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Determining Optimum Amount of Effective Factors in Automobile Brake Drum Rotatory Torque Using Response Surface Methodology

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Abstract: The present study aims at optimizing the rotatory torque of the automobile brake drum (as the response variable) getting help from the discussions in the design of experiments and response surface methodology. Considering the crucial function of Axle assy, especially the automobile brake drum due to its relevance to the safety of the automobile passengers, the study of the producing and assembling processes and conducting the quality control experiments during these stages is of great importance. With regard to the great significance of three main factors, namely, seal-oil spindle diameter, seal-oil internal diameter and nut lock torque as independent variables, the present research attempts to optimize the rotatory torque of the automobile brake drum. In the following parts the amount of the optimum torque has been calculated using the non-linear programming model.

Key words: Design of experiment, quality control, axle assy, response variable, response surface methodology

INTRODUCTION

The design of experiment is one of the powerful means for refining and improving the producing processes operations. The design starts with determining the goals of the experiment and selecting the process factors. In mathematical words, the goal is to find the operational conditions or factor levels (X_1, X_2, \dots, X_k), so that r response variables (Y_1, Y_2, \dots, Y_r), depending on the type of the problem, become minimized or maximized. In other words, the goals in the design of experiments and response surface methodology which are often conducted sequentially follow this order: extracting the elements or variables which are effective on response and adjusting their boundaries so that this may lead to the optimum response value. Investigations on the previous studies reveal that despite the salient developments in the design of experiments and in the analysis of statistical data as effective methods in improving the quality of productions of nutrition, chemical and medicine industries, the aforementioned optimization methods have not been sufficiently taken into account in other industries such as automobile industry. What follows will be an example of such cases:

In one study, Sobieszczanski-Sobieski *et al.* (2001) optimized the automobile trunk design concerning three limitations of sound intensity, vibration and inappropriateness of the environment getting help from the discussions in RSM. Marklund and Neilsen (2001) in

an article worked on the adjustment of the effective factors in automobile trunk design using the response surface methodology and the design of experiment. Chun *et al.* (2002) conducted statistical analysis and the design of experiment in order to recognize the parameters of the samples of inflexible materials. They described the scattered effects of the experiments data on the recognition of the materials parameters. The experiments results derived from doing three types of experiments on stainless steel AINSI SS 316 in 0° includes traction test, tension test in fixed pressure with an entr'acte and periodical pressure and traction test. Each experiment has been conducted with 12 samples in various pressure rates. For a statistical analysis to be done a large number of experiments is required. Random Simulation Method (RSM) is an acceptable technique in engineering which does not increase the complexity of materials recognition process. On the other hand, it requires a short time for recognition of materials and examines the safety coefficient on the most appropriate parameters of the materials. Rezzogh *et al.* (2005) in an article worked on the adjustment of the independent and effective factors in oil extraction using the response surface methodology. Also in this study, they optimized the oil extraction concerning three factors of processing pressure, the processing time and the moisture content getting help from the discussions in RSM. Dalvi *et al.* (1999) to obtain of optimum operation conditions in food industries getting help from the discussions in RSM. Also they determined

production maximum and spent cost minimum using the response surface methodology. Amiri and Salehi (2007) presented a methodology for optimizing statistical multi-response problems using goal programming. This method implemented through an algorithm which is tested in lingo programming environment. Also in this study, they used of discussions in RSM to model of response variables. Pasandideh and Niaki (2006) modeled a multi-response statistical optimization problem through the desirability function approach. Noorolsana and Sultanpanah (2003) offered a method for extracting and using decision function in optimizing multi-response problems. Amiri *et al.* (2007) conducted two case studies on the traction process of the rear fender sheet of the Pride 141 (name of an automobile). In their first case study, they explored the influential factors in the productions' quality (number of pieces in a sound sample) and the fender's traction mold lifetime by means of designing the experiments in the form of two factorial designs of 2^3 and 2^4 . In their second case study, they investigated the effective factors in development of the plastic injection mold process and they specified the dependent factors in the framework of the experiment's design objectives and examined them in a factorial design frame hereafter, they calculated and determined the acceptable values of the factors and dependent and independent variables by means of the response surface methodology and multi-purpose decisions.

MATERIALS AND METHOD

A brief description of the brake drum assembly line process: The brake drum spindle is one of the essential pieces in this process. The spindle consists of 4 bolts. These bolts put the spindle on the back plate. After the spindle is set on the back plate, the brake cylinder, the regulator lever and the shoe assy R.R. brake will be placed on it as well. After this stage, the cup bearing will set the small and big roller bearings in the brake drum and the seal-oil will be located on the brake drum assy. After making sure about the symmetric location of the shoes in the middle of the back plate and adjusting the brake cylinder furrow, we will put the brake drum assy on the back plate. Finally, all these pieces will be assembled by means of a washer and a nut lock. The measured torque of the brake drum's lock and rotatory nut at this stage should be in the range of 0.4-0.7 kg m and 6.5-10 kg cm⁻¹, respectively. After all these stages, we will install the nut lock cap and after setting the pin-cotter, we will bend the cap by a metal bar. After filling the nut lock cap with grease and drifting it, we will then tighten it with hammer strike.

Selection of effective factors and determination of their levels: According to the conducted investigations and the experts' consensus, the effective factors are as follows.

Seal-oil spindle diameter: As it is shown in Fig. 1, the spindle is made of steel following the NES standard or it is made of 1.07447 following the DIN standard. The spindle has 4 holes on its base and the thread features are M16×1.5-2 mm. The spindle will be set in the left and right brake back plate assy, one on each side. The connection of the brake back plate assy to the crust will be done by 4 spindle bolts that will be placed in each of the spindle's trunk that do the function of the brake drum; however, the spindle will remain fixed in the brake back plate assy. The smallness or the greatness of the seal-oil spindle diameter will cause more involvement and friction between the seal-oil and the spindle and consequently, this will lead to the increase or decrease of the brake drum rotatory torque. According to the above discussions, the size of the seal-oil spindle diameter is very important in determining the brake drum rotatory torque. Fig. 1 shows this piece. Moreover, the lower and upper levels of it are presented in Table 1 according to the technical map.

Seal-oil internal diameter: According to the KES-T-1168 standard, the seal-oil should be made of N.B.R. with the B code. One seal-oil will be used in the brake drum assy in order to prevent the leakage and penetration of grease into the brake back plate assy. The smallness or the greatness of the seal-oil internal diameter will cause more involvement and friction between the seal-oil and the spindle and consequently, this will lead to the increase or decrease of the brake drum rotatory torque. Figure 2 depicts this piece. Moreover, the lower and upper levels of it are presented in Table 2 according to the technical map.

Table 1: Lower and upper levels for the Seal-oil spindle

Effective factor	Lower level (-)	Upper level (+)
Seal-oil spindle diameter (mm)	38.938	39



Fig. 1: Seal-oil spindle

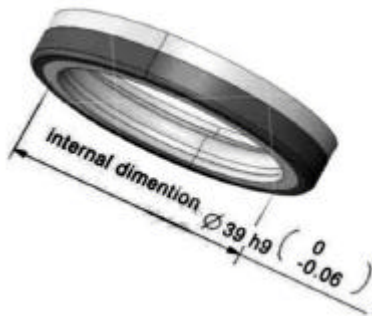


Fig. 2: Seal-oil

Table 2: Lower and upper levels for the seal-oil

Effective factor	Lower level (-)	Upper level (+)
Seal-oil internal diameter (mm)	38.938	39

Table 3: Lower and upper levels for the Nut lock torque

Effective factor	Lower level (-)	Upper level (+)
Nut lock torque (kg.m)	0.4	0.7

Nut lock torque: After assembling the nut lock, the tight or loose closure of the nut will determine the roller bearing's and the spindle's involvement and consequently, the brake drum rotatory torque. In a way that the smallness or the greatness of the nut lock torque will cause more and lesser involvement between the spindle and the roller bearing and consequently, this will lead to the increase or decrease of the brake drum rotatory torque. The lower and upper levels of this factor are shown in Table 3.

Response variable selection: Considering the fact that the response variable should contain significant characteristics of the under study process (Noorosana, 2005) and due to the researches findings, on one hand, the greatness of the brake drum rotatory torque is one of the influential factors in locking of the automobiles' rear wheels and reducing the roller bearing's lifetime and on the other hand, the smallness of this torque will lead in the looseness of the wheels. As a result, the aforementioned factors will threaten the safety of the automobile's passengers. Because of these reasons, in this study, the response variable of the brake drum rotatory torque has been taken into account.

Experiment design selection: Another significant step in conducting research is selection of an experiment design. The fitness and analysis of the response surface will be very easy if a suitable experiment design is selected.

Most of the problems related to RSM will use poly nominals and linear functions for fitness of models. If a linear function of independent variables gives a sound response, then we can say that the model's approximative function is first order. If there are curves in the system,

then poly nomials of higher orders like second order model should used. This article will also use a second order model which is shown below (Rezzoug *et al.*, 2005):

$$\hat{Y} = \beta_0 + \sum_{i=1}^3 \beta_i x_i + \sum_{i=1}^3 \beta_{ii} x_i^2 + \sum_{i=1}^3 \sum_{j=1, j \neq i}^3 \beta_{ij} x_i x_j \quad (1)$$

where,, \hat{Y} is the response variable, $\beta_0, \beta_i, \beta_{ii}, \beta_{ij}$ are the coefficients and x_1, x_2, x_3 are independent encoded factors. The soil-oil spindle diameter, the soil-oil internal diameter and the nut lock torque will be defined, respectively by the following linear equation:

$$x_i = \frac{2(X_i - \bar{X}_i)}{d_i} \quad (2)$$

where, X_i is the real value of i factor in its relevant dimension and d_i is the mean of the top and down levels of X_i . Also, d_i refers to the distance between the top and down levels of X_i (Rezzoug *et al.*, 2005; Dalvi *et al.*, 1999).

Rotation is considered as a very important factor in selection of response surface design. In this design, when rotation is done around a center $(0,0,\dots,0)$ the \hat{Y} variance will remain fixed. Because the purpose of RSM is optimization and since the location of the optimum point is not determined before doing the experiment, a design with the same degree of precision throughout will be selected. In this model, R^2 (manifold determining coefficient) has a greater value than the first order model or the first order model with corresponding relations. Also, in this model, MSE (Error Square Mean) has a smaller value than the previous two models. One of the designs used for fitness of second order model is the center composite design which includes 2^k (encoded with the common symbol of ± 1) joint with 2^k axial point $(\pm\alpha, 0, 0, \dots, 0), (\dots, 0, 0, \pm\alpha, 0)$ and \dots and $(0, 0, \dots, 0, \pm\alpha)$ and n_c number of central points $(0, 0, \dots, 0)$. The center composite design will become rotatory if α is given a sound value. The value of α to become rotatory depends on the number of the points in the factorial section of the design. In fact, by assuming that $\alpha = (n_f)^{1/4}$ it will be clear that the center composite design is rotatory. Here, n_f refers to the points in the factorial section of the design. In this article, three factors have been studied. Therefore, the factorial section consists of $n_f = 2^3 = 8$ points. Thus, the value of α should be $\alpha = (8)^{1/4} = 1.68$ to make the design rotatory. We can control other factors of the center composite design by selecting n_c number of central points. If the number of n_c is right, it is possible to make the center composite design reliable or use the n_c to make a design with fixed precision (Neter *et al.*, 1996). In this research, the number of central points required for making a design with fixed precision has been considered 5. The boundary of the

Table 4: The boundary of the levels, the central and axial points

Uncoded value factor	Factors	Coded level (x_i)				
		$-\alpha$	-1	0	1	α
Seal-oil spindle diameter (mm)	x_1	38.9169	38.938	38.969	39.0	39.021
Seal-oil internal diameter (mm)	x_2	38.9169	38.938	38.969	39.0	39.021
Nut lock torque (kg m l)	x_3	0.3	0.4	0.55	0.7	0.8

$\alpha = 1.6818$, N is the number of experiments of orthogonal design, i.e. of the factorial design. In this case $\sqrt[4]{N} = (\text{Axial distance}) \alpha$

Table 5: Experimental data and the observed response value with different combination of seal-oil diameter (x_1) seal-oil internal diameter (x_2) and nut lock torque (x_3) used in the randomized Central composite rotatable second order design for the response surface methodology

No. of experiment	Run	Coded variable level			Brake drum rotatory torque (Y) (kg cm^{-1})
		x_1	x_2	x_3	
1	16	-1	-1	-1	9.0
2	3	-1	1	-1	7.0
3	6	-1	-1	1	12.0
4	8	-1	1	1	9.0
5	9	1	-1	-1	18.0
6	1	1	1	-1	7.0
7	12	1	-1	1	23.5
8	7	1	1	1	11.0
9	2	$-\alpha$	0	0	6.5
10	14	α	0	0	21.0
11	5	0	$-\alpha$	0	19.0
12	15	0	α	0	6.0
13	13	0	0	$-\alpha$	7.0
14	18	0	0	α	12.0
15	10	0	0	0	9.0
16	19	0	0	0	8.5
17	11	0	0	0	9.5
18	17	0	0	0	8.0
19	4	0	0	0	9.0

levels, the central and axial points for the three composites mentioned above are showed in Table 4.

Conducting the experiment: Regarding the fact that the brake drum torque measurement is one of the operations that will be done during the assembling process and throughout this period it is impossible to halt the assembly line in respect of changing the boundaries of the effective factors, thus this experiment will be conducted in the laboratory. The method of conducting the experiment is in a way that first, we get the appropriate spindle and nut from the storehouse unit and give it to the laboratory unit. (Because spindle and spindle nut featured by α and $-\alpha$ do not exist in the laboratory, first of all, the order to produce these pieces should be offered to the producing company and the company will send these pieces to the laboratory for conducting experiments). We should notice that one of the most significant requirements of statistical analysis section is randomization of the experiments. Suppose that the experiments are numbered in the form of Table 5. Then a number between 1 and 19 is selected randomly, this process will continue till the 19th experiment is conducted. In this article the random numbers which will be used in conducting the experiments are as follows: 6, 9, 2, 19, 11, 3, 8, 4, 5, 15, 17, 7, 13, 10, 12, 1, 18, 14, 16. For example, in

experiment No. 1 with the features of -1,-1,-1, the seal-oil spindle diameter, the seal-oil internal diameter and the nut lock torque will all be considered at their lowest levels and then we will measure the response variable value.

The same procedures will be followed for all the experiments with 5 central points and 6 axial (environmental) points. The data on the following Table 5 showed the brake drum torque (response variable).

RESULTS AND DISCUSSION

Model curvature: The curve's total squares with the degree of freedom of 1 will be determined via the following expression (Myers and Montgomery, 2002):

$$SS_{\text{curvature}} = \frac{n_f n_c (\bar{y}_F - \bar{y}_C)^2}{n_f + n_c} \tag{3}$$

where, n_f is the number points on the factorial design and n_c is the number of central points, y_F and y_C are respectively the means of the factorial points and the central points.

The curvature test examines the following assumptions:

Table 6: Analysis of variance (ANOVA) to examine the existence of curvature in the model

Source	SS	df	MS	f-value
Curvature	32.75	1	32.75	100.77
Error	1.3	4	0.325	

Table 7: Test of significance for regression coefficients

Variable	df	Coefficient	SE	F-value	p> F
x ₁	1	3.43428	0.66472	26.69*	0.0003
x ₂	1	-3.68932	0.66472	30.80*	0.0002
x ₃	1	1.6783	0.66472	6.37*	0.0282
x ₁ x ₂	1	-2.3125	0.86811	7.10*	0.0220
x ₁ x ₃	1	0.5625	0.86811	0.42*	0.5303
x ₂ x ₃	1	-0.3125	0.86811	0.13	0.7257

*Significant

Table 8: Analysis of variance and regression for brake drum rotatory torque

Source	Sum of squares	df	Mean square	F-value	p> F
Model	484.7391	7	69.2484	59.57	<0.0001
Error	12.7872	11	1.1625		
Total	497.5263	18			

Coefficient of correlation (R²), 0.974298, Coefficient of determination (adjusted R²), 0.9579, Coefficient of variation, 9.662939%

$$H_0 : \sum_{j=1}^k \beta_{ij} = 0 \tag{4}$$

$$H_1 : \sum_{j=1}^k \beta_{ij} \neq 0$$

Due to the results obtained from Table 6 and the point that F_{0.05,1,4} = 12.22 and F* > F_{0.05,1,4} therefore the null hypothesis is rejected.

$$n_F = 2^3 = 8, n_C = 5, \bar{y}_F = 12.0625, \tag{5}$$

$$\bar{y}_C = 8.8, SSE = \sum_{i=1}^c (y_i - \bar{y}_c)^2, df_{curvature} = n_c - 1$$

Statistical analysis: The statistical results of the design, the fit regression model by SAS software and the distribution diagram which is presented in front of the response variable values in Fig. 3 and the normal probability diagram of Y in Fig. 4 is presented by MINITAB 14 software.

In Table 7 the results of the tests with the assumption that there are major and corresponding effects if we compare the statistic F* and F_{0.05,1,11} = 6.72 are calculated. And Table 8 shows the features of the fit model response variable.

$$\alpha = 0.05, F_{0.05,1,11} = 6.72 \tag{6}$$

The regression model for the response variable Y: The regression model for the brake drum rotatory torque is as follows:

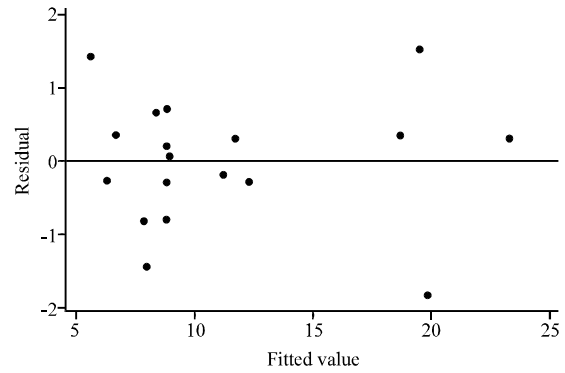


Fig. 3: The error distribution diagram

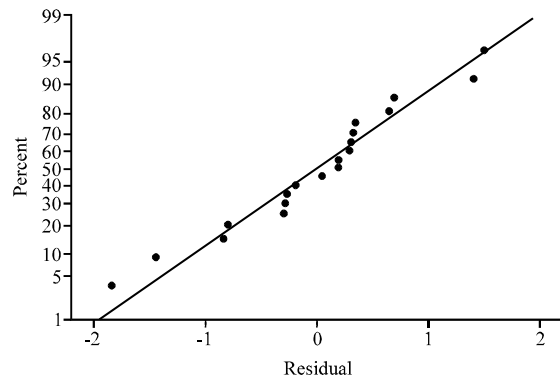


Fig. 4: The residuals' normal probability diagram

$$Y = 8.802 + 1.743x_1^2 + 1.3x_2^2 + 0.237x_3^2 + 3.434x_1 + 3.689x_2 + 1.678x_3 - 2.312x_1 \times x_2 \tag{7}$$

Study of error variance: If the model is sound and the fitted values are established, the residuals should be unstructured and not dependent on any other variable including the response variable. With regards to the error distribution diagram in front of the predicted values, the errors are also variance (Fig. 3).

In view of the variance analysis and the normal probability diagram, in the response variable Y the main effects x₁, x₂, x₃ and the paired counter-effect of x₁ × x₂ as effective factors will influence the brake drum rotatory torque. Thus, the regression model will be fit with the above mentioned effective factors.

Study of the model's efficiency: With the assumption that there is normal distribution of errors with the mean of zero, by drawing the residuals' normal probability diagram similar to a direct line, we may get to the model's efficiency (Fig. 4).

Obtaining an optimum response: In order to obtain an optimum response for the fit model, the equation's

response or the brake drum rotatory torque in the form of a mathematical programming problem and is written find an acceptable response if we consider $i = 1, 2, 3, x_i$ as encoded variables between -1 and 1.

General scheme of the problem's mathematical programming: Considering the fact that the brake drum torque value should be maximized in the range of 6.5-10 kg.cm⁻¹, therefore, we can formulize its mathematical programming model as follows:

$$\max Y = 8.802 + 1.743x_1^2 + 1.3x_2^2 + 0.237x_3^2 + 3.434x_1 - 3.689x_2 + 1.678x_3 - 2.312x_1 \times x_2 \quad (8)$$

s.t:

$$x_1 \geq -1 \quad (9)$$

$$x_2 \geq -1 \quad (10)$$

$$x_3 \geq -1 \quad (11)$$

$$x_1 \leq 1 \quad (12)$$

$$x_2 \leq 1 \quad (13)$$

$$x_3 \leq 1 \quad (14)$$

$$8.802 + 1.743x_1^2 + 1.3x_2^2 + 0.237x_3^2 + 3.434x_1 - 3.689x_2 + 1.678x_3 - 2.312x_1 \times x_2 \geq 6.5 \quad (15)$$

$$8.802 + 1.743x_1^2 + 1.3x_2^2 + 0.237x_3^2 + 3.434x_1 - 3.689x_2 + 1.678x_3 - 2.312x_1 \times x_2 \leq 10 \quad (16)$$

After solving the problem by using Lingo software the following responses will be obtained:

$$x_1 = 0.75 \Rightarrow X_1 = 38.992 \text{ mm} \quad (17)$$

$$x_2 = 0.75 \Rightarrow X_2 = 38.992 \text{ mm} \quad (18)$$

$$x_3 = 0.65 \Rightarrow X_3 = 0.65 \text{ kg.m} \quad (19)$$

The value of the predicted response (i.e., the brake drum rotatory torque) obtained through the Lingo software, $\hat{Y} = 10 \text{ kg.cm}$ is derived from the results of an empirical experiment. The obtained brake drum rotatory torque value was 10.5 kg cm which had a trivial difference of 0.05 compared with the Lingo software predicted value.

Alternatively, due to the fact that the diagram's model is cave, we can claim the absoluteness of the optimum response which is obtained through solving the above mentioned mathematical programming model.

CONCLUSION

In this article, in addition to studying and determining the effective factors in the brake drum rotatory torque value, these factors (seal-oil spindle diameter, seal-oil internal diameter and nut lock torque) were adjusted in such a way that the response variable value becomes optimum. The top and down levels and the central and axial points of the effective factors were presented in the experiments that were discussed in the previous sections and with regards to F value and level of significance $\alpha = 0.05$, all the main factors x_1, x_2, x_3 and the counter-effect of x_1, x_2 on the response variable was deemed as significant. The second order model which was presented in this study, due to its trivial MSE and its suitable determining coefficient and also the normal values with the mean of zero and the same variance has an appropriate efficiency for description of the data. In the next part, the amount of the effective factors were calculated by solving a non-linear programming model and by utilizing the results in an empirical experiment, the brake drum rotatory torque value was 10.5 kg.cm which had a trivial difference of 0.05 compared with the predicted value. Furthermore, this final result, because the model's shape is cave, is deemed as the absolute optimum value.

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