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Nonlinear Tracking Control on a Robot Manipulator in the Task Space with Uncertain Dynamics

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Abstract: This study was presented the design of a nonlinear robust controller based on Lyapunov method for tracking control of a robot manipulator in the task space. First of all, the dynamics of manipulator was divided to the known and unknown parts. The feedback linearization was used to remove the known nonlinear terms from the closed loop system which was advantageous to reduce the estimated uncertainties. Then the robust controller was designed to overcome uncertainties using their estimated maximum values. The proposed controller can guarantee the globally asymptotical stability. This control approach was studied on a two links elbow manipulator and can be applied for n degrees of freedom robot manipulators.

Key words: Robot manipulator, uncertainty, feedback linearization, robust nonlinear, control, task space

INTRODUCTION

It is well known that the dynamics of robots are highly nonlinear with large couplings and uncertainties in model. In spite of this, the most adopted controller in industrial settings is still the Proportional-Integral-Derivative (PID), with additional features like filters, feedforward actions and so on. Although simple controllers such as PID controllers are effective for regulating purposes in robotic applications, they cannot work well for tracking purposes. Instead, model-based robot controllers can work perfectly using exact models of the robot manipulator.

However, most research on robot control has assumed that the exact dynamics and kinematics of the manipulator from the joint space to the task space are known. This assumption leads us to several open problems in the development of robot control laws today. In free motion (Arimoto, 1999), this implies that the exact lengths of the links, joint offsets and the object which the robot is holding, must be known. Unfortunately, no physical parameters can be derived exactly. Moreover, when the robot picks up objects or tools of different lengths, unknown orientations and gripping points, the overall parameters are changing and, therefore, difficult to derive exactly. Therefore, the robot is not able to manipulate the tool to a desired position if the length or gripping point of the tool is uncertain. When the control problem is extended to the control of multifingered robot hands (Bicchi, 2000), such assumption also limits its potential applications because the kinematics is uncertain in many applications of robot hands. For example, the contact points of the robot fingers are uncertain and changing during manipulation. Similarly, in hybrid position force control (Yoshikawa, 2000), the assumption of exact kinematics also leads us to an open problem on how to control the robot if the dynamics and constraint are uncertain.

Robust control methods have been used for designing of control on robot manipulator in joint space. In these articles, it is proved that closed-loop system has asymptotic stability. But in these researches, problem of unstructured uncertainties remains, because, in design method of these controllers, only parameter uncertainties have been considered by Koo and Kim (1994), Liu and Goldenberg (1997) and Mnif *et al*. (1999).

With regard to the discussed case, even if controllers are designed that guarantee precise tracking in joint space, one can not state that this precise tracking is available in task space, because, in control method of robot in joint space, kinematical model is used for determination of position and end-effector. In presence of uncertainty, this model is not precise and therefore, the resulting error is not observed and compensated due to lack of feedback from position of end-effector (Cheah *et al*., 2003a, b; Dixon, 2004; Cheah and Liaw, 2005).

The most effective method for prevention from this error is to control robot in task space without inverse kinematical model (Cheah *et al*., 2003a, b; Dixon, 2004; Cheah and Liaw, 2005). In this method, information of task space is feed backed directly to the input control. Therefore, tracking error is directly observed and corrected.

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In order to compensate kinematical uncertainty, approximate Jacobin controllers have been provided for setpoint control in task space. Using the approximate Jacobin control approach, other open problems such as force control with uncertainties and control of robot fingers with uncertain contact points can be resolved in a unified formulation (Cheah *et al*., 1998, 2003a, b). Since tracking is important in most of the works done by robots, some of the researchers have concentrated their researches on trajectory control of robot manipulator.

Recently, adaptive Jacobian controllers have been provided for trajectory control of robot manipulator in task space (Cheah *et al*., 2004, 2005, 2006). These controllers don't need precise information about kinematics and Jacobin matrix. But in these articles, it is assumed that dynamics of robot have structured uncertainties. Therefore, the way in which unstructured uncertainties are dealt with is a great challenge and is considered as one of the research open problems.

In this study, by use of feedback linearization through direct Lyapunov method, robust nonlinear controller has been provided for trajectory control of robot manipulator in the task space with presence of structured and unstructured uncertainties.

ROBOT DYNAMICS

The dynamics of the robot with n degrees of freedom are nonlinear and can be expressed as (Qu and Dawson, 1996):

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d = u(t) \qquad (1)$$

where, $q(t)\in R^n$ denotes the joint angles of the manipulator, $\dot{q}(t)$ and $\ddot{q}(t)$ are the vectors of joint velocity and joint acceleration, respectively. $M(q)\in R^{n\times n}$ is the inertia matrix which is symmetric and positive definite, $V_m(q,\dot{q})\dot{q}\in R^n$ is a vector function containing coriolis and centrifugal forces, $G(q)\in R^n$ is a vector function consisting of gravitational forces. $F_d\in R^{n\times n}$ is a diagonal matrix of viscous and dynamic friction coefficients, $F_s(\dot{q})\in R^n$ is the vector of unstructured friction effects such as static friction terms. $T_d\in R^n$ is the vector of any generalized input due to disturbances or un-modeled dynamics and $u(t)\in R^n$ is the vector function consisting of applied generalized torques.

For simplicity Eq. 1, $H(q,\dot{q})$ can be shown as:

$$H(q,\dot{q}) = V_m(q,\dot{q})\dot{q} + G(q) + F_d\dot{q} + F_s(\dot{q}) + T_d \qquad (2)$$

By substituting Eq. 2 into 1 we have:

$$M(q)\ddot{q} + H(q,\dot{q}) = u(t) \qquad (3)$$

UNCERTAINTIES AND BOUNDING FUNCTION

In the presence of uncertainty such as unknown parameters, frictions, load variation, disturbances and un-model dynamics, dynamics (3) of robotic systems are usually not totally known. All the terms in Eq. 3 can be reduced without loss of any generality into two parts:

$$\begin{aligned} M(q) &= M_k(q) + M_u(q) \\ H(q,\dot{q}) &= H_k(q,\dot{q}) + H_u(q,\dot{q}) \end{aligned} \qquad (4)$$

Where:

$$H_k(q,\dot{q}) = V_{m,k}(q,\dot{q})\dot{q} + G_k(q) + F_{d,k}\dot{q} \qquad (5)$$

$$H_u(q,\dot{q}) = V_{m,u}(q,\dot{q})\dot{q} + G_u(q) + F_{d,u}\dot{q} + F_s(\dot{q}) + T_d \qquad (6)$$

where, $M_k(q), H_k(q,\dot{q}), V_{m,k}(q,\dot{q}), G_k(q)$ and $F_{d,k}$ are the known parts and $M_u(q), H_u(q,\dot{q}), V_{m,u}(q,\dot{q}),$ $G_u(q)$ and $F_{d,u}$ denote the unknown parts of $M(q), H(q,\dot{q}), V_m(q,\dot{q})$, $G(q)$ and F_d, respectively. For design of robust nonlinear controller, the following assumptions should be established.

Assumptions:

- $\underline{m}I_n \le M(q) \le \bar{m}(q)I_n \quad ,\forall q\in R^n$
- $\|V_m(q,\dot{q})\| \le \xi_c(q)\|\dot{q}\| \quad ,\forall q,\dot{q}\in R^n$
- $\|F_d y + F_s(y)\| = \xi_{f_0} + \xi_{f_1}\|y\| \ ,\forall y\in R^n$
- $\|G(y)\| \le \xi_g(y) \ ,\forall y\in R^n$
- $\|T_d\| \le \xi_t$

where, $\underline{m}$, ξ_{f_0}, ξ_{f_1} and ξ_t are positive constant and it is assumed that these are known constant. $\bar{m}(q)$ and $\xi_c(q)$ are known, positive definite function of q and ξ_g is a known positive definite function. For a revolute-joint robot, matrix M(q) is not only positive definite but also its dependence on q is in the form of the trigonometric functions, sine and cosine. This implies that, for revolute-joint robots $\bar{m}(q)=\bar{m}$, $\xi_c(q) = \xi_c$ and $\xi_g(q) = \xi_g$ are all constants.

ROBOT DYNAMICS IN THE TASK SPACE

For design of robust nonlinear controller in task space, the dynamics equation of manipulator in task space should be established there for Eq. 3 is simplified as:

$$\ddot{q} = M^{-1}(q)\left(u(t) - H(q,\dot{q})\right) \qquad (7)$$

The task space velocity $\dot{X}(t)$ is related to joint space velocity $\dot{q}(t)$ as (Cheah *et al*., 2006):

$$\dot{X} = J(q)\dot{q} \tag{8}$$

where, $J(q)\in R^{n\times n}$ is the Jacobian matrix from joint space to task space. The derivative of Eq. 8 respect to time can be written as:

$$\ddot{X} = J(q)\ddot{q} + \dot{J}(q)\dot{q} \tag{9}$$

$\dot{j}(q)$ exists if the desired path is smooth. Assuming there are no singular points in the desired path in task space such that the jacobian matrix is of full rank. Eq. 7 is substituted into Eq. 9:

$$\ddot{X} = J(q)M^{-1}(q)\big(u(t) - H(q,\dot{q})\big) + \dot{J}(q)\dot{q} \tag{10}$$

Eq. 10 is rearranged as:

$$M(q)J^{-1}\ddot{X} + H(q,\dot{q}) - M(q)J^{-1}\dot{J}\dot{q} = u(t) \tag{11}$$

$J^{-1}(q)$ is the inverse of Jacobian matrix. Torque is related to force as (Cheah *et al*., 2006):

$$u(t) = J^{T}(q)f(t) \tag{12}$$

$J^{T}(q)\in R^{n\times n}$ is the transpose of jacobian matrix and $f(t)\in R^{n}$ is the input force of manipulator. Equation 12 is substituted into Eq. 11 and it is rearranged as:

$$J^{-T}M(q)J^{-1}\ddot{X} + J^{-T}H(q,\dot{q}) - J^{-T}M(q)J^{-1}\dot{J}\dot{q} = f(t) \tag{13}$$

To simplify Eq. 13, the following equations are expressed:

$$A(q) = J^{-T}M(q)J^{-1} \tag{14}$$

$$N(q,\dot{q}) = J^{-T}H(q,\dot{q}) - J^{-T}M(q)J^{-1}\dot{J}\dot{q} \tag{15}$$

Substituting Eq. 14 and 15 into Eq. 13, the equations of motion can be expressed as:

$$A(q)\ddot{X} + N(q,\dot{q}) = f(t) \tag{16}$$

According to Eq. 4, M(q) and $H(q,\dot{q})$ have a known and an unknown part therefore A(q) and $N(q,\dot{q})$ have the same parts:

$$\begin{aligned} A(q) &= A_k(q) + A_u(q) \\ N(q,\dot{q}) &= N_k(q,\dot{q}) + N_u(q,\dot{q}) \end{aligned} \tag{17}$$

where, $A_k(q)$ and $N_k(q,\dot{q})$ are the known parts and $A_u(q)$ and $N_u(q,\dot{q})$ denote the unknown parts of A(q) and $N(q,\dot{q})$, respectively.

ROBUST NONLINEAR CONTROL

e(t) is the trajectory error and is defined as:

$$X_d(t) - X(t) = e(t)$$

where, $X_d(t)$is a desired path and X(t) is the position of manipulator in task space. In Eq. 16, a control law is presented as:

$$\begin{aligned} f(t) &= A_k(q)\ddot{X}_d + N_k(q,\dot{q}) + \alpha A_k(q)\dot{e}(t) + \\ &K A_k(q)e(t) + \alpha A_k(q)\big(\alpha e(t) + \dot{e}(t)\big) + u_r(t) \end{aligned} \tag{18}$$

where, α and K are the positive constants and $u_r(t)$ is the new robust control input. For computational simplicity, $A_k(q)$ and $N_k(q,\dot{q})$ can be simplified versions of the known parts of A(q) and $N(q,\dot{q})$, respectively. In the worst situation when there is no knowledge about the controlled robotic manipulator, one can choose $A_k(q) = I$ and $N_k(q,\dot{q}) = 0$. By defining $r(t) = \alpha e + \dot{e}$ and substituting Eq. 18 into Eq. 16, the following equation can be expressed as:

$$\begin{aligned} A(q)\ddot{X} &= A_k(q)\ddot{X}_d + N_k(q,\dot{q}) + \alpha A_k(q)\dot{e}(t) + \\ &K A_k(q)e(t) + \alpha A_k(q)r(t) - N(q,\dot{q}) + u_r(t) \end{aligned} \tag{19}$$

Equation 17 is substituted into Eq. 19 and $A(q)\ddot{X}_d$ is added and subtracted:

$$\begin{aligned} A(q)\ddot{X} &= A(q)\ddot{X}_d - A(q)\ddot{X}_d + A_k(q)\ddot{X}_d + \\ &\alpha A_k(q)\dot{e}(t) + K A_k(q)e(t) + \alpha A_k(q)r(t) - N_u(q,\dot{q}) + u_r(t) \end{aligned} \tag{20}$$

Equation 20 is rearranged as:

$$\begin{aligned} -A(q)\ddot{e}(t) &= -A_u(q)\ddot{X}_d + \alpha A_k(q)\dot{e}(t) + \\ &K A_k(q)e(t) + \alpha A_k(q)r(t) - N_u(q,\dot{q}) + u_r(t) \end{aligned} \tag{21}$$

$A_k(q) = A(q) - A_u(q)$ is used into Eq. 21 and 22 as:

$$\ddot{e}(t) = A^{-1}(q)\left\{\begin{aligned} &A_u(q)\ddot{X}_d + \alpha A_u(q)\dot{e}(t) + K A_u(q)e(t) + \alpha A_u(q)r(t) \\ &-\alpha A(q)\dot{e}(t) - K A(q)e(t) - \alpha A(q)r(t) + N_u(q,\dot{q}) - u_r(t) \end{aligned}\right\} \tag{22}$$

To simplify Eq. 22, the following equation can be expressed:

$$\begin{aligned} \Delta A &= A_u(q)\ddot{X}_d + \alpha A_u(q)\dot{e}(t) + \\ &K A_u(q)e(t) + \alpha A_u(q)r(t) + N_u(q,\dot{q}) \end{aligned} \tag{23}$$

Equation 23 is substituted into Eq. 22:

$$\ddot{e}(t) = A^{-1}(q)\{\Delta A - \alpha A(q)\dot{e}(t) - K A(q)e(t) - \alpha A(q)r(t) - u_r(t)\} \tag{24}$$

By definingand, $e(t) = Z_1(t)$ and $\dot{e}(t) = Z_2(t)$ Eq. 24 is rearranged as:

$$\begin{aligned} &\dot{Z} = DZ + B\left[\Delta A - \alpha A(q)\dot{e}(t) - K A(q)e(t) - \alpha A(q)r(t) - u_r(t)\right] \\ &Z = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}, D = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ A^{-1}(q) \end{bmatrix} \\ &r(t) = [\alpha I_n \quad I_n]Z \end{aligned} \tag{25}$$

In Eq. 25, most of the uncertainties exist in ΔA and matrix B. To design the robust control, the maximum size of uncertainties should be available. The boundaries of uncertainties are discussed further.

MAXIMUM SIZE OF THE UNCERTAINTIES

According to assumptions mentioned earlier, Eq. 23 can be expressed as:

$$\begin{aligned} \|\Delta A\| \le \bar{A}_u(q)\sup_{t\ge 0}\|\ddot{X}_d\| + \alpha\bar{A}_u(q)\|\dot{e}(t)\| + K\bar{A}_u(q)\|e(t)\| \\ +\alpha\bar{A}_u(q)\|r(t)\| + \xi_c(q)\|\dot{q}\|^2 + \xi_g(q) + \xi_{f_0} + \xi_{f_1}\|\dot{q}\| + \xi_t \end{aligned} \tag{26}$$

$$\begin{aligned} \rho(Z,t) = \bar{A}_u(q)\sup_{t\ge 0}\|\ddot{X}_d\| + \alpha\bar{A}_u(q)\|\dot{e}(t)\| + K\bar{A}_u(q)\|e(t)\| \\ +\alpha\bar{A}_u(q)\|r(t)\| + \xi_c(q)\|\dot{q}\|^2 + \xi_g(q) + \xi_{f_0} + \xi_{f_1}\|\dot{q}\| + \xi_t \end{aligned} \tag{27}$$

$\bar{A}_u(q)$ is the maximum size of $A_u(q)$.

Robust control: To prove the stability of system presented by Eq. 25, the Lyapunov function candidate is presented as:

$$V(Z) = \frac{1}{2}K Z_1^T Z_1 + \frac{1}{2}r^T r \tag{28}$$

The derivative of Eq. 28 respect to time can be written as:

$$\dot{V}(Z) = K\dot{Z}_1^T Z_1 + \dot{r}^T r \tag{29}$$

By using of Eq. 24 and 25, Eq. 29 is simplified as follows:

$$\dot{V}(Z) = -\alpha\left(K\|Z_1\|^2 + \|r\|^2\right) + r^T A^{-1}(q)\{\Delta A - u_r(t)\} \tag{30}$$

If uncertainty ΔA is known, control law $u_r(t) = \Delta A$ will stabilize the system but If uncertainty ΔA is unknown for $\dot{V} \le 0$, u_r should be chosen properly.

Lemma: Let V be a Lyapunov function candidate for any given continuous time system. Suppose that along the trajectory of the system:

$$\dot{V}(Z) = -\lambda_v\bar{\lambda}(Z)\|Z\|^2 + \varepsilon\phi(t) \qquad \forall Z \in R^n \tag{31}$$

where, $\bar{\lambda}(Z) \ge \underline{\lambda} \succ 0$ for all Z and λ_v, $\varepsilon \succ 0$ are constants and is uniformly continuous and satisfies the properties that, for all $t \ge t_0$:

$$0 \prec \phi(t) \prec \infty, \qquad \int_{t_0}^{\infty}\phi(s)ds = C_\phi \prec \infty \quad ,\forall t_0 \tag{32}$$

Then, the system is asymptotically convergent to the origin of the whole.

Proof : It follows that:

$$\dot{V}(Z) \le -\lambda_v V(Z) + \varepsilon\phi(t) \tag{33}$$

Now, define a new variable as:

$$\omega(t) = \dot{V}(Z) + \lambda_v V(Z) - \varepsilon\phi(t) \tag{34}$$

Note that the preceding differential equation is scalar, of first order and time invariant. It follows that $\omega(t) \le 0$. Solving the equation, we have:

$$\begin{aligned} V(Z) &= V(Z(t_0))e^{-\lambda_v(t-t_0)} + \int_{t_0}^{t} e^{-\lambda_v(t-s)}\left(\varepsilon\phi(s) + \omega(s)\right)ds \\ &\le V(Z(t_0))e^{-\lambda_v(t-t_0)} + \varepsilon\int_{t_0}^{t} e^{-\lambda_v(t-s)}\phi(s)ds \\ &= V(Z(t_0))e^{-\lambda_v(t-t_0)} + \varepsilon\int_{t_0}^{\frac{t}{2}} e^{-\lambda_v(t-s)}\phi(s)ds + \varepsilon\int_{\frac{t}{2}}^{t} e^{-\lambda_v(t-s)}\phi(s)ds \end{aligned} \tag{35}$$

Note that by the mean value theorem there exists $s^* \in \left[t_0, \frac{t}{2}\right]$ such that:

$$0 \le \int_{t_0}^{\frac{t}{2}} e^{-\lambda_v(t-s)}\phi(s)ds = e^{-\lambda_v(t-s^*)}\int_{t_0}^{\frac{t}{2}}\phi(s)ds \le e^{-\lambda_v(t-s^*)}C_\phi \tag{36}$$

Which implies:

$$\lim_{t\to\infty}\int_{t_0}^{\frac{t}{2}} e^{-\lambda_v(t-s)}\phi(s)\,ds = 0 \tag{37}$$

Similarly, there exists $s^* \in \left[\frac{t}{2}, t\right]$ such that:

$$\int_{\frac{t}{2}}^{t} e^{-\lambda_v(t-s)}\phi(s)ds = \phi(t^*)\int_{\frac{t}{2}}^{t} e^{-\lambda_v(t-s)}ds \le \frac{2}{\lambda_v}\phi(t^*) \tag{38}$$

which implies that:

$$\lim_{t\to\infty}\int_{\frac{t}{2}}^{t} e^{-\lambda_m(t-s)}\phi(s)\,ds = 0 \tag{39}$$

Since $t^* \to \infty$ as $t \to \infty$ and since, by uniform continuity and the barbalat lemma (Khalil, 2001), $\lim_{t\to\infty}\phi(t) = 0$. Consequently the state $||Z||$ converges to zero asymptotically for any initial condition. According to the above mentioned assumptions, the system Eq. 25 has global asymptotically stability that in Eq. 30, the following equation should be expressed:

$$r^T A^{-1}(q)\Delta A - r^T A^{-1}(q)u_r \le \varepsilon\phi(t) \tag{40}$$

According to Eq. 26 and 27, Eq. 40 can be expressed as:

$$\|r\|\bar{A}^{-1}(q)\rho(Z,t) - r^T A^{-1}(q)u_r \le \varepsilon\phi(t) \tag{41}$$

$\rho(Z, t)$ and $\bar{A}^{-1}(q)$ are the maximum size of ΔA and $A^{-1}(q)$, respectively. According to Eq. 41, control law u_r should be presented as:

$$\begin{aligned} u_r &= \frac{\gamma(Z,t)\rho(Z,t)\bar{A}^{-1}(q)}{\underline{A}^{-1}(q)\left(\|\gamma(Z,t)\| + \varepsilon\phi(t)\right)} \\ \gamma(Z,t) &= r\bar{A}^{-1}(q)\rho(Z,t) \end{aligned} \tag{42}$$

$\bar{A}^{-1}(q)$ and $\underline{A}^{-1}(q)$ are the maximum and minimum sizes of $A^{-1}(q)$, respectively. Equation 42 is substituted into left side of Eq. 41 and it can be expressed as:

$$= \|r\|\bar{A}^{-1}(q)\rho(Z,t) - r^T A^{-1}(q)\left(\frac{\gamma(Z,t)\rho(Z,t)\bar{A}^{-1}(q)}{\underline{A}^{-1}(q)\left(\|\gamma(Z,t)\| + \varepsilon\phi(t)\right)}\right) \tag{43}$$

$$= \|r\|\bar{A}^{-1}(q)\rho(Z,t) - A^{-1}(q)\left(\frac{\gamma^T(Z,t)\gamma(Z,t)}{\underline{A}^{-1}(q)\left(\|\gamma(Z,t)\| + \varepsilon\phi(t)\right)}\right) \tag{44}$$

$$= \|\gamma(Z,t)\| - A^{-1}(q)\left(\frac{\|\gamma(Z,t)\|^2}{\underline{A}^{-1}(q)\left(\|\gamma(Z,t)\| + \varepsilon\phi(t)\right)}\right) \tag{45}$$

According to Eq. 45, it is clear that the following equation can be expressed:

$$\le \|\gamma(Z,t)\| - \frac{\|\gamma(Z,t)\|^2}{\|\gamma(Z,t)\| + \varepsilon\phi(t)} \tag{46}$$

Equation 46 is simplified as:

$$\le \frac{\|\gamma(Z,t)\|\varepsilon\phi(t)}{\|\gamma(Z,t)\| + \varepsilon\phi(t)} \le \varepsilon\phi(t) \tag{47}$$

From Eq. 47 it is derived that by choosing robust control input Eq. 42, 40 is established hence system presented by Eq. 25 has global asymptotically stability by proposed control.

CASE STUDY OF TWO-LINK ELBOW ROBOT MANIPULATOR

Controller which has been studied in this study was assembled on two link elbow robot manipulator in Fig. 1. Dynamic equations of this robot are reported by Spong *et al.* (2006) presented as follows:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} h_1(q,\dot{q}) \\ h_2(q,\dot{q}) \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{48}$$

$$M_{11} = \left(l_2^2 m_2 + 2l_1 l_2 \cos(q_2) + l_1^2(m_1 + m_2)\right) \tag{49}$$

$$M_{12} = M_{21} = \left(l_2^2 m_2 + l_1 l_2 m_2 \cos(q_1)\right) \tag{50}$$

$$M_{22} = l_2^2 m_2 \tag{51}$$

$$h_1(q,\dot{q}) = \left\{\begin{aligned} &-m_2 l_1 l_2 \sin(q_2)\dot{q}_2^2 - 2m_2 l_1 l_2 \sin(q_2)\dot{q}_1\dot{q}_2 \\ &+m_2 l_2 g\cos(q_1 + q_2) + (m_1 + m_2)l_1 g\cos(q_1) \\ &+F_{d_1}\dot{q}_1 + F_{s_1}(\dot{q}_1) + T_{d_1} \end{aligned}\right\} \tag{52}$$

$$h_2(q,\dot{q}) = \left\{\begin{aligned} &\left(m_2 l_2 g\cos(q_1 + q_2)\right) + \left(m_2 l_1 l_2 \sin(q_2)\dot{q}_1^2\right) \\ &+F_{d_2}\dot{q}_2 + F_{s_1}(\dot{q}_2) + T_{d_2} \end{aligned}\right\} \tag{53}$$

where, l_1 and l_2 are lengths of the first and second links respectively, m_1 and m_2 are masses of the first and second links, respectively, g is the gravitational force, F_d is

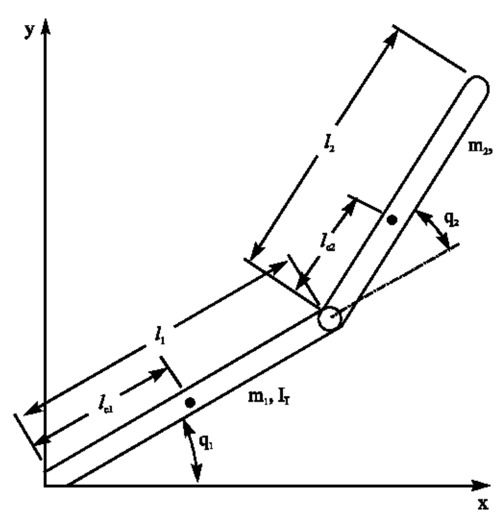

Fig. 1: Two link elbow robot manipulator

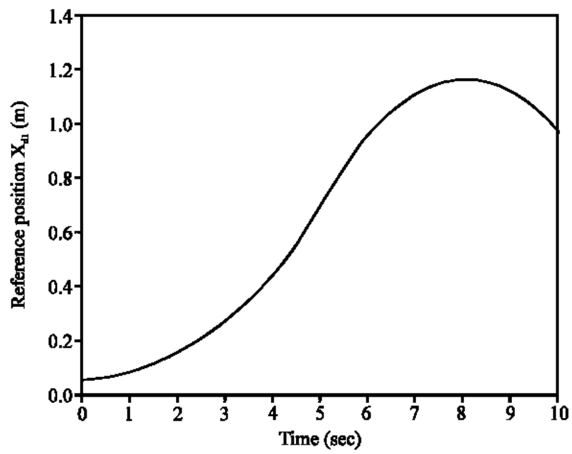

Fig. 2: Desired path X_{d1} in the task space

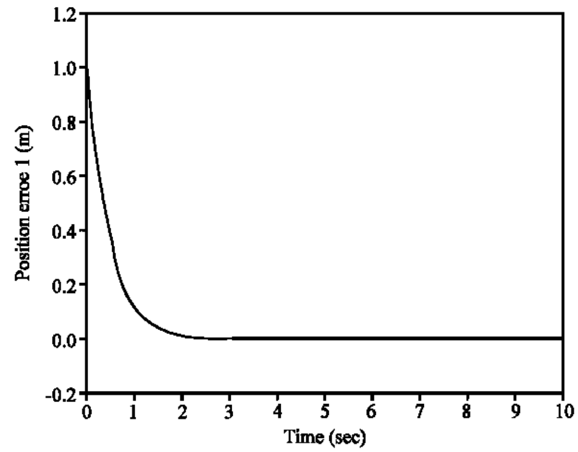

Fig. 4: Tracking error X_{d1} in the task space

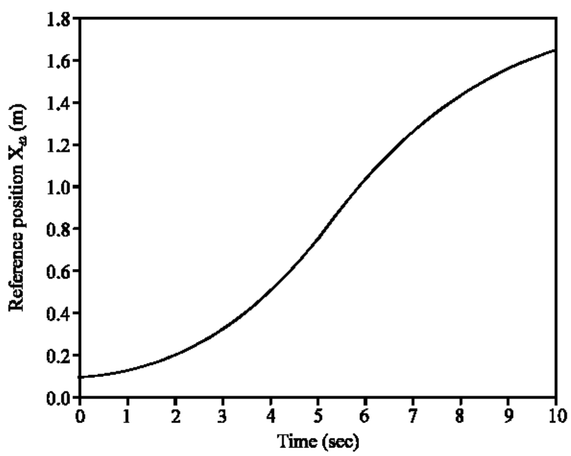

Fig. 3: Desired path X_{d2} in the task space

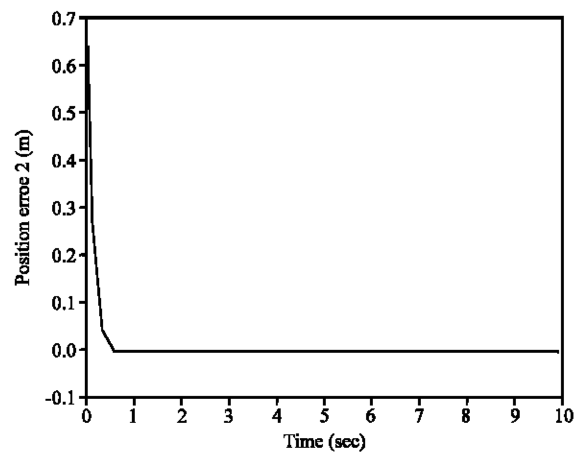

Fig. 5: Tracking error X_{d2} in the task space

Table 1: Parameters of two link elbow robot

$l_1 = 1$ m	$l_2 = 1$ m
$m_1 = 10$ kg	$m_2 = 5$ kg
$g = 9.8$	$F_{d1} = F_{d2} = 2$
$F_{s_1}(\dot{q}_1) = F_{s_2}(\dot{q}_2) = 1$	$T_{d1} = T_{d2} = 5$

dynamic friction. $F_s(\dot{q})$ is static friction and T_d is disturbance and un-modeled dynamics. u_1 and u_2 are input torques of the first and second links, respectively. Robot parameters which have been used in this simulation are shown in Table 1.

For the design of robust nonlinear control, it is assumed that system parameters are unknown but bounded as:

$$0.5 \le m_1,\ \ l_2, l_1, m_2 \le 1.5 \tag{54}$$

In the control design parameter and function are selected to be $K = 1$, $\alpha = 0.3$, $\varepsilon = 30$ and $\phi(t) = e^{-0.2t}$. Then, from bounding parameters, ξ_c, ξ_g, ξ_t, ξ_{f0} and ξ_{f_1}, we choose bounding function $\rho(Z, t)$ to be:

$$\rho(Z,t) = 300 + 65\left(\|Z\| + \|Z\|^2\right) \tag{55}$$

Desired path in task space are given in Fig. 2 and 3. According to Fig. 4 and 5, in the presence of structured and unstructured uncertainties, the proposed control performs appropriately and trajectory errors X_{d1} in 2 sec and X_{d2} in 1 sec converge to zero.

CONCLUSION

In this study, we proposed a robust nonlinear control for tracking of robot manipulator. The proposed control was designed in task space to overcome structured and unstructured uncertainties presence in dynamical model of robot. First we presented a method to determine dynamical equation by which robot manipulator in task space was modeled. In design of controller we used the method of eliminating known dynamics to reduce the non-linear effects. Finally to overcome the remaining uncertainties, we used straight Lyapunov method and determined maximum value for uncertainties. By this strategy a robust controller was designed which performs the best even at worst condition. Mathematically reasoning confirms that by applying the proposed controller, the closed loop system has asymptotically stability. The experimental results illustrate that the designed controller performs well in presence of uncertainties and the tracking errors converges to zero rapidly. In order to reduce the computational complexity of controller, the considered system dynamics can be reduced, instead studying uncertainties can be considered more precisely.

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