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Capacity of MIMO Channels at Different Antenna Configurations

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Abstract: This research aims to study the effect of Channel State Information (CSI) at the transmitter on the overall channel capacity. The capacity of MIMO correlated Rayleigh channels for different antenna configurations with and without channel knowledge (CSI) at the transmitter is simulated. When CSI is available at the transmitter (i.e., informed transmitter), waterfilling algorithm is used to allocate the power among the transmitter antennas. Simulations show that capacity is improved significantly when CSI is known at the transmitter. It also shows that the lack in channel knowledge (i.e., uninformed transmitter) can be compensated for by increasing the number of antennas in the receiver (M_r). When the number of antennas in the transmitter is larger than those in the receiver, using waterfilling becomes necessary to get the optimum capacity.

Key words: MIMO correlated channels, waterfilling algorithm, informed/uninformed transmitter, channel state information at the transmitter, lower SNR region, higher SNR region

INTRODUCTION

Modern wireless systems continue to demand higher data rates and better reliabilities. These demands can be fulfilled using the conventional systems which are limited by multipath fading and interference by increasing either channel bandwidth, transmitted power, or both. However, this simplistic solution is not attractive for the following reasons. Firstly, transmitted power cannot exceed a certain value for its biological hazards and beside that, the sensitivity of wireless receivers rarely exceeds 30-35 dB because of the difficulty of building linear receivers at reasonable cost (Paulraj *et al*., 2004). Secondly, frequency spectrum is a scarce resource especially below the 6 GHz. This makes it very difficult and costly to increase the channel bandwidth (Paulraj *et al*., 2004). For all these reasons, new techniques must be introduced to achieve these demands of the modern wireless systems. These techniques must be affordable in terms of cost and biologically unharmful.

Multiple-Input Multiple-Output (MIMO) system represents one of these genius solutions that improve the bandwidth efficiency and system reliability without need to use extra bandwidth or transmitting more power into the channel (Foschini and Gans, 1998).

In MIMO systems, the transmitter uses more than one antenna to transmit the data and the receiver uses more than one antenna to receive these data. The capacities achieved by MIMO systems are very high compared with the conventional systems (SISO, SIMO and MISO) given that the underlying channel is rich of scatterers with independent spatial fading. This gain in capacity and reliability depends on the number of antennas at both sides, the statistics of the channel and the channel knowledge at the transmitter (Gesbert *et al*., 2003) bandwidth, at the same time and this is the source of capacity gain in spatial multiplexing based MIMO systems. This extra capacity is obtained without increasing the channel bandwidth, or transmitted power.

In this study, we will investigate the capacity of MIMO Rayleigh correlated channels for different configurations and at different correlation coefficients. First we will study the capacity of MIMO systems with equal number of antennas at transmitter and receiver, when the channel is known and when it is unknown at the transmitter. Secondly, MIMO systems capacity with receive diversity also will be studied for both known and unknown channels. Thirdly, MIMO systems with larger transmit array will be used to calculate the capacity for both cases known and unknown channels at the transmitter.

MIMO SYSTEM MODEL

Since MIMO is a narrowband technology (Salous, 2003), a narrowband, flat fading Rayleigh correlated channel is to be considered here.

Single user, with multiple transmit and receive antennas will be considered in this study. The total transmit power is P, where P is independent of the number of antennas at the transmit side. This system is described as follows:

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$$y = Hx + n \tag{1}$$

where, $x = (x_1, x_2,\ldots, x_{Mt})^T$ is the $M_t{\times}1$ complex vector representing the transmitted signal with the power constraint

$$tr\,(E\,(xx^H)) \le P \tag{2}$$

$y = (y_1, y_2\ldots\ y_{Mr})^T$ is the $M_r{\times}1$ complex vector representing the received signal and $n = (n_1, n_2,\ldots, n_{Mr})^T$ is $M_r{\times}1$ complex vector representing the additive white Gaussian noise vector (AWGN) with a zero mean and covariance matrix $\sigma^2_n I_{M_r}$, where I_{M_r} is the $M_r{\times}M_r$ identity matrix. The $(.)^T$, $(.)^H$, tr(.) and E(.) denote transposition, conjugate transpose, trace and expectation, respectively. H is $M_r{\times}M_t$ MIMO channel matrix, whose entries h_{ij} represent the channel response of the channel between jth transmit antenna and the ith receive antenna.

MIMO CHANNEL MODEL

Kronecker model will be used here in this paper to describe the Rayleigh correlated channel. In this model the channel spatial correlation R_H = E [vec(H)vec(HH)] (Shiu *et al*., 2000), where vec(H) denotes the $M_tM_r{\times}1$ vector formed by stacking the columns of H. When the channel is rich with multipath and no LOS component exists, the transmit antennas correlation and receive antennas correlation can be considered independent. In such case, the channel correlation matrix R_H can be decomposed into two correlation matrices, the transmit correlation matrix R_t and the receive correlation matrix R_r, so as $R_H = R_t^{\ T} \otimes R_r$, where $\otimes$ is the kronecker product. Hence the Rayleigh correlated channel can be written as:

$$H = R_r^{1/2}\, H_{i.i.d}\, R_t^{1/2}$$

Where:
R_r = The receive correlation matrix
R_t = The transmit correlation matrix
$H_{i.i.d}$ = The uncorrelated channel matrix

CORRELATION MODEL

The exponential correlation model will be adopted in this study (Loyka, 2001). For this model, the components of the correlation matrices (R_r and R_t) are given by:

$$r_{ij} = \begin{cases} r^{j-1}, & i \le j \\ r_{ji}{}^*, & i > j \end{cases} \qquad |r| \le 1$$

where, r is the complex correlation coefficient of neighboring antenna. This model is suitable for studying the effects of correlation on the channel capacity, although it is not accurate for some real world scenarios. However, this model is physically reasonable, where the correlation between the adjacent antennas is larger than the correlation between none-adjacent antennas (ibid).

MIMO CHANNEL CAPACITY

The theoretical capacity of this system is expressed by the following formula (Telatar, 1999).

$$C = E_H\left[\log_2 \det(I_{M_r} + \tfrac{E_s}{M_t N_0} HQH^H)\right] \tag{3}$$

where, $Q = E[xx^H]$ is the input covariance matrix and E_s is the total transmit power, N_0 is the noise power in each antenna at the receive side.

In Eq. 3, the mean is taken over the random channel. The capacity depends on the number of antennas at both sides, input covariance matrix Q and the channel statistics. When channel H is Rayleigh distributed, its mean will be zero (no LOS component exists) and its covariance is 1.

The Q matrix represents the covariance matrix of the transmitted vector. This matrix is diagonal and its elements are all real numbers. The trace of this matrix should not exceed the number of transmit antennas. In other words, tr (Q) = M_t. There are two cases for this matrix, when the transmitter does not have a prior knowledge about the channel, this channel will be the identity matrix $Q = I_{M_t}$, when the instantaneous channel is available at the transmitter, therefore transmitter can optimize Q matrix to obtain the optimum capacity. We will consider both cases in what follows,

Uninformed transmitter: When the channel is unknown to the transmitter (i.e., uninformed transmitter), but perfectly known to the receiver, the optimum choice is to divide the available transmit power equally among the antenna elements of the transmitter. Assume that the components of the transmitted vector x are statistically independent, meaning that $Q = I_{M_t}$ with Gaussian distribution, then the ergodic capacity reduces to:

$$C = E_H\left[\log_2 \det(I_{M_r} + \tfrac{E_s}{M_t N_0} HH^H)\right] \tag{4}$$

Given that $HH^H = VDV^H$ (Eigen Value Decomposition theorem), the capacity of the MIMO channel can be expressed as

$$C = E_H\left[\ \log_2 \det\left(I_{M_r} + \frac{E_s}{M_t N_0}\ VDV^H\right)\ \right] \tag{5}$$

where, V is an $M_r \times M_r$ matrix (eigenvectors of the channel H) satisfying $V^H V = VV^H = I_{M_r}$ and $D = \text{diag}\{\lambda_1, \lambda_2, \ldots \lambda_{Mr}\}$ with $\lambda_i \geq 0$. The diagonal matrix D comprises the eigenvalues (λ_i) of the channel H. using the identity det $(I_m + AB) = \det(I_n + BA)$ for matrices A $(m \times n)$ and B $(n \times m)$ and $V^H V = I_{M_r}$, Eq. 3 simplifies to

$$C = E_H\left[\log_2 \det(I_{M_r} + \tfrac{E_s}{M_t N_0} D)\right] \tag{6}$$

or equivalently

$$C = E_H\left[\sum_{i=1}^{r} \log_2\left(1 + \frac{E_s}{M_t N_0}\lambda_i\right)\right] \tag{7}$$

where, $r = \text{rank}(HH^H) = \min[M_t, M_r]$ (number of parallel channels) and λ_i $(i = 1,\ldots,r)$ are the positive eigenvalues of HH^H. Equation 5 expresses the capacity of the MIMO channel as the sum of the capacities of r SISO channels, each having power gain λ_i $(i = 1,\ldots, r)$ and transmit power E_s/M_t. It is noticed that all eigenchannels in Eq. 7 are allocated the same power; this is because these eigenchannels are not accessible due to the lack of knowledge in the transmitter, so it just divides the power equally among them.

Informed transmitter: There is a possibility that transmitter learns the channel state information (CSI or channel matrix H) before it transmits the data vector. For instance, in TDD (Time Division Duplexing) systems, the channel matrix can be fed back to the transmitter from the receiver. In such an event, the capacity can be increased by resorting to the so-called waterfilling principle (Paulraj *et al*., 2003), by assigning various levels of transmit power to various transmitting antennas. This power is assigned on the bases that the better the channel is, the more power it gets and vice versa.

WATERFILLING ALGORITHM

When the channel parameters are known at the transmitter, the waterfilling algorithm can be used to maximize the channel capacity by allocating more power to the channels that are in good condition and less or none at all to the bad channels [ibid].

Assuming a narrowband channel, the system can be expressed as in Eq. 1.

Given $H = USV^H$ (Singular Value Decomposition, SVD), now system can be expressed as:

$$y = USV^H x + n \tag{8}$$

where, U is a matrix containing the eigenvectors of the receiver, V is a matrix containing the transmitter eigenvectors and the matrix S is a diagonal matrix containing the singular values (σ_i, where $\sigma_i = \sqrt{\lambda_i}$) of the matrix H. U and V matrices are unitary, satisfying $UU^H = U^H U = I_{mr}$ and $VV^H = V^H V = I_{M_t}$.

The transmitted vector is multiplied by a matrix V prior to transmission to cancel the effect of the matrix V^H contained in H. In the same way, received vector is multiplied by a matrix U^H to cancel the effect of the matrix U contained in H.

$$x' = Vx,\ y' = U^H y,\ n' = U^H n,$$

Substituting these values in Eq. 8, will produce the following

$$y' = Sx' + n' \tag{9}$$

The system modeled by Eq. 9 is representing a group of parallel SISO channels; their power gains are the-none zero diagonal elements of the matrix S.

The capacity of the MIMO channel is the sum of the individual parallel SISO channel capacities and is given by

$$C = E_H\left[\sum_{i=1}^{r} \log_2\left(1 + \frac{E_s \gamma_i}{M_t N_0}\lambda_i\right)\right] \tag{10}$$

where, γ_i is the amount of power transmitted over the eigenvalue λ_i such that:

$$\sum_{i=1}^{r} \gamma_i = M_t \tag{11}$$

Channel capacity maximization implies that transmitter accesses the individual subchannels (the eigenvalues) and allocates variable power levels to them. Hence, the mutual information maximization problem becomes,

$$C = E_H\left[\max_{\sum_{i=1}^{r}\gamma_i} \sum_{i=1}^{r} \log_2\left(1 + \frac{E_s \gamma_i}{M_t N_0}\lambda_i\right)\right] \tag{12}$$

Using Lagrangian method, the optimal energy allocated to each eigenmode is

$$\gamma_i^{opt} = \left(\mu - \frac{M_t N_0}{E_s \lambda_i}\right)_+,\ i=1,2,\ldots\ldots,r \tag{13}$$

and

$$\sum_{i=1}^{r} \gamma_i^{opt} = M_t \tag{14}$$

where, μ is a constant representing the water level and $(x)_+$ implies

$$(x)_+ = \begin{cases} x & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x \prec 0 \end{cases} \tag{15}$$

Now, the optimal energy allocation is found iteratively through the water filling algorithm as described below:

The iteration count p is set to 1 and then the constant μ in Eq. 13 is calculated based on the following formula,

$$\mu = \frac{M_t}{(r-p+1)}\left[1 + \frac{N_0}{E_s}\sum_{i=1}^{r-p+1}\frac{1}{\lambda_i}\right] \tag{16}$$

Using the obtained value of μ from Eq. 14, the power allocated to the ith sub-channel can be calculated using

$$\gamma_i = \left(\mu - \frac{M_t N_0}{E_s \lambda_i}\right), \; i = 1, 2, ..., r-p+1 \tag{17}$$

If the power allocated to the channel with the lowest gain is negative i.e., the term is $\frac{M_t N_0}{E_s \lambda_i}$ greater than μ (the sub-channel is bad), this channel is discarded and by setting $\gamma^{opt}_{r-p+1} = 0$ and the algorithm is rerun with incrementing the iteration account by 1. This algorithm is repeated until all good sub-channels are allocated the optimal power. The capacity of the MIMO channels when the channel is known to the transmitter is at least equal to that obtained when the channel is unknown to the transmitter. Ones the optimal power allocation across the spatial sub-channels is determined, the optimized input covariance matrix Q is now obtained,

$$Q^{opt} = \text{diag}\{\gamma_1^{opt}, \gamma_2^{opt}, ..., \gamma_r^{opt}\} \tag{18}$$

and the Eq. 1 will take the new form

$$C = E_H\left[\log_2 \det(I_{M_r} + \frac{E_s}{M_t N_0} H Q^{opt} H^H)\right] \tag{19}$$

RESULTS AND DISCUSSION

In this study, we study the MIMO channel capacity for different antenna configurations (different array sizes) over the SNR range from 0 to 20 dB. We investigate the capacity when the channel is known and unknown to the transmitter. Monte Carlo simulation technique is used. Channel capacity is calculated at each SNR point by generating 10,000 channel matrixes and taking the average over them. We will consider three cases here

$$Mt = Mr$$

Figure 1-3 show the capacity of (2,2), (3,3) and (6,6) MIMO channels, respectively, for different correlations (0, 0.6, 0.9), when the channel is known and unknown to the transmitter. Figure 1 show that the capacity of the channel is higher when it is known to the transmitter, for the transmitter sends only through the good sub-channels but when the channel is unknown, the transmitter divides the power equally among all the sub-channels, in this case the transmitted power through the bad sub-channels is wasted and does not contribute to the overall channel capacity. However, the extra capacity gained from the channel knowledge at the transmitter begins to disappear at high SNRs. It is also worth noting that the channel knowledge at the transmit side is more

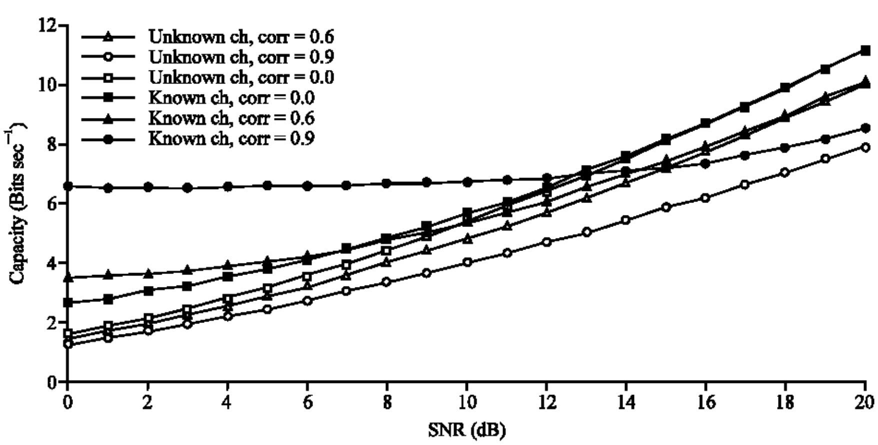

Fig. 1: Ergodic capacity variation with SNR for correlated 2*2 (Mt = 2, Mr = 2) channel

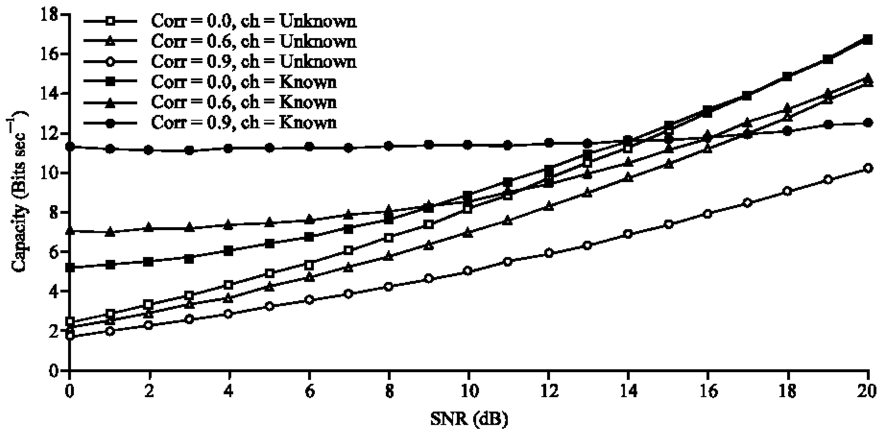

Fig. 2: Ergodic capacity variation with SNR for correlated 3*3 (Mt = 3, Mr = 3) channel

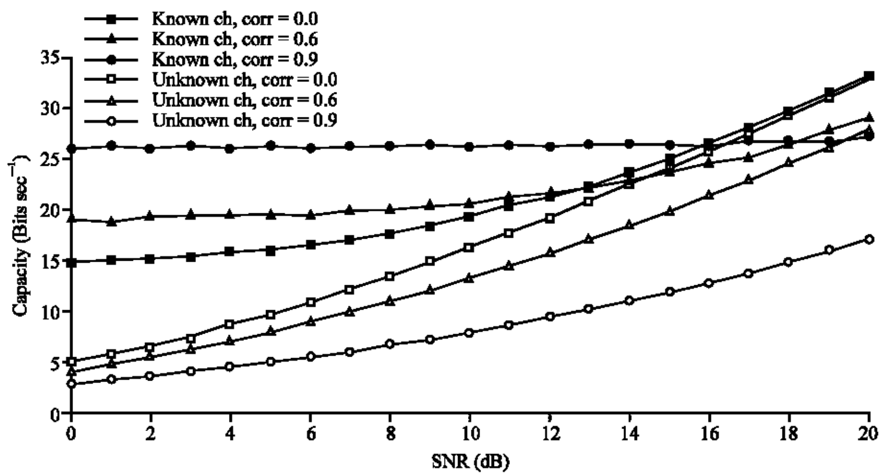

Fig. 3: Ergodic capacity variation with SNR for correlated 6*6 (Mt = 6, Mr = 6) channel

beneficial at the low SNRs, large systems and at high correlations. The figures reflect the significance of channel knowledge at the transmitter.

Mr > Mt

When the number of receive antennas is larger than the transmit antennas, the gain obtained from the channel knowledge at the transmit side becomes less. From Fig. 4, we see that for a (2, 3) channel, the gain in capacity is less than that of (2, 2) channel when the channel is known at the transmitter especially for SNR ≤12 dB. This gain becomes less when we increase the number of receive antennas by 1 and disappear when Mr ever increases, for instance, for (2, 6) channel the capacity of unknown and known channel at the transmitter is almost the same above SNR ≥12 dB (Fig. 5). Increasing the number of receive antennas compensates for the lack of channel knowledge at the transmitter when the channel is low correlated. When the correlation is high (0.9) the simulation shows that capacity decreases when the number of receive array elements increases at the low SNR region (less than 12 dB, Fig. 6).

Mt > Mr

When the number of antennas at the transmit side is larger than those at the receive side, prior knowledge of the channel at the transmit side is very important. The capacity gained from the prior knowledge of the channel at the transmitter is high in low and high SNRs. When the number of transmit antennas gets higher, the gain becomes higher as well. When the number of antennas at the transmit side is larger than the receive side, the number of antennas is also larger than the eigenvalues of the channel. Hence the channel knowledge is very necessary in this case to get better capacity, so the transmitter can get access the right eigenvalues.

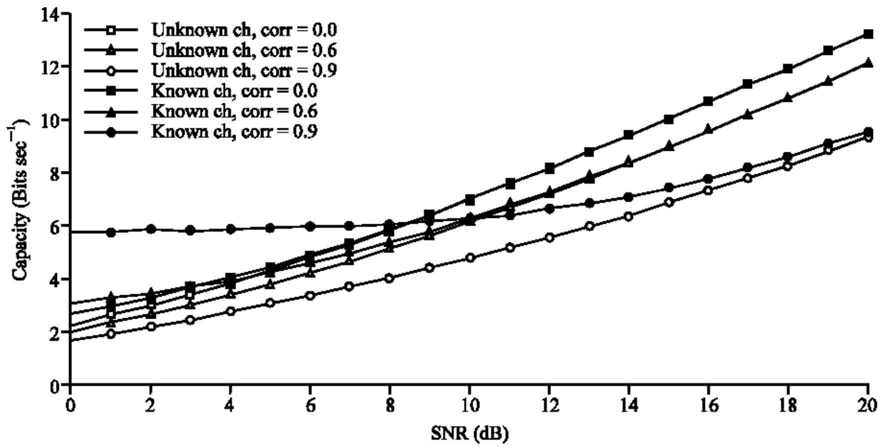

Fig. 4: Ergodic capacity variation with SNR for correlated 2*3 (Mt = 2, Mr = 3) channel

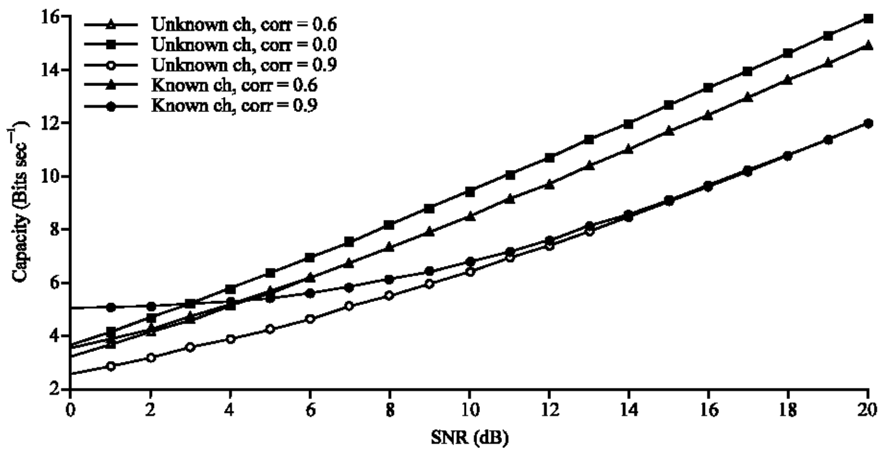

Fig. 5: Ergodic capacity variation with SNR for correlated 2*6 (Mt = 2, Mr = 6) channel

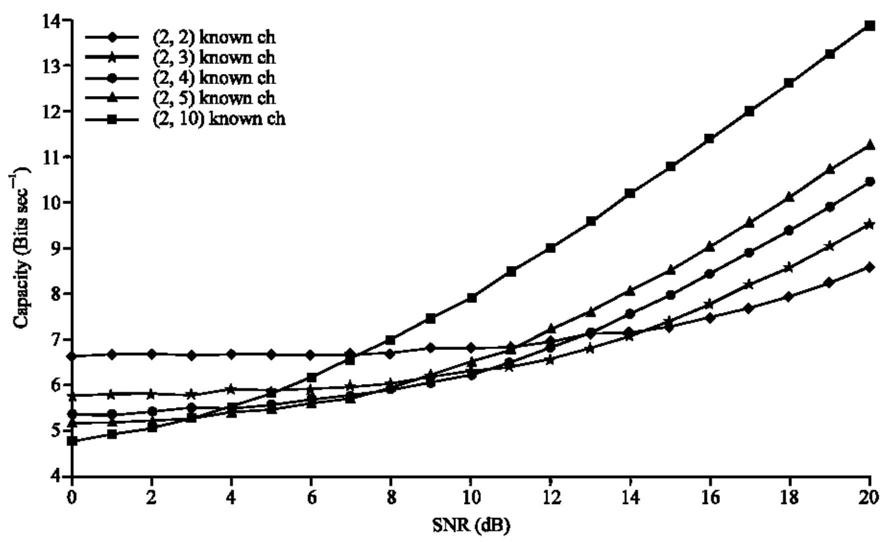

Fig. 6: Ergodic capacity variation with SNR for 0.9 correlation, Mt = 2 and Mr = 2, 3, 4, 5, 10

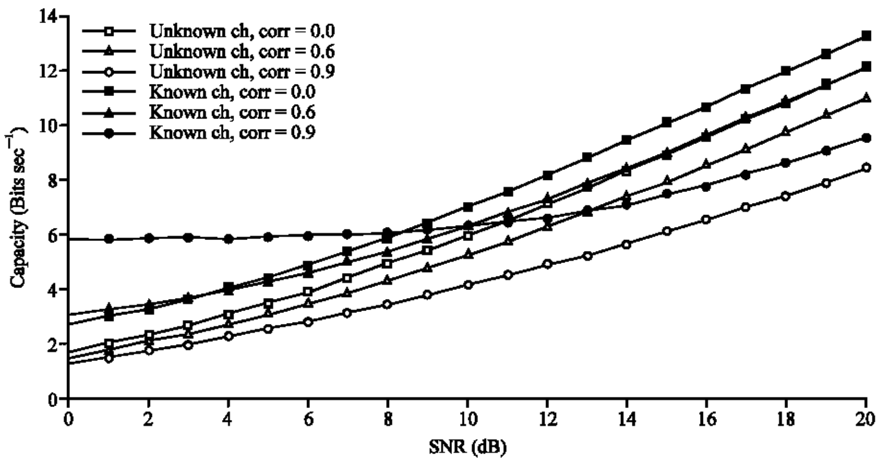

Fig. 7: Ergodic capacity variation with SNR for correlated 3*2 (Mt = 3, Mr = 2) channel

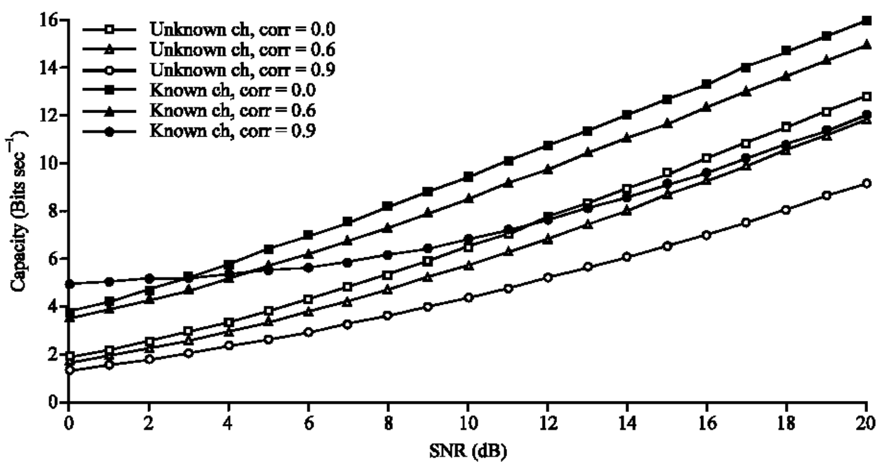

Fig. 8: Ergodic capacity variation with SNR for correlated 6*2 (Mt = 6, Mr = 2) channel

Figure 7 and 8 show the channel capacity when the number of elements at the transmit side is larger than the number of elements in the receive side. In this case, the channel knowledge is very necessary to achieve the maximum capacity.

CONCLUSION

In this study, we have studied the capacity of MIMO channels, with different antenna configurations and different correlations, when the channel is known and unknown to the transmitter. When $M_t \geq M_r$, channel capacity can be improved significantly if the channel is known to the transmitter especially in the low SNR regime, large systems and high correlations. However, in the high SNR regime channel knowledge at the transmitter will not increase the capacity of the channel. When $M_r \geq M_t$, the lack of channel knowledge at the transmitter can be compensated for by increasing the number of antennas at the receive side. It is noted that when the ratio $M_r/M_t \approx 3$, the capacity of known and unknown channel is almost the same for the SNRs greater than 0 dB. When $M_t \geq M_r$, the eigenvalues of the channel will be less than M_t so channel knowledge is necessary for obtaining the optimum capacity.

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