vertical stress of 110 kPa was reached at a suction of 440 kPa. The horizontal swelling pressure at a suction of 10 kPa was 124 kPa with the vertical swelling pressure at a value of 26.7 kPa. The vertical swelling pressure reached a value of 0.0 kPa at a suction of 373 kPa and the simulation was terminated at this point.

Experimental results: The behavior of the sample during the wetting path can be divided into two distinct phases. Initially the wetting of the sample caused a development of both vertical and horizontal net stress with a peak values of 217 and 239 kPa being reached at a suction of 348 kPa. At this level of suction the development of swelling pressure in the sample ceased. On further wetting, the sample exhibited a reduction in net stress, with vertical and horizontal net stress reducing to values of 124 and 121 kPa, respectively at a lowest suction value of 10 kPa. The horizontal and vertical net stress converged at that stage and remained at similar values during the redrying path. The redrying path showed a further reduction of net stress with a value of zero being reached in both planes at a suction of 373 kPa.

RESULTS AND DISCUSSION

Numerical example: The domain geometry of size 10×100 mm was discretised into 10 eight noded isoparametric elements equally sized (10×10 mm). The results for this mesh have been found to give spatially converged results when compared to a finer mesh. The deformation was only allowed to develop in the vertical direction. All the parameters and functions used in the formulations are shown in Table 1.

Comparison of model prediction and experimental results: Here, a comparison of the blind prediction and the experimental results is made. To assist comparison Fig. 3 shows each set of results graphically.

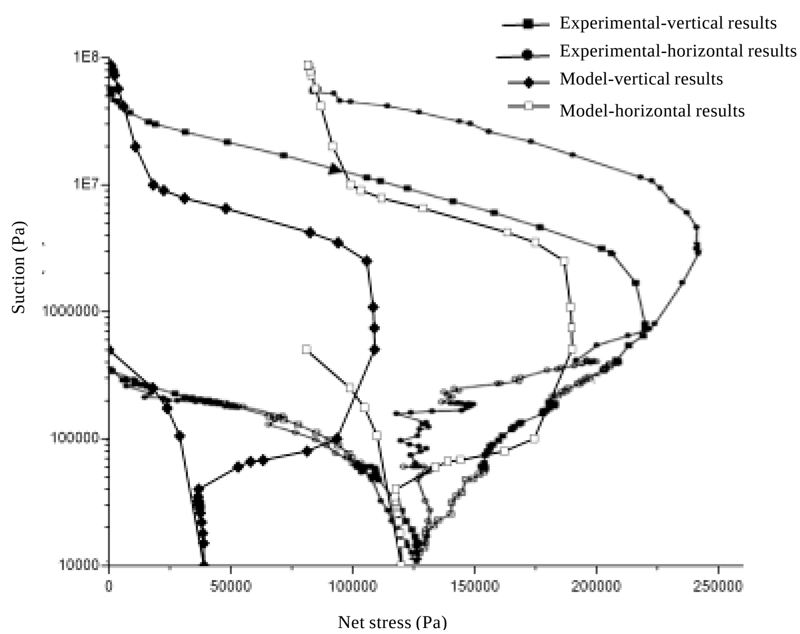

Fig. 3: Comparison of experimental and predicted results

From Fig. 3, it can be observed that the simulation correctly predicts the trends of behavior exhibited by the sample during the experiment. Significantly, the phenomenon of the swelling pressure increase and subsequent reduction during the wetting path is correctly predicted. The prediction of the redrying path correctly shows a further stress reduction within the sample. In terms of magnitude, the simulation under predicted the peak swelling pressure and predicted its occurrence at a lower suction than was experimentally observed. The swelling pressure at maximum wetting was correctly predicted for the horizontal net stress but significantly under predicted for the vertical net stress.

Experimentally the horizontal and the vertical net stresses converge during the later part of the wetting path and stay at similar values during the redrying path. The simulation predicted that the initial difference in horizontal and vertical net stress would remain constant during the test. In terms of the model this is a result of the assumption that a suction change will affect the sample isotropically.

The result of the blind prediction, in comparison with experimentally measured behavior, is very encouraging. The ability of the model to correctly predict the complex trends of behavior during a swelling pressure test has been demonstrated. Also, a qualitative comparison shows that the simulation has given a reasonable prediction of the development of net stress in the sample.

CONCLUSION

The study presented in this research describes recent research performed on the development of a model to describe the coupled flow of moisture and air in expansive soils. The representation of the development of swelling pressure within an expansive clays has been investigated via a double structure elastoplastic relationship. The blind prediction simulation performed has produced a set of very encouraging results. The model has been able to represent the experimentally measured trends of behaviour of the sample during initial wetting and drying paths.

REFERENCES

Alonso, E.E., A. Gens and A. Josa, 1990. A constitutive model for partially saturated soils. Geotechnique, 40: 405-430.

Alonso, E.E., A. Gens and W.Y.Y. Gehling, 1994. Elastoplastic Model for Unsaturated Expansive Soils. In: Num. Meth. in Geotech. Engrg., Smith, I. (Ed.0. A.A. Balkema, Rotterdam, ISBN: 90 54105100, pp: 181-186.

Alonso, E.E., A. Lloret, A. Gens and D.Q. Yang, 1995a. Experimental behaviour of highly expansive double structure clay. In: Proceedings of 1st International Conference on Unsaturated Soils, 1995 Paris, pp: 11-16.

Alonso, E.E., A. Lloret, A. Gens, C.H. Delahaye, J. Vaunat and G. Volckaert, 1995b. Coupled analysis of a backfill hydration test. Proceeding of the International Workshop Hydro-Thermo-Mechan. Eng. Clay Barriers and Geological Barriers, July 1995, McGill University, Montreal, Quebec, Canada, pp: 5-7.

Alonso, E.E., J. Vaunat and A. Gens, 1999. Modeling the mechanical behaviour of expansive clays. Eng. Geol., 54: 173-183.

Bendani, K., H. Missoum, H. Khelafi and N. Laredj, 2008. Modelling the hydro-mechanical behaviour of highly expansive clays. Asian J. Applied Sci., 1: 206-216.

Bernier, F., G. Volckaert, E.E. Alonso and M. Villar, 1997. Suction controlled experiments on Boom clay. Eng. Geol., 47: 325-338.

Cui, Y.J. and P. Delage, 1996. Yielding and plastic behaviour of an unsaturated compacted silt. Geotechnique, 46: 291-311.

Dif, A.E. and W.F. Bluemel, 1991. Expansive soils under cyclic drying and wetting. Geotech. Testing J., 14: 96-102.

Fredlund, D.G. and H. Rahardjo, 1993. Soil Mechanics of Unsaturated Soil. 1st Edn. John Wiley and Sons, Inc., New York.

Gens, A. and E.E. Alonso, 1992. A framework for the behaviour of unsaturated expansive clays. Can. Geotechnol. J., 29: 1013-1032.

Pousada, E., 1984. Deformabilidad de arcillas expansivas bajo succion controlada. Ph.D Thesis, Universidad Politecnica De Madrid, Spain.

Romero, E., 1999. Characterization and thermo-hydro-mechanical behavior of unsaturated boom clay: An experimental study. Doctoral Thesis, Universitat Politecnica de Catalunya, Barcelona, Spain, pp: 405.

Thomas, H.R. and H.M. Harb, 1990. Analysis of normal consolidation of viscous clay. J. Eng. Mech. ASCE., 116: 2035-2053.

Thomas, H.R. and M.R. Sansom, 1995. A fully coupled analysis of heat, moisture and air transfer in unsaturated soil. J. Eng. Mech. Am. Soc. Civ. Eng., 121: 392-405.

Zienkiewicz, O.C. and R.L. Taylor, 2000. The Finite Element Method. 5th Edn. Butterworth-Heinemann, Oxford, UK, ISBN-10: 0340759844, ISBN:13: 978-0340759844.