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A Methodology for Analyzing the Transient Availability and Survivability of a System with the Standby Components in Two Cases: The Identical Components and the Non-Identical Components

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Abstract: In this study, a method for transient analysis of availability and survivability of a system with the standby components is presented. The availability and survivability of the standby systems is evaluated in two cases, the standby systems with the identical components and the standby systems with the non-identical components. In this study the Markov models, eigen vectors and eigen values for analyzing the transient availability and survivability of the system are employed. The method is implemented through an algorithm which is tested in MATLAB (matrix laboratory) programming environment. The new method enjoys a stronger mathematical foundation and more flexibility for analyzing the transient availability and survivability of the system.

Key words: Markov models, reliability, standby, eigen vector, eigen value

INTRODUCTION

Reliability has been a major concern for the system designers. One of the most important systems in reliability is the standby system. In this system, the whole components are not employed at the moment, it means that in each moment there is just one part or component that is employed and as soon as the failure of the operating component, the system switch on another well component.

Many systems consist of components having various failure modes. Several studies have presented a K-out-of-N system subject to two failure modes. Moustafa (1996) presented Markov models for analyzing the transient reliability of K-out-of-N: G systems subject to two failure modes. Moustafa (1996) proposed a procedure for obtaining closed form of the transient probabilities and the reliability for non-repairable systems. Another research effort is the research of Pham and Pham (1991), which has considered [k, n-k+1]-out of-n: F systems subject to two failure modes. Shao and Lamberson (1991) presented a model for k-out-of-n: G system with load sharing.

Zhang *et al*. (2000) presented circular consecutive 2-out-of-n repairable system with one repairman. They determined rate of occurrence of failure, mean time between failures, reliability and mean time to first failure. Li *et al*. (2006) presented a k-out-of-n system with independent exponential components. They assigned that some working components are suspended as soon as the system is down, repair starts immediately when a component fails and repair times are independent and exponentially distributed. Also they determined mean time between failures, mean working time in a failure repair cycle and mean down time in a failure-repair cycle.

Another attempt is the study conducted by Sarhan and Abouammoh (2001), who applied the concept of shock model to derive the reliability function of a k-out-of-n non-repairable system with non-independent and non-identical components. Later El-Gohary and Sarhan (2005) extended Sarhan and Abouammoh (2001) study by proposing a Bayes estimator for of a three non-independent and non-identical component series system under the condition of four sources of fetal shock. Sarhan and Abouammoh (2001) supported their estimation method by presenting a simulation study and showed how one can utilize the theoretical results obtained in their study.

Azaron *et al*. (2006) introduced a new methodology, by using continuous-time Markov processes and shortest path technique, for the reliability evaluation of an L-dissimilar-unit non-repairable cold-standby redundant system. Amiri and Ghassemi (2007a) introduced a method for transient analysis of availability and survivability of a system with repairable components using Markov models, eigen values and eigen vectors. The considered system was supposed to consist of n identical components and k repairmen which components are arranged in series or in

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k-out-of-n or in parallel. they proposed a methodology for obtaining availability, survivability, $MTTF_s$ (Mean time to system failure) of the system and calculating the duration for the system to reach to its steady state. Amiri and Ghassemi (2007b) introduced a method for analyzing the transient reliability of systems with identical components and identical repairmen using Markov models, eigen values and eigen vectors and assumed that the components of the systems under consideration can have two distinct configurations, namely; that can be arranged in series, or in parallel. they also considered third case in which the system is up (good) if k-out-of-n components are good. For all three cases they proposed a procedure for calculating the transient probability of the system availability and the duration of the system to reach the steady state.

In this study, a methodology for transient analysis of availability and survivability of a system with standby components is presented in two cases: the identical components and the non-identical components. In this study, a methodology for obtaining availability, survivability, $MTTF_s$ (Mean time to system failure) of the system and calculating the duration for the system to reach to its steady state is proposed.

NOMENCLATURE AND DEFINITIONS

N (t) = No. of components failed before time t
N'(t) = No. of repaired components before time t
X(t) = No. of failed components at time t

$$X(t) = N(t) - N'(t) \quad (1)$$

$p_n(t)$ = Probability of having n failed components at time t

$$p_n(t) = P(X(t) = n) \quad (2)$$

A(t) = Probability of system to be up (good) at time t, regardless of its historical components failure and/or repair
A(∞) = Long time system availability or system reliability
$R_s(t)$ = Survivability function

Determines the probability that a system does not leave the set B of functioning states during the time interval (0 t);

$$R_s(t) = \sum_{j \in B} p_j(t) \quad (3)$$

MTTFs: Mean time to system failure;

$$MTTF_s = \int_0^{+\infty} R_s(t)dt \quad (4)$$

Definition 1: If Q considered as the state transient rate matrix and P(t) as the state transient probability in the exponential Markov chain with the continuous time, then P'(t) and $P_n(t)$ are defined as follows:

$$\begin{aligned} &1.\ P'(t) = P(t) \cdot Q \\ &2.\ P_n(t) = P_n(0) \cdot P(t) \end{aligned} \quad (5)$$

where, Q and P(t) are square matrixes and $P_n(t)$ and $P_n(0)$ are row vectors.

THE MODEL

In this study, our aim is the determining of availability, survivor function and MTTF of a system with the following assumptions:

- The components in the system are standby
- There are n independent components
- The system components are repairable
- There are k identical repairmen
- The lifetime of each component is exponentially distributed with the parameter λ
- The service time of each component by each repairman is exponentially distributed with the parameter μ
- As soon as failure of an operating component, the system switch on another well component

THE PROPOSED METHODOLOGY

To describe the proposed methodology for analyzing the system's transient reliability, consider a system having n components and k repairmen. In the case of standby system with identical components, it is considered that the system fails when all n components fail. also In case of standby system with non-identical components, it is considered that system fails when all n components fail. It is assumed that the time between two components failure is a random variable having the exponential distribution with the parameter λ. It is also assumed that there are k identical repairmen providing services to the system. The service time of a component is also an exponentially distributed random variable with the parameter μ. Present goal is to provide a methodology for analyzing the transient availability and survivability of the system and the time until the system is reached to its steady state. Considering X(t) as the number of failed components at time t, the following Markov models are considering (Fig. 1 for identical components and Fig. 2 for non-identical components):

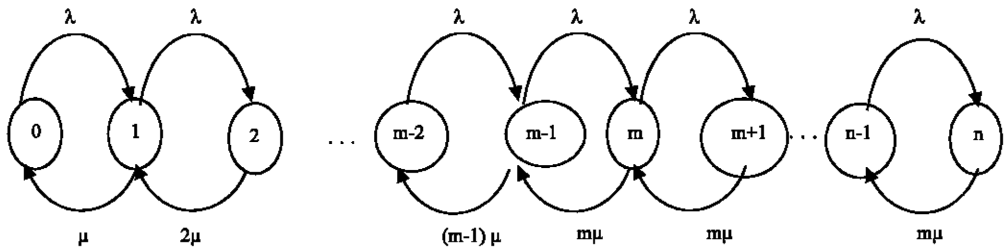

Fig. 1: State transition diagram of the system with k repairmen (identical components)

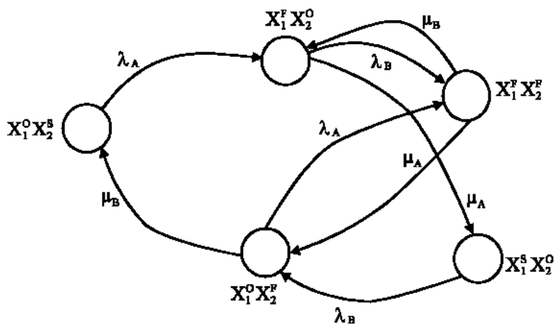

Fig. 2: State transition diagram of the system with k repairmen (non-identical components)

Figure 2 presents the standby system with two components, but the system can also be evaluated with more components.

Letter O in the Fig. 2, is the abbreviation of operating and letter F, is the abbreviation of failure and also letter S, is the abbreviation of standby.

As an example if considering n = 5 and k = 1 the Markov model is represented as follows:

The proposed methodology for obtaining the system availability and the transient probabilities are based on several theorems. These theorems are established to provide the underlying theory of our methodology. these theorems are presented as the following:

Theorem 1: Considering a continuous time exponential Markov chain in which $P'(t) = e^{Q\cdot t}$, therefore:

$$P(t) = e^{Q\cdot t}$$
$$P_n(t) = P_n(0)\cdot e^{Q\cdot t} \tag{6}$$

Proof

$$P'(t) = P(t)\cdot Q \Rightarrow \frac{dP(t)}{dt} = P(t)\cdot Q \Rightarrow \frac{dP(t)}{P(t)} = Q\cdot dt$$

$$\int \frac{dP(t)}{P(t)} = \int Q\cdot dt \Rightarrow \ln P(t) = Q\cdot t + C\cdot I \Rightarrow e^{\ln P(t)} = e^{Q\cdot t}\cdot e^{C\cdot I} \Rightarrow$$

$$P(t) = e^{Q\cdot t}\cdot e^{C\cdot I}$$

where, I is an identity matrix. Since P(0) = I then: $P(t) = e^{Q\cdot t}$ By Definition 1:

$$P_n(t) = P_n(0)\cdot P(t) = P_n(0)\cdot e^{Q\cdot t}.$$

Consider the following theorem [12].

Let consider Q as an n×n square matrix which has n non-repeating eigen values, then:

$$e^{Q\cdot t} = V\cdot e^{d\cdot t}\cdot V^{-1} \tag{7}$$

where, t represent time, V is a matrix of eigen vectors of Q, V^{-1} is the inverse of V and d is a diagonal eigen values of Q defined as follows:

$$d = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

And the matrix $e^{d\cdot t}$ is as follows:

$$e^{dt} = \begin{pmatrix} e^{\lambda_1 t} & 0 & \cdots & 0 \\ 0 & e^{\lambda_2 t} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & e^{\lambda_n t} \end{pmatrix}$$

Theorem 2: Consider $P(t) = e^{Q\cdot t}$ in which Q is the transition matrix. In matrix Q one of the eigen values is zero and the remaining eigen values are the complex number with the negative real part.

Proof: Since in every row of transition matrix the summation of row elements is zero, it can deduced that one its eigen value of matrix Q is zero. By theorem 1 and relation (7):

$$P(t) = V\cdot e^{d\cdot t}\cdot V^{-1} = (p_{ij}(t))$$
$$p_{ij}(t) = \pi_j + \sum_{k=1}^{n-1} \alpha_{ijk}\cdot e^{\lambda_k\cdot t} \tag{8}$$

where, λ_k is the kth eigen value, α_{ijk}'s are constant values and π_j is the limiting probability. Using the contradictory concept, if it is assumed that one of the eigen values of Q is a complex number with positive real part then:

$$\lim_{t\to+\infty} p_{ij}(t) = \infty$$

which contradicts $\lim_{t\to+\infty} p_{ij}(t) = \pi_j$ and therefore the eigen values of Q are complex numbers with the negative real part.

Theorem 3: Consider $P(t) = e^{Q\cdot t}$ in which Q is the transition matrix, the time elapse until system reaches to the steady state ($P(t) = \Pi$) can be calculated by the following formula:

$$t = \frac{\ln\varepsilon}{S_r} \quad (9)$$

In which ε is a very small number (i.e., ε = 0.0001), S_r is the largest real part of the eigen values excluding the zero element of matrix Q and Π is a square matrix representing the limiting probabilities. The elements of matrix P(t) and Π are shown as follows:

$$P(t) = \begin{bmatrix} p_{00}(t) & p_{01}(t) & \cdots & p_{0n}(t) \\ p_{10}(t) & p_{11}(t) & \cdots & p_{1n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ p_{n0}(t) & p_{n1}(t) & \cdots & p_{nn}(t) \end{bmatrix}$$

$$\Pi = \begin{bmatrix} \pi_0 & \pi_1 & \cdots & \pi_n \\ \pi_0 & \pi_1 & \cdots & \pi_n \\ \vdots & \vdots & \ddots & \vdots \\ \pi_0 & \pi_1 & \cdots & \pi_n \end{bmatrix}$$

Proof

$$p_{kj}(t) = \pi_j + \sum_{m=1}^{n-1} \alpha_{kjm} \cdot e^{\lambda_m \cdot t} = \pi_j + \sum_{m=1}^{n-1} \alpha_{kjm} \cdot e^{(S_m + C_m \cdot i)\cdot t}$$

By theorem 2 all S_m are negative and $i = \sqrt{-1}$ (π_j, α_{kjm}, S_m and C_m are constant numbers). Now suppose S is greater then S_m, then for large values of t:

$$\varepsilon = e^{S_r \cdot t}$$
$$S_r \cdot t = \ln\varepsilon$$
$$t = \frac{\ln\varepsilon}{S_r}$$

Based on the proof of these theorems, an algorithmic procedure for calculating the availability and survivability of the system is proposed.

Algorithm

- Let i = 0
- Determine the transition matrix Q
- Determine the eigen values and eigen vectors of the matrix Q and Let i = i+1
- Determine $P(t) = V.e^{d\,t}.\ V^{-1}$
- Determine $P(t) = P_n(0).P(t)$ and if i = 1 go to step 6 and if i = 2 go to step 7
- Determine the availability of the system according to the type of the system as follows:

*For a system with standby components (identical components):

$$A(t) = 1 - p_n(t) \quad (10)$$

*For a system with standby components (non-identical components):

$$A(t) = 1 - p_n(t) \quad (11)$$

where, n in the statement 11, is the state that all n components are failed.

Then delete the nth row and nth column of the matrix Q and go to step 3.

- Determine the survivability and $MTTF_s$ of the system with parallel components as follows:

$$R_s(t) = \sum_{j=0}^{n-1} p_j(t),\ \ MTTF_s = \int_0^{+\infty} R_s(t)dt \quad (12)$$

First numerical example (identical components): Consider a system having five identical components. There is one repairman for repairing this system. It is assumed that time to failure of repaired component is a random variable with exponential distribution function with the mean of 1/2 h. The repair time is also considered to be a random variable distributed exponentially with the mean of 1/10 h. The availability, survivability of the system at any given time is calculated. It is assumed that the components of the system are identical and as soon as the failure of the operating component, the system switch on another well component.

Solution: For determining of the system availability, the graphical Markov model can be presented as the Fig. 3.

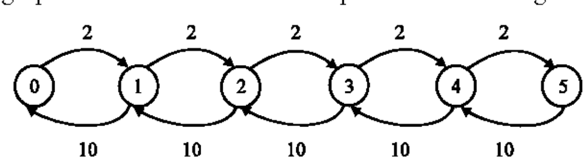

Fig. 3: State transition diagram of the system with 1 repairmen (identical components)

According to the algorithm:

$$Q = \begin{bmatrix} -2 & 2 & 0 & 0 & 0 & 0 \\ 10 & -12 & 2 & 0 & 0 & 0 \\ 0 & 10 & -12 & 2 & 0 & 0 \\ 0 & 0 & 10 & -12 & 2 & 0 \\ 0 & 0 & 0 & 10 & -12 & 2 \\ 0 & 0 & 0 & 0 & 10 & -10 \end{bmatrix}$$

$P_n(0) = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$

$P_n(t) = P_n(0).\ V\ .\ e^{dt}\ .\ V^{-1} = (p_0(t)\ \ p_1(t)\ p_2(t)\ p_3(t)\ p_4(t)\ p_5(t))$

$$\begin{aligned}
&p0(t)=0.8000+0.008440e^{-19.7t}+0.0555e^{-12t}+0.03917e^{-4,25404t} +0.06642e^{-7.527t}+0.03035e^{-16.47t}\\
&p1(t)=0.16001-0.01497e^{-19.7t}-0.05555e^{-12t}-0.008831e^{-4.254t}-0.036716e^{-7.527t}-0.043929e^{-16.472t}\\
&p2(t)=0.032002+0.009914e^{-19.74t}-0.01111e^{-12t}-0.01467e^{-4.254t}-0.02970e^{-7.527t}+0.0135e^{-16.472t}\\
&p3(t)=0.0064004-0.004683e^{-19.74t}+0.01111e^{-12t}-0.009601e^{-4.254t}-0.005940e^{-7.52t}+0.002714e^{-16.47t}\\
&p4(t)=0.001280+0.001645e^{-19.74t}+0.002222e^{-12t}-0.004502e^{-4.254t}+0.003283e^{-7.527t}-0.003929e^{-16.47t}\\
&p5(t)=0.0002560-0.0003376e^{-19.74t}-0.002222e^{-12t}-0.001567e^{-4.25t}+0.00265e^{-7.52t}+0.001214e^{-16.47t}
\end{aligned} \tag{13}$$

Now the system availability is calculated as follows:

$$\begin{aligned} &A(t) = 1 - p_5(t) \\ &A(\infty) = 0.999 \end{aligned} \tag{14}$$

Table 1 represents the probability of the system to be up (good) at time t, for different values of t.

Table 1: Elapse until system reaches to the steady state

t	$p_5(t)$	A(t)
0.05	4.85342E-08	0.999999951
0.10	1.01345E-06	0.999998987
0.15	4.86899E-06	0.999995131
0.20	1.31857E-05	0.999986814
0.25	2.62784E-05	0.999973722
0.30	4.33822E-05	0.999956618
0.35	6.31775E-05	0.999936823
0.40	8.42521E-05	0.999915748
0.45	0.000105380	0.999894620
0.50	0.000125639	0.999874361
0.70	0.000189428	0.999810572
1.00	0.000235168	0.999764832
1.20	0.000246824	0.999753176
1.50	0.000253395	0.999746605
2.00	0.000255700	0.999744300
3.00	0.000256012	0.999743988
4.00	0.000256016	0.999743984
5.00	0.000256016	0.999743984
6.00	0.000256016	0.999743984
7.00	0.000256016	0.999743984
8.00	0.000256016	0.999743984
9.00	0.000256016	0.999743984

By the following closed form formula, the time elapse until system reaches to the steady state can be calculated as follow:

$$t = (LN(0.00001))/(-4.254) = 2.706376$$

The limiting probability can also calculated as follows:

$\pi_0 = 0.800057$, $\pi_1 = 0.160012$, $\pi_2 = 0.032002$, $\pi_3 = 0.0064$, $\pi_4 = 0.00128$, $\pi_5 = 0.000256$

The amounts of $p_n(t)$ for different t values can be calculated. The results are represented in Table 2.

For determining of the system survivability and $MTTF_s$, according to the algorithm:

$$Q' = \begin{bmatrix} -2 & 2 & 0 & 0 & 0 \\ 10 & -12 & 2 & 0 & 0 \\ 0 & 10 & -12 & 2 & 0 \\ 0 & 0 & 10 & -12 & 2 \\ 0 & 0 & 0 & 10 & -12 \end{bmatrix}$$

$$\begin{aligned}
&P_0(t)=0.79937e^{-0.0020t}+0.069153e^{-5.225t}+0.012153e^{-19.42t}+0.042791e^{-15.33t}+0.074421e^{-10.01t}\\
&P_1(t)=0.16093e^{-0.00205t}-0.022491e^{-5.2256t}-0.021048e^{-19.420t}-0.05700e^{-15.338t}-0.060236e^{-10.013t}\\
&P_2(t)=0.031587e^{-0.00205t}-0.028536e^{-5.2256t}+0.01325e^{-19.4207t}+0.010399e^{-15.3382t}-0.026640e^{-10.0133t}\\
&P_3(t)=0.006186e^{-0.00205t}-0.01512e^{-5.2256t}-0.005609e^{-19.4207t}+0.007919e^{-15.338t}+0.0066195e^{-10.013t}\\
&P_4(t)=0.001026e^{-0.00205t}-0.004456e^{-5.2256t}+0.001511e^{-19.420t}-0.004745e^{-15.338t}+0.006663e^{-10.013t}
\end{aligned} \tag{15}$$

Now the system survivability and $MTTF_s$, can be calculated. The results are represented in Table 3.

$$R_s(t) = \sum_{j=0}^{4} p_j(t), \quad MTTF_s = 487.5 \tag{16}$$

Second numerical example (non-identical components): Consider a system having two non-identical components. There is one repairman for repairing this system. It is assumed that time to failure of repaired component is a random variable with exponential distribution function with the mean of 1/2 h for the first component and 1/3 h for the second component. The repair time is also considered to be a random variable distributed exponentially with the mean of 1/10 h for the first component and 1/15 h for the second component. The availability, survivability of the system at any given time

Table 2: Probability of having n failed components at time t

t	$p_0(t)$	$p_1(t)$	$p_2(t)$	$p_3(t)$	$p_4(t)$	$p_5(t)$
0.05	0.9242710	0.0723243	0.0033045	0.0001052	2.56E-06	4.853E-08
0.10	0.8806987	0.1096728	0.0090632	0.0005464	2.57E-05	1.013E-06
0.15	0.8544137	0.1298054	0.0144691	0.0012310	8.357E-05	4.869E-06
0.20	0.8378555	0.1411356	0.0188301	0.0020000	0.0001735	1.319E-05
0.25	0.8270190	0.1477830	0.0221508	0.0027445	0.0002843	2.628E-05
0.30	0.8196913	0.1518378	0.0246215	0.0034101	0.0004038	4.338E-05
0.35	0.8145978	0.1543999	0.0264461	0.0039784	0.0005225	6.318E-05
0.40	0.8109751	0.1560704	0.0277940	0.0044501	0.0006341	8.425E-05
0.45	0.8083485	0.1571897	0.0287941	0.0048352	0.0007351	0.0001054
0.50	0.8064134	0.1579577	0.0295408	0.0051462	0.0008242	0.0001256
0.70	0.8024059	0.1593606	0.0311001	0.0058836	0.0010683	0.0001894
1.00	0.8006497	0.1598664	0.0317779	0.0062609	0.0012179	0.0002352
1.20	0.8003027	0.1599540	0.0319098	0.0063415	0.0012532	0.0002468
1.50	0.8001242	0.1599966	0.0319772	0.0063841	0.0012725	0.0002534
2.00	0.8000649	0.1600102	0.0319994	0.0063985	0.0012792	0.0002557

Table 3: System survivability at time t

t	$p_0(t)$	$p_1(t)$	$p_2(t)$	$p_3(t)$	$p_4(t)$	$R_s(t)$
0.05	0.9221313	0.0726405	0.0033121	0.0001029	1.954E-06	0.99819
0.10	0.8785339	0.1101185	0.0090155	0.0005505	2.484E-05	0.99824
0.15	0.8522271	0.1303461	0.0143756	0.0012396	8.166E-05	0.99827
0.20	0.8356504	0.1417432	0.0187000	0.0020112	0.0001686	0.99827
0.25	0.8247988	0.1484371	0.0219919	0.0027562	0.0002735	0.99826
0.30	0.8174587	0.1525237	0.0244394	0.0034197	0.0003838	0.99823
0.35	0.8123551	0.1551071	0.0262446	0.0039828	0.0004899	0.99818
0.40	0.8087235	0.1567910	0.0275753	0.0044465	0.0005863	0.99812
0.45	0.8060885	0.1579179	0.0285590	0.0048208	0.0006702	0.99806
0.50	0.8041444	0.1586886	0.0292895	0.0051189	0.0007410	0.99798
0.70	0.8000783	0.1600673	0.0307821	0.0057941	0.0009159	0.99764
1.00	0.7981107	0.1604803	0.0313677	0.0060932	0.0010006	0.99705
1.20	0.7975393	0.1604951	0.0314553	0.0061431	0.0010155	0.99665
1.50	0.7969444	0.1604303	0.0314787	0.0061619	0.0010215	0.99604
2.00	0.7961015	0.1602739	0.0314568	0.0061611	0.0010221	0.99502
3.00	0.7944668	0.1599459	0.0313931	0.0061489	0.0010201	0.99297
4.00	0.7928374	0.1596178	0.0313288	0.0061363	0.0010180	0.99094
5.00	0.7912114	0.1592905	0.0312645	0.0061237	0.0010159	0.98891
6.00	0.7895887	0.1589638	0.0312004	0.0061111	0.0010138	0.98688
7.00	0.7879693	0.1586378	0.0311364	0.0060986	0.0010117	0.98485
8.00	0.7863533	0.1583124	0.0310726	0.0060861	0.0010097	0.98283
9.00	0.7847406	0.1579877	0.0310088	0.0060736	0.0010076	0.98082

is calculated. It is assumed that the components of the system are identical and as soon as the failure of the operating component, the system switch on another well component.

Solution: For determining of the system availability, the graphical Markov model can be presented as the Fig. 3 with $\lambda A = 2$; $\lambda B = 3$; $\mu A = 10$; $\mu B = 15$;

According to the algorithm:

$$Q = \begin{bmatrix} -2 & 2 & 0 & 0 & 0 \\ 0 & -13 & 0 & 3 & 10 \\ 5 & 0 & -7 & 2 & 0 \\ 0 & 5 & 10 & -15 & 0 \\ 0 & 0 & 3 & 0 & -3 \end{bmatrix}$$

$$P_n(0) = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$$

$$P_n(t) = P_n(0).\ V\ .\ e^{dt}\ .\ V^{-1} = (p_0(t)\ \ p_1(t)\ p_2(t)\ p_3(t)\ p_4(t)\ p_5(t))$$

$P_0(t)=0.499072+0.00490144e^{-29.6318t}-0.0125423e^{-15.0475t}-0.386867e^{-9.10368t}+0.895162e^{-6.217t}$

$P_1(t)=0.0964194-0.0098725e^{-29.6318t}-0.00115655e^{-15.0475t}-0.40175e^{-9.10368t}+0.31671e^{-6.217t}$

$P_2(t)=0.0665683-0.00903142e^{-29.6318t}+0.0109279e^{15.0475t}+0.183268e^{-9.10368t}-0.251723e^{-6.217t}$

$P_3(t)=0.0169387+0.0102971e^{-29.6318t}+0.00181965e^{15.0475t}-0.0527501e^{-9.10368}+0.0236833e^{-6.217t}$

$P_4(t)=0.321202+0.00370844e^{-29.6318t}+0.000949792e^{-15.0475t}+0.658218e^{-0.10368t}-0.984042e^{-6.217t}$

Now the system availability can be calculated as follows:

$A(t) = 1 - p_3(t)$

$A(\infty) = 0.983$

Table 4 represents the probability of the system to be up (good) at time t, for different values of t.

Table 4: Elapse until system reaches to the steady state

t	$p_3(t)$	A(t)
0.05	0.004031136	0.995968864
0.10	0.009367964	0.990632036
0.15	0.013106628	0.986893372
0.20	0.015345637	0.984654363
0.25	0.016575087	0.983424913
0.30	0.017191606	0.982808394
0.35	0.017456578	0.982543422
0.40	0.017530297	0.982469703
0.45	0.017507235	0.982492765
0.50	0.017441174	0.982558826
0.70	0.017153760	0.982846240
1.00	0.016980085	0.983019915
1.20	0.016951378	0.983048622
1.50	0.016940749	0.983059251
2.00	0.016938794	0.983061206
3.00	0.0169387	0.9830613
4.00	0.0169387	0.9830613
5.00	0.0169387	0.9830613
6.00	0.0169387	0.9830613
7.00	0.0169387	0.9830613
8.00	0.0169387	0.9830613
9.00	0.0169387	0.9830613

Table 5: Probability of having n failed components at time t

t	$p_0(t)$	$p_1(t)$	$p_2(t)$	$p_3(t)$	$p_4(t)$
0.05	0.9049	0.0709	0.0014	0.0040	0.0189
0.10	0.8216	0.1041	0.0071	0.0094	0.0580
0.15	0.7514	0.1183	0.0153	0.0131	0.1021
0.20	0.6940	0.1226	0.0242	0.0153	0.1440
0.25	0.6482	0.1221	0.0324	0.0166	0.1809
0.30	0.6124	0.1193	0.0396	0.0172	0.2117
0.35	0.5846	0.1158	0.0456	0.0175	0.2367
0.40	0.5634	0.1122	0.0505	0.0175	0.2566
0.45	0.5472	0.1090	0.0543	0.0175	0.2722
0.50	0.5350	0.1063	0.0573	0.0174	0.2842
0.70	0.5099	0.0998	0.0636	0.0172	0.3096
1.00	0.5008	0.0970	0.0661	0.0170	0.3193
1.20	0.4996	0.0966	0.0664	0.0170	0.3206
1.50	0.4992	0.0964	0.0665	0.0169	0.3211
2.00	0.4991	0.0964	0.0666	0.0169	0.3212

By the following closed form formula, the time elapse until system reaches to the steady state can be calculated.

$$t = (LN(0.00001))/(-6.217) = 1.852$$

The limiting probability can also calculated as follows:

$\pi_0 = 0.499072$, $\pi_1 = 0.096419$, $\pi_2 = 0.066568$, $\pi_3 = 0.016939$, $\pi_4 = 0.321202$

The amounts of $p_n(t)$ for different t, can be calculated. The results are represented in Table 5.

For determining of the system survivability and $MTTF_s$, according to the algorithm:

$$Q' = \begin{bmatrix} -2 & 2 & 0 & 0 \\ 0 & -13 & 0 & 10 \\ 5 & 0 & -7 & 0 \\ 0 & 0 & 3 & -3 \end{bmatrix}$$

$P_0(t)=0.541465e^{-0.370399t}-0.279643e^{-10.9019t}+0.0398818e^{-17.8014t}+0.698258e^{-5.9263t}$

$P_1(1)=0.0857736e^{-0.370399t}-0.266552e^{-10.9019t}-0.0166127e^{-17.8014t}+0.197411e^{-5.9263t}$

$P_2(t)=-0.182774e^{-5.9263t}+0.0588278e^{-0.370399t}-0.0420119e^{-17.8014t}+0.165959e^{-10.9019t}$

$P_4(t)=0.326094e^{-0.370399t}+0.337334e^{-10.9019t}+0.011221e^{-17.8014t}-0.674644e^{-5.9263t}$

The system survivability and $MTTF_s$ can be calculated. The results are represented in Table 6.

$$R_s(t) = p_0(t) + p_1(t) + p_2(t) + p_4(t) \qquad MTTF_s = 2.7346$$

Table 6: System survivability at time t

t	$p_0(t)$	$p_1(t)$	$p_2(t)$	$p_4(t)$	$R_s(t)$
0.05	0.904964585	0.069620344	0.000816	0.018666	0.994066742
0.10	0.820544826	0.099394258	0.004342	0.056533	0.980813881
0.15	0.747507876	0.109191174	0.009948	0.097654	0.964300921
0.20	0.685771920	0.109398815	0.016318	0.135031	0.946520047
0.25	0.634421358	0.105397978	0.022467	0.166152	0.928438317
0.30	0.592090678	0.099909719	0.027856	0.190656	0.910511664
0.35	0.557289722	0.094247491	0.032281	0.209123	0.892940588
0.40	0.528603037	0.088989601	0.035735	0.222472	0.875799306
0.45	0.504786541	0.084340510	0.038313	0.231661	0.859101502
0.50	0.484798434	0.080323672	0.040148	0.237564	0.842833761
0.70	0.428688115	0.069171227	0.042586	0.241128	0.781573666
1.00	0.375717301	0.059744983	0.040134	0.223361	0.698956314
1.20	0.347733744	0.055154919	0.037569	0.208528	0.648986212
1.50	0.310750277	0.049237972	0.033726	0.186997	0.580710822
2.00	0.258139384	0.040892539	0.028044	0.155455	0.482530871
3.00	0.178231186	0.028233646	0.019364	0.107339	0.333167490
4.00	0.123061278	0.019494167	0.013370	0.074113	0.230038419
5.00	0.084968740	0.013459918	0.009231	0.051172	0.158832047
6.00	0.058667413	0.009293519	0.006374	0.035332	0.109666982
7.00	0.040507431	0.006416792	0.004401	0.024395	0.075720531
8.00	0.027968711	0.004430530	0.003039	0.016844	0.052281905
9.00	0.019311242	0.003059099	0.002098	0.011630	0.036098501

CONCLUSION

In this study, a methodology for analyzing the transient availability and survivability of a system with the standby components was presented in two cases: the identical components and the non-identical components. We employed the Markov models, eigen vectors and eigen values concepts to develop the methodology for the transient reliability of such systems. The proposed methodology can also be employed for determining $MTTF_s$ and the time elapse until system reaches to the steady state.

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