# Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 8 (23): 4440-4444, 2008
ISSN 1812-5654


# PAPR Reduction Based on Weighted OFDM with Product Block Codes for Wireless Communication

[1]A. Seddiki, [1]M. Djebbouri and [2]A. Taleb-Ahmed
[1]Laboratoire de Télécommunication et Traitement Numérique du Signal,
Université de Sidi-Bel-Abbès, 22000, Algérie
[2]LAMIH, Université de Valenciennes, UMR/CNRS 8530, France

**Abstract:** In this study, we investigate the potential of some of the well know linear block codes in reducing the peak-to-average power ratio (PAPR). The influence of combined weighting and product block coding on the PAPR is investigated. A class of constructed product block codes based on BCH block codes capable of both error correction and PAPR reduction together with several weighting functions is considered. We investigate the interplay of various weighting functions with product codes in order to minimize PAPR. Proposed scheme reduce PAPR significantly as is evident from the simulation results.

**Key words:** OFDM, PAPR, BCH code, product code, weighting function

## INTRODUCTION

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) (Zou and Wo, 1995; Nee and Prasad, 2000) has becomes a good candidate for wireless multimedia communication by virtue of its excellent properties in frequency-selective fading environment. In OFDM, data is transmitted over several parallel low data rate channels. Thus it provides data integrity due to fading, relative to modulation methods that employ single channel for high data rate transmission. Among other benefits of OFDM is that it fully exploits the advantages of digital signal processing concepts (Bingham, 2000).

OFDM transmitter and receiver structures are shown in Fig. 1. The serial input data is encoded using channel encoder before being sent to a serial-to-parallel (S/P) converter that partitions the input data arriving at the rate R into N parallel information symbols each at a reduced data rate of R/N. The number of bits in each of the N output sequences of the S/P converter is determined by the constellation of the signal mapper. In this research, we have chosen BPSK as mapping technique. The output of the signal mapper is then fed to N-point IFFT, which is the most important block of an OFDM system as it outputs orthogonal signals on its N sub channels. After up converting (modulator block) the resulting signal to desired carrier frequency $f_c$, the signal is transmitted.

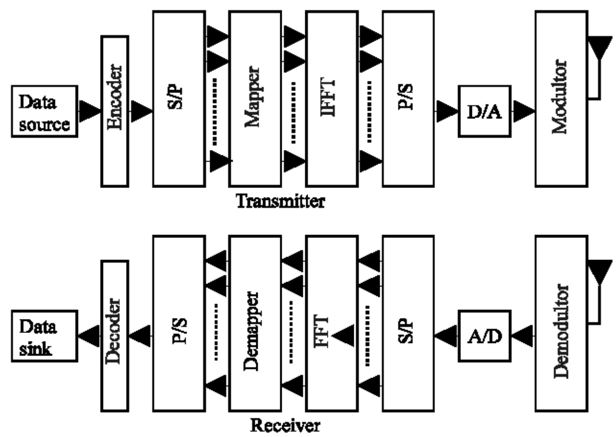


Fig. 1: Typical OFDM system

At the receiver, the process described above is reversed. As shown in Fig. 1, the process starts with down converting the received signal and passing through several blocks to eventually obtain the transmitted data sequence. In the absence of noise and fading, transmitted data is recovered without errors.

In OFDM, as the number of subcarriers increase, the effective waveform approaches that of a sample function from a Gaussian process. This results in occasional peaks in the transmitted signal. Peak-to-average power ratio (PAPR) is a good measure of these peaks.

**Corresponding Author:** A. Seddiki, Laboratoire de Télécommunication et Traitement Numérique du Signal, Université de Sidi-Bel-Abbès, 22000, Algérie

A Baseband OFDM signal with N sub channels has a PAPR = N (Cimini and Sollenberger, 2000). When passed through a non linear device, such as a transmit power amplifier, the signal may suffer significant spectral spreading and in-band distortion (Junsong and Kavehrad, 1999). It is desired to reduce PAPR because the power amplifiers used at the transmitter have a linear behavior up to a certain range. Beyond this range they become nonlinear causing signal distortion. Hence, the problem of PAPR has received widespread attention in recent years. Many methods have been suggested to alleviate the problem of PAPR. Among these, one of the suggestions is to employ block codes (Jones *et al*., 1994; Ochiai and Imai, 1998; Fragiacomo *et al*., 1998; Zhang *et al*., 1999; Jiang and Zhu, 2005). Yet another technique is to reduce PAPR through weighting of OFDM signal (Nikoobar and Prasad, 2000).

The intent of this study is to explore the potential of product coding in reducing PAPR. In literature, block codes and weighting functions have been considered separately for the reduction of PAPR. Since weighted OFDM also reduces PAPR at the expense of BER, we investigate methods to compensate for the degradation of BER by employing these linear block codes to construct product block codes achieving high capability of errors correction. These product codes have not been examined with weighting functions in OFDM systems in literature. Hence, we address the problem of jointly reducing PAPR and BER using product block codes and weighting functions.

We consider a class of weighting functions such as Gaussian, Half Sine, Raised Cosine, Shannon etc. and investigate their interplay with product block codes based on linear BCH block codes to jointly optimize PAPR and BER.

## PAPR AND CODING SCHEME

A typical OFDM symbols is represented by:

$$x(t) = \sum_{m=0}^{N-1} X_m e^{j2\pi f_m t} \quad , \qquad 0 \le t \le NT \tag{1}$$

$X_m, \{m = 0,1,\ldots,N-1\}$, are the outputs of the mapper which are ±1 for BPSK; $f_m$ is the frequency of the nth carrier;

$$f_m = f_o + \frac{m}{t_s} \tag{2}$$

where, $f_0$ is the lowest frequency of the carriers and $t_s$ is the OFDM symbols duration.

The PAPR of the envelope of the transmitted signal can be written as (Cimini and Sollenberger, 2000):

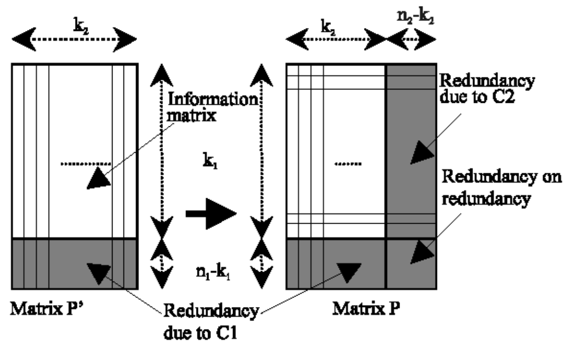


Fig. 2: Product coding scheme

$$PAPR = \frac{\max\{\,|x(t)|^2\,\}}{E\{\,|x(t)|^2\,\}} \tag{3}$$

where, x(t) denotes the complex modulating signal.

We investigate four linear block codes to construct product block codes.

These are BCH(7,4), (15,11), (15,7) and (15,5). The first two codes can correct one bit errors each while BCH(15,7) and (15,5) can correct up to 2 and 3 errors respectively (Proakis and Salehi, 2002).

The product codes have been introduced by Pundiah (1998). They permit the construction of long correcting code with high correction capability using two elementary codes.

Let us consider two systematic linear block codes $C_1(n_1,k_1,d_1)$ and $C_2(n_2,k_2,d_2)$ where $n_i$, $k_i$ and $d_i$ (i = 1,2) stand for codeword length, number of information bits and minimum Hamming distance, respectively. The product coding $P = C_1{*}C_2$ is obtained by placing $k_1k_2$ information bits in an array of $k_1$ rows and $k_2$ columns. A first coding is operated (by $C_1$ code) on the $k_2$ columns each one with the length $k_1$ which lead to the matrix P', (Fig. 2).

A second coding (by $C_2$ code) is done on the $n_1$ rows of matrix P' (with length $n_1k_2$) to give the final matrix P (with length $n_1n_2$).

Thus the code rate of a product code is given by:

$$R = R_1 \times R_2 = \frac{k_1k_2}{n_1n_2} \tag{4}$$

where, $R_i$ is the code rate of code $C_i$.

With a global Hamming distance $d_1d_2$, product codes are able to detect and correct all combinations of t random errors and able to correct all configurations of burst errors of length l (Pundiah, 1998):

$$t = \frac{d_1d_2 - 1}{2} \quad ; \qquad l = \max(n_1t_2, n_2t_1) \quad ; \quad t_i = \frac{d_i - 1}{2} \tag{5}$$

Using this technique to construct product block codes, we have simulated the following product codes noted as follow:

$$BCH(7,4) \otimes BCH(7,4) = BCH(7,4)^2$$
$$BCH(15,11) \otimes BCH(15,11) = BCH(15,11)^2$$
$$BCH(15,7) \otimes BCH(15,7) = BCH(15,7)^2$$
$$BCH(15,5) \otimes BCH(15,5) = BCH(15,5)^2$$

## WEIGHTING FUNCTIONS

A novel scheme of using weighting functions to reduce PAPR has been introduced (Nikoobar and Prasad, 2000). In this scheme, each complex number is weighted by a factor $w_m, m = 0,1,...,N-1$ before taking the IFFT.

Thus weighted complex OFDM modulated signal can be written as:

$$x_w(t) = \sum_{m=0}^{N-1} X_m w_m e^{j2\pi f_m t} \quad , \quad 0 \le t \le NT \qquad (6)$$

And the PAPR of the envelope is given by:

$$PAPR = \frac{\max\{\,|x_w(t)|^2\,\}}{E\{\,|x_w(t)|^2\,\}} \qquad (7)$$

In this research, we consider the following weighting functions:

- **Barlett:** This weighting function has a triangular shape and expressed by

$$w_m = \begin{cases} A\left(1 - \frac{|m - \frac{N}{2}|}{\frac{N}{2}}\right) & ; 0 \le m \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

- **Rectangular:** This weighting function has a rectangular shape

$$w_m = \begin{cases} A & ; 0 \le m \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

- **Raised cosine:** This weighting function is explained by

$$w_m = \begin{cases} A\sin^2\left(\pi\frac{m}{N}\right) & ; 0 \le m \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

- **Half-sine:** The shape of this weighting function is described by

$$w_m = \begin{cases} A\sin\left(\pi\frac{m}{N}\right) & ; 0 \le m \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (11)$$

- **Shannon:** The shape of these weighting factors is the sinc function, i.e.,

$$w_m = \begin{cases} A\,\mathrm{sinc}\left(\frac{2m - N}{N}\right) & ; 0 \le m \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

- **Gaussian:** These factors are generated based on the Gaussian function, i.e.,

$$w_m = \begin{cases} A\exp\left(-\frac{(m - \frac{N}{2})^2}{2\sigma^2}\right) & ; 0 \le m \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad (13)$$

where, σ is the spread or standard deviation of the weighting factors around N/2. We have considered σ = 3N/16.

- **Chebyshev:** The shape of this weighting function is exponentially increasing,

$$w_m = \begin{cases} A\left(\sin^{-1}\left(h\frac{m}{N}\right)\right) & ; 0 \le m \le N-1; \\ & h = 0.1, 0.2, ..., 0.8 \\ 0 & \text{otherwise} \end{cases} \qquad (14)$$

For performance comparison of OFDM systems, the amplitude A in 8-14 is selected in such a way that the power of all weighting factors is constant, i.e.,

$$\sum_{m=0}^{N} w_m^2 = 1 \qquad (15)$$

A block diagram for generating of coded and weighted OFDM signal is shown in Fig. 3.

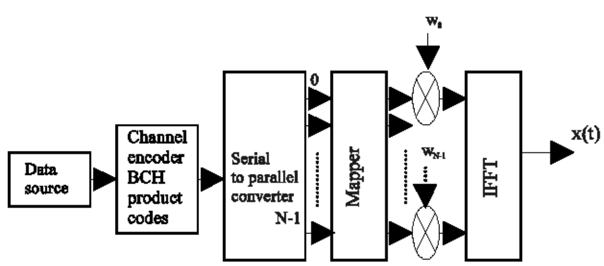


Fig. 3: Generation of weighted and coded OFDM signal

## NUMERICAL RESULTS

Table 1 and 2 show achievable PAPR as a function of product BCH codes, with several weighting functions. It is observed that for 8 carriers, product BCH(7,4) code yield to nearly of 0.9 dB PAPR reduction than the case when a biased BCH(7,4) code (Tasadduq and Rao, 2001) is used without weightings. However, we obtain significant improvements with weightings and significant PAPR reduction. For Gaussian and Chebyshev weightings, PAPR improves with this product code.

In case of 16-carriers product BCH(15,11) coded and Gaussian weighting, it is possible to achieve an improvement of nearly 6.8 dB relative to 16-carriers uncoded system without weighting. For identical weightings, both product BCH(15,7) and product BCH(15,5) are inferiors to product BCH(15,11) code. However, these systems give good PAPR reduction with Gaussian weighting, relative to 16-carriers uncoded system, by nearly 4 and 2 dB, respectively. It is observed that in case of product BCH(15,11) system, the best PAPR is achieved when either Chebyshev or Gaussian weighting is employed.

In order to illustrate the influence of weighting and product coding on PAPR of OFDM signals, we compute the complementary cumulative distribution function (CCDF = Pr ($PAPR>PAPR_o$)) (Cimini and Sollenberger, 2000).

Table 1: Maximum PAPR for 8 carrier system

| Weighting functions | Maximum PAPR (dB) Uncoded | BCH biased code (7,4) | BCH product code $(7,4)^2$ |
|---|---|---|---|
| Non wtd. | 9.00 | 5.20 | 4.32 |
| Chebyshev | 6.50 | 5.39 | 4.25 |
| Gaussian | 7.20 | 5.68 | 4.10 |
| Rectangular | 9.00 | 5.35 | 4.90 |
| Raised cos. | 7.36 | 5.83 | 5.29 |
| Barlett | 7.65 | 5.32 | 5.37 |
| Shannon | 7.84 | 5.31 | 5.78 |
| Half sine | 8.00 | 5.32 | 4.82 |

Table 2: Maximum PAPR for 16 carrier system

| | Maximum PAPR (dB) | | | |
|---|---|---|---|---|
| | | BCH product codes | | |
| Weighting functions | Uncoded | $(15,11)^2$ | $(15,7)^2$ | $(15,5)^2$ |
| Non wtd. | 12.0 | 5.72 | 6.35 | 7.66 |
| Chebyshev | 8.60 | 5.31 | 6.10 | 9.70 |
| Gaussian | 10.30 | 5.11 | 6.12 | 8.35 |
| Rectangular | 12.00 | 5.58 | 6.80 | 9.50 |
| Raised cos. | 10.45 | 5.70 | 6.80 | 9.97 |
| Barlett | 11.00 | 5.60 | 6.51 | 9.98 |
| Shannon | 11.13 | 5.78 | 6.51 | 9.97 |
| Half sine | 11.41 | 5.79 | 6.54 | 9.91 |

Figure 4 shows the CCDF plots for an 8 carrier coded and uncoded OFDM system. A 4.4 dB improvement is achieved when product BCH(7,4) code is used relative to uncoded and non weighted system and nearly 0.4 dB improvement relative to biased BCH(7,4) coded system. From Fig. 4 we observe that using OFDM product coding system appears more interesting to improve PAPR reduction. Figure 5 shows the performance of a 16-carriers product BCH(15,7) coded OFDM system. A reduction of 5 dB in PAPR is achieved relative to uncoded and non weighted system. It is observed that weighting plays a little gain of 0.2 dB in reducing PAPR in this case (relative to product coded system).

For product BCH(15,11) coded system along with weighting functions gives the best performance for 16-carrier OFDM system. Figure 6 shows a gain of almost 6.5 dB when product BCH(15,11) is used along with Chebyshev weighting relative to uncoded and non weighted system. We observe that weighting improves

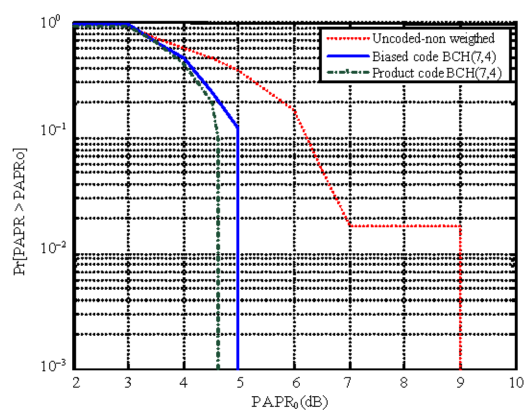


Fig. 4: CCDF for N = 8 when biased block code and product block codes BCH(7,4) are used

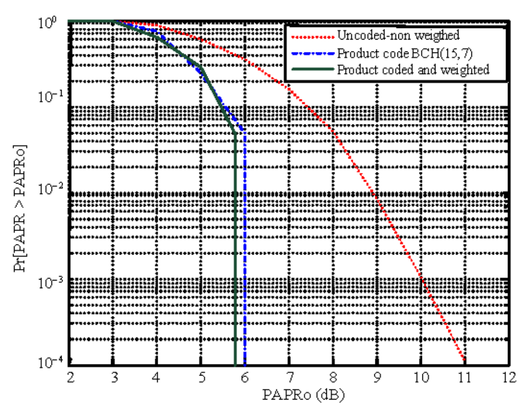


Fig. 5: CCDF for N = 16 and product block codes BCH(15,7) with Gaussian weighting

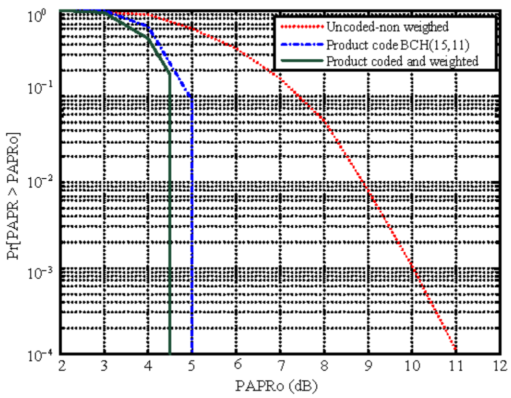


Fig. 6: CCDF for N = 16 and product block codes BCH(15,11) with Chebyshev weighting

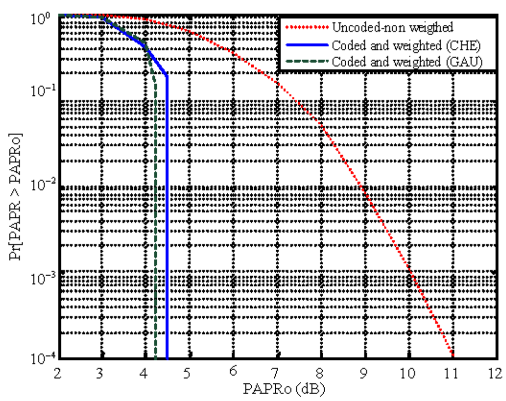


Fig. 7: CCDF for N=16 and BCH(15,11) product code with Gaussian and Chebyshev weightings

0.5 dB PAPR reduction with this product code. From Fig. 7, best performance are obtained when using Gaussian weighting with BCH(15,11) product coded OFDM system.

## CONCLUSION

In this study we have investigated the problem of reduction of PAPR in OFDM systems by jointly employing coding and weighting. The coding scheme that we have considered is based on the construction of product block codes using elementary BCH block codes. It is shown that product BCH(15,11) code, with Gaussian weighting, achieves a PAPR improvement of approximately 7 dB. This proposed scheme, gives both achievable PAPR reduction and BER improvement as shown in result simulations.

## REFERENCES


Bingham, J.A.C., 2000. ADSL, VDSL and Multicarrier Modulation. 1st Edn., Wiley, New York, ISBN: 9780471200727 .

Cimini, L.J. and N.R. Sollenberger, 2000. Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences. IEEE Commun. Lett., 4: 86-88.

Fragiacomo, S., C. Matrakidis and J.J. O'Reilly, 1998. Multicarrier transmission peak-to-average power reduction using simple block code. IEE Elect. Lett., 34: 935-954.

Jiang, T. and G. Zhu, 2005. Complement Block coding for reduction in peak-to-average power ratio of OFDM signals. IEEE Commun. Mag., 43: S17-S22.

Jones, A.E., T.A. Wilkinson and S.K. Barton, 1994. Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes. IEE Elect. Lett., 30: 2098-2099.

Junsong, L. and M. Kavehrad, 1999. OFDM-CDMA systems with nonlinear power amplifier. IEEE Wireless Commun. Network. Conf., 13: 1167-1171.

Nee, R.V. and R. Prasad, 2000. OFDM for Wireless Multimedia Communications. 1st Edn., Artech House Publishers., Norwood, MA, USA., pp: 280 ISBN: 0890065306.

Nikoobar, H. and R. Prasad, 2000. Weighted OFDM for wireless multipath channels. IEICE Trans. Commun., E83-B: 1864-1872.

Ochiai, H. and H. Imai, 1998. Block codes for frequency diversity and peak power reduction in multicarrier systems. IEEE Int. Symposium on Information Theory, Aug. 16-21, Tokyo University, 192-192.

Proakis, J.G. and M. Salehi, 2002. Communication Systems Engineering. 2nd Edn., Englewood Cliffs, Prentice Hall, Upper Saddle River, pp: 801, ISBN 0-13-061793-8.

Pundiah, R., 1998. Near optimum decoding of product codes. IEEE Trans. Comm., 4: 1003-1010.

Tasadduq, I.A. and R.K. Rao, 2001. Weighted OFDM with Block codes for wireless communication. IEEE Pacific Rim Conference Communication, Aug. 26-28, University of Western Ontario, London, 441-444.

Zhang, Y., A. Yongacoglu, J.Y. Chouinard and L. Zhang, 1999. OFDM peak power reduction by sub-Bock-coding and its extended versions. Vehicular Technology Conference, IEEE 49th, July 1999, Ottawa University, pp: 695-699.

Zou, W.Y. and Y. Wu, 1995. COFDM: An overview. IEEE Trans. Broadc., 41: 1-8.