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An Economic Order Quantity Under Joint Replenishment Policy to Supply Expensive Imported Raw Materials with Payment in Advance

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Abstract: An economic order quantity (EOQ) model with payment in advance is developed to purchase high-price raw materials. A joint policy of replenishments and pre-payments is employed to supply the materials. The rate of demand and lead time are taken to be constant and it is assumed that shortage does not occur in the cycles. The cycle is divided into three parts; the first part is the time between the earlier replenishment-time to the next order-time (t_0), the second part is the period between t_0 to a payment-time (t_k) and the third part is the period between t_k to the next replenishment-time. At the start of the second part (t_0), α% of the purchasing cost is paid. The (1-α)% remaining purchasing cost is paid at the start of the third part (t_k). The cost of the model is purchasing under incremental discount for each order, clearance cost, fixed-order cost, transportation cost, holding and capital costs. Holding cost is for on hand inventory and capital cost is for capital that is paid for the next order. The constraints of the problem are space, budget and upper limit for the number of orders per year. Also lead-time is considered less than a cycle time. The model of this problem is shown to be an integer-nonlinear-programming type and in order to solve it, a hybrid harmony search algorithm approach is used. At the end, a numerical example is given to demonstrate the applicability of the proposed methodology in real world inventory control problems.

Key words: EOQ Model, joint replenishment, payment in advance, imported raw materials, harmony search algorithm

INTRODUCTION

Since its formulation in 1915, the square-root-formula for the economic order quantity (EOQ) has been used in the inventory literature for a long time. This formula is based on the assumption of a constant demand. The discrete case of the dynamic version of EOQ was first discussed by Wagner and Whitin (1958). Regarding the continuous-time dynamic EOQ models, Silver and Meal (1969) were the first to suggest a simple modification of the classical square-root-formula in the case of time-varying demand. Later, Silver and Meal (1973) developed an approximate solution procedure, known as the Silver–Meal-heuristic, for general case of a deterministic, time-dependent demand pattern. However, Donaldson (1977) discussed, for the first time, the classical no-shortage inventory policy for the case of a linear, time-dependent demand. His treatment was fully analytical and needed extensive computational effort to obtain the optimal solution.

The question of inventory shortages and backlogging were not considered at all by the aforementioned researchers. Deb and Chaudhuri (1987) were the first to incorporate shortages into the inventory lot-sizing problem with a linearly increasing time-varying demand. EOQ models for deteriorating items with a trended demand were considered by Goswami and Chaudhuri (1991, 1992), Xu and Wang (1990), Chung and Ting (1993, 1994), Kim (1995), Hariga (1995, 1996), Benkherouf (1995), Jalan *et al.* (1996), Jalan and Chaudhuri (1999), Giri and Chaudhuri (1997) and Lin *et al.* (2000) etc. In the model of Deb and Chaudhuri (1987), shortages are allowed in all cycles except the last one. Each of the cycles in which shortages are permitted starts with replenishment and ends with a shortage which is backlogged in the next cycle. Numerous studies have been carried out to address the problems of imperfect quality EMQ model with rework (Hayek and Salameh, 2001; Chiu, 2003; Chiu *et al.*, 2004; Jamal *et al.*, 2004). Chiu and Chiu (2006) studied optimal replenishment policy

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for an imperfect quality EMQ model with backlogging and failure in repair using conventional approach. That is to derive the optimal lot size by utilizing differential calculus on the expected cost function with the need to prove optimality first. Grubbstrom and Erdem (1999) first introduced an algebraic method to solve the classic EOQ and economic production quantity (EPQ) model without the use of derivatives. A few researches have then been carried out using the same method (Cardenas-Barron, 2001; Wee and Chung, 2007). In these researches that extend the algebraic approach to an imperfect quality EMQ model examined by Chiu and Chiu (2006), the use of differential calculus is replaced with an algebraic method and the optimal lot size solution is obtained under the expected cost minimization.

In this study, the EOQ model is extended to purchase expensive imported raw material in a joint replenishment policy. In addition, holding costs (including capital, storage and insurance cost for on hand inventory), purchasing cost under incremental discount, clearance and fixed order costs, transportation cost and capital costs for the next order are employed to the problem. All parameter of the problem are crisp and the quantity of the orders must be integer multiples of packets, each containing more than one product. Also we assume that the lead time is less than a cycle length.

PROBLEM DEFINITION

Inventory holding as a tactical-level decision against non-secure situations to increase confidence level of responding to customer demands and also to resist against all kind of non-deterministic situations in receiving raw materials made inventory control and management an important concept in supply chain management. The model that is proposed in this research is an applied model developed based on the real constraints and environments of manufacturing companies. In these companies, the annual demands of different products are first estimated at the start of the year. Then, based on these estimates, the inventory and planning departments proceed to material planning. In some cases, improper material planning and control policies and loss of sufficient raw materials in proper times and quantities, result to customer complaints and loss of customer and market share. In some instances gaining the lost-market-shares needs more promotion and advertising costs and causes increased production cost. To confront with these instances and to minimize raw-material shortages, in this research a model for material planning and control is developed.

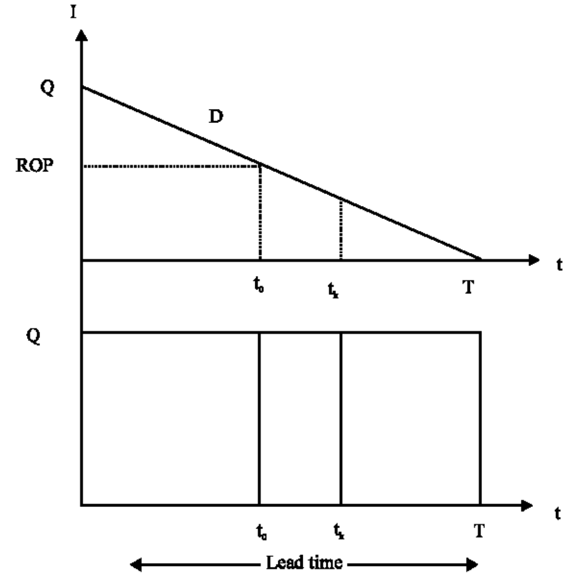

Fig. 1: Inventory control picture

In a manufacturing company, the steps involved in the ordering process of the materials are:

- Forecast the number of finished products
- Forecast the required raw-materials
- Order the required materials that consist of
  - Releasing orders of the materials to a supplier
  - Paying a percentage α% of the purchasing cost at t_0
  - Paying the remaining payment (1-α) at t_k
  - Transporting materials and receiving them at T

Figure 1 depicts the inventory control cycle of a material.

There are two payment methods in a material ordering process; (1) the credit transaction with a specific lead time and (2) cash method that has smaller lead time. In the cash method the purchasing cost of the total ordered material is paid to the supplier at the ordering time. However, in the credit transaction method α% of the material purchasing cost is paid at the order release time t_0 and the remaining (1-α)% is paid at the starting time of the material transportation t_k. The ordered materials are received at T.

Let us assume that the credit transaction method is used for payments in which the lead-time is deterministic and constant. Materials ordering and their transportation are done in batch form and the amount of raw materials in each batch and the number of batches in each vehicle is deterministic. Based on the product groups' consistency, the ordered materials can be carried together by finite-capacity vehicles with maximum capacity of F (in volume). In this study, we plan to determine the optimal order quantity of each material in a joint replenishment policy such that the total cost is minimized. The costs are (1) purchasing cost under incremental discount for each order, (2) holding cost for on hand inventory (including capital, warehouse and insurance costs), (3) capital cost of the next order, (4) transportation cost, (5) clearance cost and (6) fixed-order costs. Furthermore, we assume that the lead times are less than the cycle times of the materials.

The parameters and the variables required to model the problem are introduced further.

PROBLEM MODELING

The parameters and the variables: For i = 1,2,...,n and j = 1,2,...,P the parameters and the variables of the model are:

- P : Number of the products
- D_j : Annual constant demand of the jth product
- n_j : Number of items in the packets of the jth product
- h_{1j} : Holding cost per unit of on hand inventory of the jth product during a period
- h_{2j} : Capital cost per unit of the jth ordered product during the period before the payment of the remaining purchasing cost (10% of the total purchasing cost)
- h_{3j} : Capital cost per unit of the jth ordered product during the period after the payment of the remaining purchasing cost
- C^t_j : Transportation cost for each unit of the jth product
- C^c_j : Clearance cost for each unit of the jth product
- C^p_{ij} : Purchasing cost of the jth product in the ith discount break point
- q_{ij} : Ith discount break point of the jth product
- ROP_j : Reorder point of the jth product
- f_j : Space required for each packet of the jth product
- L : Constant joint lead time for each order
- A : Fixed order cost per each order
- A_T : Fixed transportation cost per each shipment.
- $\hat{f}$: Total available space in each trunk
- Q_j : A decision variable representing the order quantity of the jth product
- m_j : A decision variable representing the number of packets that have been ordered for the jth product
- TB : Total available budget
- T : Decision variable representing the joint cycle length
- N : Number of orders in each year (N = 1/T)
- N_T : Upper limit for number of orders
- C_H : Annual total holding cost of the products
- C_c : Annual total clearance cost
- C_P : Annual total purchasing cost of the products
- C_T : Annual total transportation cost of the products
- Z : Annual total costs

The objective function: The objective function of the model is to minimize the total cost of the joint replenishment problem given in Eq. 1.

$$Z = C_T + C_P + C_C + C_A + C_H \tag{1}$$

The terms in the right hand side of Eq. 1 are derived as follows.

Transportation cost (C_T): The transportation cost is calculated based on Eq. 2, in which $f_j m_j$ is the required space to ship the order of jth product from the supplier.

$$C^t_j = \begin{cases} A_T & ; \quad 0 < f_j m_j \le \hat{f} \\ 2A_T & ; \quad \hat{f} < f_j m_j \le 2\hat{f} \\ \vdots & \quad\quad \vdots \\ KA_T & ; \quad (K-1)\hat{f} < f_j m_j \le K\hat{f} \end{cases} \tag{2}$$

By introducing the binary variables Y_k; k = 1, 2,...,K, the annual total transportation cost (occurring N times per year) can be incorporated with the mathematical model of the problem as:

$$\begin{aligned} & C_T = N\sum_{k=1}^{K} kA_T Y_k = \frac{1}{T}\sum_{k=1}^{K} kA_T Y_k \\ & 0 < \sum_{j=1}^{P} f_j m_j \le \hat{f}Y_1 \\ & \hat{f}Y_2 < \sum_{j=1}^{P} f_j m_j \le 2\hat{f}Y_2 \\ & \vdots \\ & (K-1)\hat{f}Y_k < \sum_{j=1}^{P} f_j m_j \le K\hat{f}Y_k \\ & Y_1 + Y_2 + \cdots + Y_k = 1 \\ & Y_k = 0,1 \quad \forall k = 1, 2, \cdots, K \end{aligned} \tag{3}$$

Purchasing cost under incremental discount (C_p): The purchasing cost of the company for the jth product at each period and each order can be calculated using the incremental discount policy. It is necessary to indicate that, discount from supplier is considered for each order and not during a year. Let the incremental discount policy be:

$$C_j^P = \begin{cases} C_{1j}Q_j & ; \quad 0 < Q_j \le q_{1j} \\ C_{1j}q_{1j} + C_{2j}(Q_j - q_{2j}) & ; \quad q_{1j} < Q_j \le q_{2j} \\ \vdots & \\ C_{1j}q_{1j} + C_{2j}(q_{2j} - q_{1j}) + \ldots + C_{nj}(Q_j - q_{n-1,j}); & Q_j \ge q_{nj} \end{cases} \quad (4)$$

where, q_{ij} and C_{ij}; i = 1, 2,...,n are the discount points and the purchasing costs for each unit of the jth product that corresponds to its lth discount break point, respectively. In order to include the incremental discount policy in the inventory model, we use Eq. (5) to model the incremental discount policy.

$$\begin{aligned} & C_j^P = C_{1j}Q_{1j} + C_{2j}Q_{2j} + \ldots + C_{nj}Q_{nj} \\ & Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\ & q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j} \\ & (q_{2j} - q_{1j})\lambda_{3j} \le Q_{2j} \le (q_{2j} - q_{1j})\lambda_{2j} \\ & \vdots \\ & 0 \le Q_{nj} \le M\lambda_{nj} \\ & \lambda_{1_j} \ge \lambda_{2_j} \ge \cdots \ge \lambda_{n_j} \\ & \lambda_{i_j} = 0,1 \quad \forall i, \quad i = 1,2,\cdots,n, \text{ M is a very big No.} \end{aligned} \quad (5)$$

Finally the total annual purchasing cost will be (occurring N times per year):

$$\begin{aligned} & C_P = N\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} = \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \\ & Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\ & q_{1j}\lambda_{2j} \le Q_{1j} \le q_{1j}\lambda_{1j} \\ & (q_{2j} - q_{1j})\lambda_{3j} \le Q_{2j} \le (q_{2j} - q_{1j})\lambda_{2j} \\ & \vdots \\ & 0 \le Q_{nj} \le M\lambda_{nj} \\ & \lambda_{1j} \ge \lambda_{2j} \ge \cdots \ge \lambda_{nj} \\ & \lambda_{ij} = 0,1; \quad \forall j,\ j = 1,2,\cdots,P,\ \forall i,\ i = 1,2,\cdots,m \end{aligned} \quad (6)$$

Clearance cost (C_c): The clearance cost of the company for the jth product at each period is C^c_j, the order quantity is Q_j and the number of order per year is N. Hence, the total annual clearance cost of the company will be:

$$C_C = N\sum_{j=1}^{P} C_j^c Q_j = \sum_{j=1}^{P} C_j^c D_j \quad (7)$$

Fixed order cost (C_A): The fixed order cost of each order per period is A and the number of orders per year is N. Hence and the total annual fixed order cost in a disjoint ordering policy will be NA. However, since we are using the joint replenishment policy, the fixed order cost will change to:

$$C_A = A/T \quad (8)$$

Holding cost (C_H): According to Fig. 1, the holding cost during a cycle is:

$$\sum_{j=1}^{P} \frac{h_j^1 Q_j T}{2} \quad (9)$$

The first part of the capital cost occurs between the α% and (1-α)% payment times and is derived as:

$$\sum_{j=1}^{P} h_j^2 Q_j (t_k - t_0) \quad (10)$$

The other part of the capital cost occurs between $[t_k, T]$ and is calculated as:

$$\sum_{j=1}^{P} h_j^3 Q_j [L - (t_k - t_0)] \quad (11)$$

So, the total holding cost during a year will be:

$$C_H = N\left[\sum_{j=1}^{P} \frac{h_j^1 Q_j T}{2} + \sum_{j=1}^{P} h_j^2 Q_j (t_k - t_0) + \sum_{j=1}^{P} h_j^3 Q_j [L - (t_k - t_0)]\right] \quad (12)$$

Also, according to Fig. 1 we have;

$$Q_j = D_j T_j \quad (13)$$

knowing that,

$$N = \frac{1}{T} \rightarrow NT = 1 \quad (14)$$

Eq. (12) can be written as (15) for a joint order case.

$$C_H = \sum_{j=1}^{P} \frac{h_j^1 D_j T}{2} + \sum_{j=1}^{P} h_j^2 D_j (t_k - t_0) + \sum_{j=1}^{P} h_j^3 Q_j [L - (t_k - t_0)] \quad (15)$$

The constraints: As the total available warehouse space is F, the space required for each unit of the jth product is f_j and the order quantity of the jth product is Q_j, the space constraint will be:

$$\sum_{j=1}^{P} f_j Q_j \leq F \tag{16}$$

Furthermore, since the total available budget is TB and the purchasing cost that is calculated in earlier section is;

$$C_P = \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij}$$

the budget constraint will be:

$$\frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \leq TB \tag{17}$$

For the sake of convenience in ordering, clearance, transportation and some other activities, we assume an upper limit for the number of orders per year. In other words

$$N \leq N_T \rightarrow \frac{1}{T} \leq N_T \rightarrow T \geq \frac{1}{N_T} \tag{18}$$

Moreover, the quantities of the orders must be integer multiples of packets, each containing more than one product. That is

$$Q_j = n_j m_j \tag{19}$$

knowing that;

$$Q_j = D_j T \tag{20}$$

We have

$$TD_j = n_j m_j \tag{21}$$

Finally the model of the problem becomes:

$$\begin{aligned}
\text{Min}: Z &= C_T + C_P + C_C + C_A + C_H \\
&= \frac{1}{T}\sum_{k=1}^{K} kA_T Y_k + \frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} + \sum_{j=1}^{P} C_j^c D_j + \frac{A}{T} + \sum_{j=1}^{P}\frac{h_j^1 D_j T}{2} \\
&+ \sum_{j=1}^{P} h_j^2 D_j (t_k - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_k - t_0)] \\
&= \frac{1}{T}\left[A + \sum_{k=1}^{K} kA_T Y_k + \sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij}\right] + \left[\sum_{j=1}^{P}\frac{h_j^1 D_j}{2}\right]T \\
&+ \left[\sum_{j=1}^{P} C_j^c D_j + \sum_{j=1}^{P} h_j^2 D_j (t_k - t_0) + \sum_{j=1}^{P} h_j^3 D_j [L - (t_k - t_0)]\right]
\end{aligned}$$

s.t:

$$\begin{gathered}
\frac{1}{T}\sum_{j=1}^{P}\sum_{i=1}^{n} C_{ij}Q_{ij} \leq TB \\
\sum_{j=1}^{P} f_j Q_j \leq F \\
T \geq \frac{1}{N_T} \\
0 < \sum_{j=1}^{P} f_j m_j \leq \hat{f}Y_1 \\
\hat{f}Y_2 < \sum_{j=1}^{P} f_j m_j \leq 2\hat{f}Y_2 \\
(K-1)\hat{f}Y_k < \sum_{j=1}^{P} f_j m_j \leq K\hat{f}Y_k \\
TD_j = n_j m_j \\
Q_j = Q_{1j} + Q_{2j} + \ldots + Q_{nj} \\
q_{1j}\lambda_{2j} \leq Q_{1j} \leq q_{1j}\lambda_{1j} \\
(q_{2j} - q_{1j})\lambda_{3j} \leq Q_{2j} \leq (q_{2j} - q_{1j})\lambda_{2j} \\
\vdots \\
0 \leq Q_{nj} \leq M\lambda_{nj} \\
Y_1 + Y_2 + \cdots + Y_k = 1 \\
\lambda_{1j} \geq \lambda_{2j} \geq \cdots \geq \lambda_{nj} \\
\lambda_{ij} = 0,1; Y_k = 0,1 \quad \forall k = 1,2,\cdots,K, \; \forall j, j = 1,2,\cdots,P, \forall i, i = 1,2,\cdots,m \\
T \geq 0 \\
m_j, Q_j \geq 0 \ \text{integer}
\end{gathered} \tag{22}$$

A solution algorithm: Since the model in Eq. (22) is integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). Hence, we need to employ a meta-heuristic search algorithm to solve it.

A harmony search algorithm: New ways have been found to optimize problems for less than a century, but nature has used various ways of optimization for millions of million years. Recently scientists mimicked nature to solve different kinds of complex optimization problems. Most of these problems are so complicated and time consuming that we cannot use an exact algorithm to solve them. Thus, typically some non-precise algorithms are used to find a near optimum solution in a shorter period. We call these algorithms meta-heuristic (Dorigo and Stutzle, 2004). Many researchers have successfully used meta-heuristic methods to solve complicated optimization problems in different fields of scientific and engineering disciplines. Some of these meta-heuristic algorithms are simulating annealing (Aarts and Korst, 1989; Kirkpatrick *et al*., 1994; Taleizadeh *et al*., 2008), threshold accepting (Dueck and Scheuer, 1990). Tabu search (Joo and Bong, 1996), genetic algorithms

(Al-Tabtabai and Alex, 1999), neural networks (Gaiduk *et al*., 2002) ant colony optimization (Dorigo and Stutzle, 2004), fuzzy simulation (Taleizadeh *et al*., 2009), evolutionary algorithm (Laumanns *et al*., 2002) and harmony search (Lee and Geem, 2004; Geem *et al*., 2001).

Lee and Geem (2004) and Vasebi *et al*. (2007) showed that the Harmony Search (HS) algorithm outperforms Genetic Algorithm (GA) (Goldberg, 1989) because of its multi-vector consideration and fast computation. One of the main advantages of HS versus GA is its simple implementation. Unlike GA that has genetic operators like crossover and mutation, the HS algorithm does not have these types of operators for new generations. This causes an iteration to be faster in HS than that in GA. Therefore, we employ a HS algorithm to solve the models under different criteria.

The HS algorithm (Geem *et al*., 2001), which is inspired from the act of musician groups, was introduced in an analogy with music improvisation process where musicians in an ensemble continue to polish their pitches in order to obtain better harmony. Similar to musician groups when several notes from different musical instruments are played simultaneously by set of the pitch adjusting on a random basis to achieve pleasant harmony in several practices, this algorithm seeks the optimum solution by generating random vector solutions in a Harmony Memory (HM) that are improved iteration by iteration with some pitch adjusting and updating methods to reach global optimum. In summary, according to the analogy of improvisation and optimization, fantastic harmony is considered as global optimum, aesthetic standard is determined by the objective function, pitches of instruments are desired values of the variables and each practice is the same in each iteration.

The HS optimization method has been applied successfully to various engineering problems such as satellite heat pipe design (Geem and Hwangbo, 2006), vehicle routing (Geem *et al*., 2005a), water network design (Geem *et al*., 2002, 2005a,b) and structural design (Lee and Geem, 2004). Mahdavi *et al*. (2007) described an Improved Harmony Search (IHS) algorithm for solving optimization problems. IHS employs a novel method for generating new solution vectors that enhances accuracy and convergence rate of HS algorithm. They discussed the impacts of constant parameters on HS algorithm and presented a strategy for tuning these parameters.

Although HS algorithm has proven its ability of finding near global regions within a reasonable amount of time, it is comparatively inefficient in performing local search. Fesanghari *et al*. (2008) proposed a Hybrid Harmony Search (HHS) algorithm to solve engineering optimization problems with continuous design variables and employed a Sequential Quadratic Programming (SQP) model to speed up local search and improve precision of the HS solutions.

The HS optimization algorithm applied in this study is performed by the following steps.

Initialization: The process of initialization has two parts; parameter initialization and HM initialization as described below.

Parameter Initialization: The constant parameters of the HS algorithm include Harmony Memory Size (HMS), Harmony Memory Considering Rate (HMCR), Pitch adjusting Rate (PAR), Number of decision variables (N) and the maximum Number of Improvisations (NI).

The HMS is the number of simultaneous solution vectors in HM. Based on the frequently used HMS values in other HS applications available in the literature (Geem, 2006; Geem and Hwangbo, 2006; Geem *et al*., 2001, 2002, 2005b) it seems that using a small HMS is a good and logical choice with the added advantage of reducing space requirements. Furthermore, since HM resembles the short-term memory of a musician and since the short-term memory of the human is known to be small, it is logical to use a small HMS. In this study, the numbers 10, 20 and 30 are chosen as different values of HMS.

The HMCR is the probability of choosing HM. Choosing a very small HMCR decreases the algorithm efficiency and the HS algorithm behaves like a pure random search, with less assistance from the HM. Hence, it is generally better to use a large value for the HMCR (i.e., ≥ 0.9). In this research 0.93, 0.95 and 0.99 have been used for HMCR.

The pitch adjustment is similar to the adjustment of each musical instrument in a jazz so that pleasing harmony can be achieved. The efficiency of the algorithm lies within this pitch adjustment because of the fact that once a feasible design is determined, it searches new solution vectors around this design vector rather than generating arbitrary design vectors. Thus, this operation prevents stagnation and improves the HM for diversity with a greater chance of reaching the global optimum. The PAR is the probability of pitch adjustment where its typical value ranges from 0.3 to 0.99. In this research, 0.3, 0.7 and 0.9 have been utilized for PAR.

The value of N, the number of variables for optimization, is fully depended on the characteristics of the problem. For the proposed HS algorithm of this research, since the main variable is T and the other variables depend on T, the value of N has been chosen one.

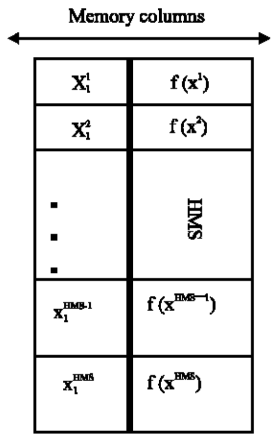

Fig. 2: The sample harmony memory

Finally, NI is the maximum number of iterations of the objective function evaluations. In this research, 100, 500 and 1000 are chosen as different iteration numbers.

Harmony memory initialization: The HM is a two-dimensional matrix with HMS rows and N+1 columns. The last column of HM is specified by the value of the objective function for each solution vector. Figure 2 shows a sample HM in which X^j_i is one of the decision variables used in HS algorithm, $f(x^j)$ is the value of the objective function for jth vector solution, i indicates the index of the decision variable in vector X and j is used as an index for the vector solution in HM.

The HM is initialized with randomly generated solutions in a specific range limited by upper and lower bounds determined by the problem at hand. However, because of the constraints, only those solution vectors that satisfy the constraints are included in HM.

New harmony generation: New Harmony improvisation is based on three rules (i) random selection (ii) HM consideration and (iii) pitch adjustment. In random selection rule, the new value of each decision variable $x'_i$is randomly chosen within the allowable range of the vector solutionX^j. Then, $X' = [x_1, x_2,...,x_n]$ will represent the new vector solution. In HM algorithm, the random choosing from HM occurs with probability HMCR and the random selection is performed with probability (1-HMCR). Algorithm (1) shows the choosing and the selection processes.

For $i = 1 : N(\text{here } N = 1)$

If $Rand_i < HMCR \quad ; \quad Rand_i \sim Uni(0,1)$

$$x'_i \leftarrow x'_i \in [x_i^1, x_i^2, \cdots, x_i^{HMS}]$$

Else

$x'_i \leftarrow$ generate a new one within the allowable range

End If

End For

Algorithm (1): The choosing and selection processes of HM algorithm

In pitch adjustment, every component obtained by the memory consideration, is examined to determine whether it should be pitch adjusted or not. The value of the decision variable is changed by Eq. (23) with probability of PAR and this value is kept without any change with probability 1-PAR. In Eq. (23) the BW stands for band width and denotes the amount of change for pitch adjustment. Also, rand is a uniform random number between 0 and 1. In this equation, for each component of the vector the selection for increasing or decreasing are carried out with the same probability.

$$X' = X' \pm (rand)(BW)\ ; \quad rand \sim U[0,1] \qquad (23)$$

Harmony memory update: The constraint handling part of the algorithm is performed before the HM update. There are three constraints in the proposed model. The constraint handling part checks whether these constraints are satisfied or not. If they are satisfied, then the HM update action occurs. In this stage, by the objective function evaluation, if the new fitness value is better than the worst case in the HM, the worst harmony vector is replaced by the new solution vector. The remaining steps of the HM algorithm are performed after the HM update.

Stopping criterion: The last step in a HS method is to check if the algorithm has found a solution that is good enough to meet the user's expectations. Stopping criteria is a set of conditions such that when satisfied a good solution is obtained. Different criteria used in literature are: 1) Stopping the algorithm after a specific number of iterations, 2) no improvement in the objective function and 3) Reaching a specific value of the objective function. In this research, we stop when a predetermined number of consecutive iteration is reached. The number of sequential iterations depends on the specified problem and the expectations of the user.
In summary, the steps involved in the HS algorithm used in this research are:

1: Initialize both the parameters and the HM of the HS algorithm

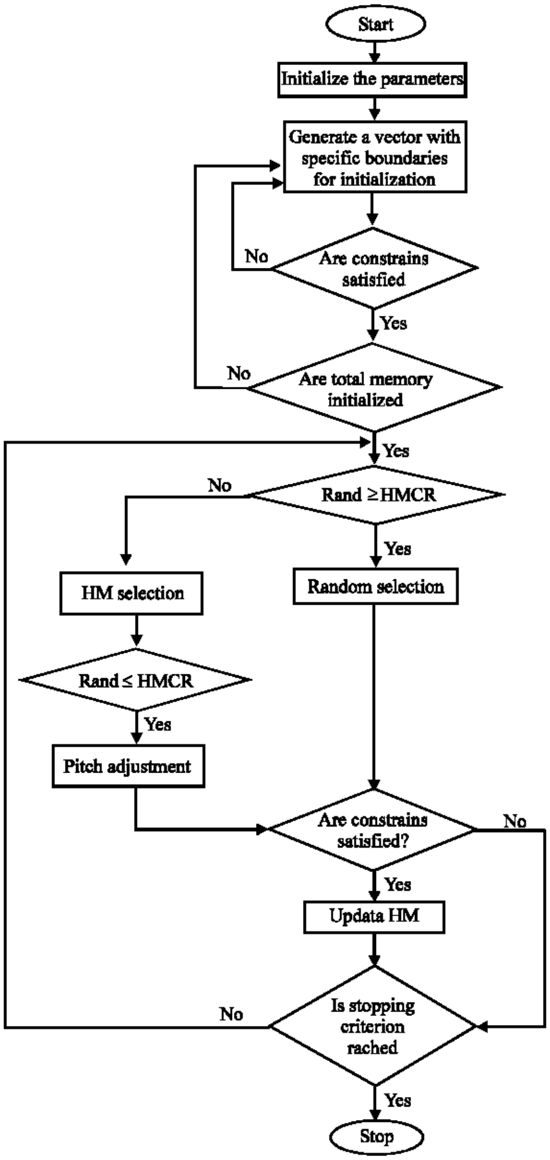

Fig. 3: Flow-chart of the proposed HS algorithm

2: Make a new vector $X^{'}$. For each component $x^{'}$:

- With probability HMCR pick the component from memory
- With probability 1-HMCR pick a new random value in the allowed range

3: Pitch adjustment: For each component $x^{'}_{i}$:

- With probability PAR, a small change is made to $x^{'}_{i}$
- With probability 1-PAR do nothing

4: If $X^{'}$ is better than the worst X^{j} in the memory, then replace X^{j} by $X^{'}$
5: Go to step 2 until a maximum number of iterations has been reached

The detailed flow-chart of the proposed HS algorithm is shown in Fig. 3.

NUMERICAL EXAMPLE

Consider a multi-product inventory control problem with sixteen products and general data given in Table 1. The total available warehouse space, the total budget and the maximum No. of order are F = 1000, TB = 400,000,000 and N_T = 6, respectively. Also, L = 2.5 months, A = 2000, A_T = 300,000, $\hat{f}$ =200, (t_0) = 0 and (t_k) = 1.5 months. Table 2 shows different values of the HS parameters used to obtain the solution. In this research, all of the possible combinations of the HS parameters are employed and using the min (min) criterion the best combination of the parameters has been selected. Table 3 shows the best combination of the HS algorithm and the best result is shown in Table 4. Furthermore, the convergence path of the best result of the objective function is shown in Fig. 4.

Table 1: Multi-product inventory control problem with sixteen products and general data

Product	D_j	n_j	h_{1j}	h_{2j}	h_{3j}	C^t_j	C^c_j	C^p_{1j}	C^p_{2j}	C^p_{3j}	q_{1j}	q_{2j}	q_{3j}	f_j
1	900000	30000	40	3	30	4	75	70	65	60	150000	250000	350000	4
2	850000	30000	40	3	30	5	75	70	65	60	150000	250000	350000	5
3	800000	30000	40	3	30	6	75	70	65	60	150000	250000	350000	6
4	750000	30000	40	3	30	4	75	70	65	60	150000	250000	350000	4
5	700000	30000	40	3	30	5	75	70	65	60	150000	250000	350000	5
6	650000	30000	50	4	40	6	100	95	90	85	75000	125000	200000	6
7	600000	30000	50	4	40	4	100	95	90	85	75000	125000	200000	4
8	550000	15000	50	4	40	5	100	95	90	85	75000	125000	200000	5
9	500000	15000	50	4	40	6	100	95	90	85	75000	125000	200000	6
10	450000	15000	50	4	40	4	100	95	90	85	75000	125000	200000	4
11	400000	15000	60	5	50	5	145	140	135	130	30000	60000	90000	5
12	350000	15000	60	5	50	6	145	140	135	130	30000	60000	90000	6
13	300000	5000	60	5	50	4	145	140	135	130	30000	60000	90000	4
14	250000	5000	60	5	50	5	145	140	135	130	30000	60000	90000	5
15	200000	5000	60	5	50	6	145	140	135	130	30000	60000	90000	6
16	150000	5000	60	5	50	4	145	140	135	130	30000	60000	90000	4

Table 2: The parameters of the HS algorithm

NI	PAR	HMCR	HMS
100	0.30	0.93	10
500	0.70	0.95	20
1000	0.90	0.99	30

Table 3: The best combination of the HS parameters

NI	PAR	HMCR	HMS
1000	0.70	0.95	10

Table 4: The best result of the HS algorithm

Product	T	Z	m_j	Q_j
1	0.2383 of Year	124,820,000	6	143010
2			7	154920
3			8	166840
4			8	178760
5			9	190680
6			5	202590
7			6	214510
8			6	59590
9			6	71500
10			7	83420
11			7	95340
12			8	107260
13			8	119170
14			10	131090
15			12	35750
16			15	47670

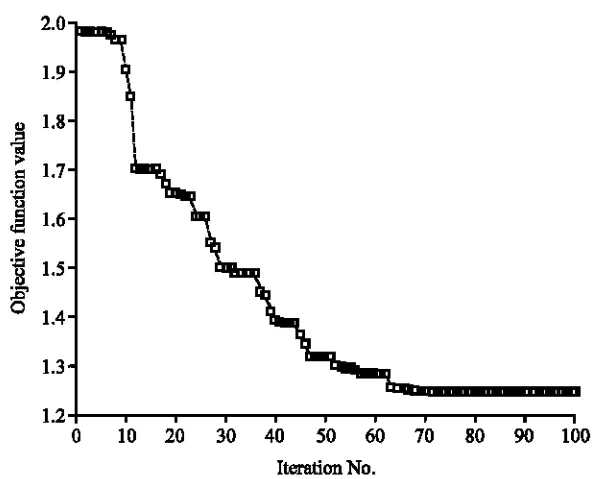

Fig. 4: Convergence path of the best result of objective function

CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this study, a joint replenishment multi-product multi constraint inventory model based on EOQ model to purchase high price raw materials was investigated. A mathematical modeling by incremental discount and transportation, clearance, fixed order, holding and shortage costs was introduced and shown to be integer-nonlinear programming problems. Then, a meta-heuristic algorithm (Harmony Search) has been proposed to solve the integer non-linear problem.

Some recommendations for future study follow:

- Shortage can be included in the model
- Stochastic demands can be taken into account
- Stochastic or fuzzy lead-time can be considered

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