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Maximum Diversity and Full Rank Space-Time Block Code Based on Orthogonal Polynomials

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Abstract: In this research a new coding technique for Space Time Block Code (STBC) in terms of code operator, that could effectively handle the multipath fading (over rayleigh fading channel) is proposed. The proposed coding technique has been built around a set of carefully chosen orthogonal polynomials. The proposed coding scheme exploits the maximum diversity order for a given number of transmit and receive antennas subject to the constraints of having a simple decoding algorithm. The proposed scheme is similar to the generalized STBC. In the simulation work Phase Shift Key (PSK) and Quadrature Amplitude Shift Key (QASK) are used and perfect channel knowledge is assumed. At the receiver end, we use Maximum Likelihood (ML) decoding. This proposed coding technique results in a full diversity code with high coding advantage.

Key words: Code diversity, fading channels, orthogonal polynomials, PSK, QASK

INTRODUCTION

For wireless communication systems, the principal radio design challenges arise from the harsh radio frequency (RF) propagation environment characterized by channel fading, due to diffuse and specular multipath and CoChannel Interference (CCI) due to the aggressive reuse of radio resources. Interleaved coded modulation on transmit and multiple antennas on receive are standard methods used by wireless communication system designers to combat time-varying fading and to mitigate interference. Both are examples of diversity techniques, which can be provided using temporal, frequency, polarization and spatial resources. However, the wireless channel is neither significantly time-variant nor highly frequency selective. Hence the communication engineers consider the possibility of deploying multiple antennas at both the transmitter and receiver to achieve spatial diversity and also provide high performance.

The Space-Time Codes first used in Tarokh *et al.* (1998), achieve coding gain, with inputs mapped on the vectors rather than scalars. Teletar *et al.* (1995) and Foshini and Gans (1998) had shown independently that the rich scattering wireless channel can support higher data rates when multiple antennas are used at the transmitter and the receiver. Alamouti (1998) gave a simple, single symbol decoding STBC for two transmitter antennas from the complex orthogonal design matrix. The simple design rule and an easy decoding technique stimulated a lot of researchers to work in this area and since then many more STBCs, are proposed. Tarokh *et al.* (1999) generalized the Alamouti scheme and gave STBC from Orthogonal design. Then many more complex Orthogonal design matrices with rates less than 1 like those in Su and Xiz (2003) were identified. Sethuraman *et al.* (2003) proposed STBC from diversion Algebra and extension field concepts to identify code matrices. Damen *et al.* (2002) proposed a diagonal algebraic space-time code that by construction are of full rank and of rate 1. But these codes require some parameters to optimize over for diversity and coding gain. Mukkavilli *et al.* (2000) used the equal eigenvalue criterion to maximize the coding gain and identified some codes using this strategy.

The purpose of this research is to construct a new Space-Time Block Codes from Orthogonal Polynomials of full rank, full rate and low complexity. Simulation results conform that, the proposed space-time block code with multiple transmit antennas, a significant performance gain can be achieved with less processing expense, as the proposed code scheme is configured as integer only.

STBC DESIGN AND SYSTEM MODEL

STBC design: In Space Time Block Code design, the essential design criteria are the provided transmit (Tx) diversity, the symbol rate of the code and the delay. The degree of Tx diversity is characterized by the number of

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independently decodable channels. For full diversity it equals the number of transmit antennas. If multiple receive (Rx) antennas are deployed the total diversity degree is the product of the Tx and Rx diversity degrees. The number of Rx antennas is however, irrelevant for the design of orthogonal STBC. The symbol rate of the code is the number of symbols transmitted by the code per time. The delay is the length of STBC frame. Depending on the underlying modulation scheme, the proposed orthogonal polynomial based STBC is aimed to maximize the rate and minimize the delay, keeping the full diversity. In general if transmitting antenna is one and receiving antenna is one then the pairwise error probability is inversely proposanal to signal to noise ratio ($N_t = 1$, $N_r=1$ then PEP α SNR^{-1}). If the transmitting antenna is one and receiving antennas are more than one the pairwise error probability is inversely proposanal to signal to noise ratio of power of N_r ($N_t = 1$, $N_r>1$ then PEP α SNR^{-Nr}) If the transmitting antenna is more than one and receiving antenna is more than one the pairwise error probability is inversely proposanal to signal to noise ratio of power of N_r and N_t. ($N_t > 1$, $N_r>1$ then PEP α SNR^{-NrNt}) and Channel capacity C is defined as min $\{N_t, N_r\}$ log (SNR). So the rate and diversity is only based on STBC.

General STBC model: We consider single-user wireless communication links consisting of N_t transmit antennas and N_r receive antenna. The received symbol r_{jk} can be given as,

$$r_{jk} = \sum_{i=1}^{N_t} h_{jk}\, u_{jk} + n_{jk}$$

Where, $j = 1,2,..,N_r$ denote the receive antenna and $k = 1,2,\ldots T$ the time at which the symbol was sent, u_{ik} is the code symbol transmitted from antenna $I = 1,2,\ldots,N_t$ at time k and h_{ji} the complex channel gain between the i^{th} transmit antenna and the j^{th} receive antenna. The noise symbols n_{jk} are complex Gaussian with mean zero and variance σ^2. In matrix formulation, this system can be represented as,

$$R = U.H + N$$

Where:

H	= The channel matrix of dimension $N_t \times N_r$
U (or) M	= The code matrix of size $T \times N_t$
R	= The received matrix of size $T \times N_r$
N	= The noise matrix of size $T \times N_r$

Here M is the space-time block code matrix. The space-time block codes (STBC) spans a matrix of size $N_t \times T$, where the i^{th} row vector is transmitted by the i^{th} transmit antenna and the t^{th} column vector is transmitted during the t^{th} time slot. We assume quasi-static fading channels where the channel matrix remains constant over the code duration T. Perfect channel estimation at the receiver end is also assumed and the systems have no feedback. Channel estimation is done with training/pilot sequences in regular intervals during the transmission. We focus on full diversity designs that have a simple and effective decoding strategy.

PROPOSED ORTHOGONAL POLYNOMIALS BASED STBC

The proposed space time block coding is considered around a cartezian coordinate separable, blurring, code operator in which the signal I results in the super position of point source of impulse weighted by the value of the object function f. Expressing the object function f in terms of derivatives of the signal function I relative to the cartezian coordinates and time is very useful for analyzing the signal in order to achieve the diversity. Hence, the initial requirements to analyse the diversity may be stated as follows: Since the diversity can be achieved based on the local properties of the signal, a local code operator is required to be devised such that it is cartezian separable and denoising operator. The two dimensional code function M(x,y) can be considered to be a real valued function for $(x,y) \in X \times Y$ where X and Y are ordered subsets of real values. In our case the x is modeled to represent the space and y represents time slot and consisting of a finite set, which for convenience can be labeled as $\{0,1,2\ldots,n-1\}$, the functions M (x, y) reduces to a sequence of functions

$$M(i,t) = u_i(t),\ I=0,\ 1,\ 2,\ \ldots n-1 \tag{1}$$

As shown in Eq. 2 the process of space -time block codes analysis can be viewed as the linear two dimensional transform coding defined by code operator, M (x, y) M (I, t) = u_i (t)

$$\beta'(\zeta,n) = \int_{x \in X} \int_{y \in Y} M(\zeta,x) M(\eta,y) I(x,y) Dxdy \tag{2}$$

Considering both X and Y to be finite set of values $\{0,1,2\ldots,n-1\}$ Eq. 2 can be written in matrix notation as follows

$$|\beta'_{ij}| = (|M| \otimes |M|\ )^t\, |I| \tag{3}$$

Where, the code operator |M| is

$$|M| = \begin{vmatrix} u_0(t_0) & u_1(t_0) & \dots & u_{n-1}(t_0) \\ u_0(t_1) & u_1(t_1) & \dots & u_{n-1}(t_1) \\ & M & \dots & \\ u_0(t_{n-1}) & u_t(t_{n-1}) & \dots & u_{n-1}(t_{n-1}) \end{vmatrix} \quad (4)$$

$\otimes$ is the outer product $|\beta`_{ij}|$ and $|I|$ are the $n^2$ matrices arranged in the dictionary sequence. $|I|$ is the signal to be transmitted and $|\beta`_{ij}|$are the coefficients of transformation.

We consider a set of orthogonal polynomials $u_0(t)$, $u_1(t),\dots,u_{n-1}(t)$ of degrees 0,1,2,..., n-1, respectively. The generating formula for the polynomials is as follows.

$$u_{i+1}(t) = (t-\mu)\, u_i(t) - b_i(n)\, u_{i-1}(t) \text{ for } i \geq 1, \quad (5)$$
$$u_1(t) = t - \mu \text{ and } u_0(t) = 1,$$

Where,

$$b_i(n) = \frac{<u_i, u_i>}{<u_{i-1}, u_{i-1}>} = \frac{\sum_{t=1}^{n} u_i^2(t)}{\sum_{t=1}^{n} u_{i-1}^2(t)}$$

and $$\mu = \frac{1}{n}\sum_{t=1}^{n} t$$

Considering the range of values of t to be t = i, i = 1,2,3,...,n, we get

$$b_i(n) = \frac{i^2(n^2 - i^2)}{4(4_i^2 - 1)}, \mu = \frac{1}{n}\sum_{t=1}^{n} t = \frac{n+1}{2}$$

We construct code operators $|M|$s of different sizes from the above orthogonal polynomials as follows.

$$|M| = \begin{vmatrix} u_0(t_0) & u_1(t_0) & \dots & u_{n-1}(t_0) \\ u_0(t_1) & u_1(t_1) & \dots & u_{n-1}(t_1) \\ & M & & \\ u_0(t_{n-1}) & u_t(t_{n-1}) & \dots & u_{n-1}(t_{n-1}) \end{vmatrix}$$

for $n \geq 2$ and $t_i = i+1$

Note: For the convenience of code operations, the elements of are scaled to make them integers.

Construction of code operator: Here we present the construction of the code operator of size n. It can be noted at this juncture that some of the STBC which are proposed in the literature are related with the code operator of even size only. Also special techniques are devised to use Hadmard matrix for construction of odd size code operator. But our proposed orthogonal polynomial based STBC is designed to have any width. For the sake of computational simplicity, the finite cartezian coordinate set X,Y are labeled as {1,2,3} to model the space and time slots respectively. The code operator in Eq. 4 that defines the linear transform of signals can be obtained as $\otimes$ where M is computed as scaled from Eq. 5 as

$$|M| = \begin{vmatrix} u_0(t_0) & u_1(t_0) & u_2(t_0) \\ u_0(t_1) & u_1(t_1) & u_2(t_1) \\ u_0(t_2) & u_1(t_2) & u_2(t_2) \end{vmatrix} = \begin{vmatrix} 1 & -1 & \frac{1}{3} \\ 1 & 0 & -\frac{2}{3} \\ 1 & 1 & \frac{1}{3} \end{vmatrix} = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix} \quad (7)$$

The set of 9 two dimensional basis operators O_{ij}, $(o \leq i,j \leq 2)$ can be computed as follows.

$$O_{ij} = \hat{u}_i \otimes \hat{u}_j^t$$

Where, $\hat{u}_i$ is the (i +1) st column vector of. The complete set of the basis operators of size (2×2) and (3×3) are given below:

Polynomial basis operators of size (2×2) are

$$\left[O_{00}^2\right] = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \left[O_{01}^2\right] = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$
$$\left[O_{10}^2\right] = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \left[O_{11}^2\right] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Where, $|M| = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Polynomial basis operators of size (3×3) are

$$\left[O_{00}^3\right] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \left[O_{01}^3\right] = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix},$$
$$\left[O_{02}^3\right] = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\left[O_{10}^3\right] = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \left[O_{11}^3\right] = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix},$$
$$\left[O_{12}^3\right] = \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\left[O_{20}^{3}\right]=\begin{bmatrix}1 & 1 & 1\\ -2 & -2 & -2\\ 1 & 1 & 1\end{bmatrix},\left[O_{21}^{3}\right]=\begin{bmatrix}-1 & 0 & 1\\ 2 & 0 & -2\\ -1 & 0 & 1\end{bmatrix},$$

$$\left[O_{22}^{3}\right]=\begin{bmatrix}1 & -2 & 1\\ -2 & 4 & -2\\ 1 & -2 & 1\end{bmatrix}$$

The following symmetric finite differences for estimating partial derivatives at (x,y) position of the signal I are analogous to the eight finite difference operators O_{ij}^{3} excluding O_{00}.

$$\begin{aligned}
&\frac{\partial I}{\partial y}\bigg| x,y=\sum_{i=-1}^{1}\left[I(x-i,y+1)-I(x-i,y-1)\right]\\
&\frac{\partial I}{\partial x}\bigg| x,y=\sum_{i=-1}^{1}\left[I(x+1,y-i)-I(x-1,y-i)\right]\\
&\frac{\partial^2 I}{\partial y^2}\bigg| x,y=\sum_{i=-1}^{1}\left[I(x-i,y-1)-2I(x-i,y)+I(x-i,y+1)\right]\\
&\frac{\partial^2 I}{\partial X^2}\bigg| x,y=\sum_{i=-1}^{1}\left[I(x-i,y-I)-2I(x,y-i,)+I(x+1,y+i)\right]
\end{aligned} \tag{8}$$

and so on
In general.

$$\begin{aligned}
&\frac{\partial^{i+j}}{\partial x^i\,\partial y^j}=\left|O_{ij}\right| \text{and} \frac{\partial^{i+j} I}{\partial x^i\,\partial y^j}=\left(\left|O_{ij}\right|,\left|I\right|\right)\\
&=\beta'_{ij}\, 0\le i,j\le 2 \text{ and } i=j\ne 0
\end{aligned} \tag{9}$$

Where, || indicates the arrangement in dictionary sequence and (,) indicates the inner product and β'_{ij} are the coefficients of the linear transformations defined as follows:

$$|\beta'_{ij}|=|M|^t|I| \tag{10}$$

Where, |M| is the 2-D code operator defined as $|M| = |M|\otimes |M|$.
Theorem : The orthogonal transformation (Eq.10) defined by the orthogonal system |M| is complete.
Proof: We obtain an orthogonal system |H| by normalizing |M| as follows

$$|H|=|M|(|M|^t|M|)^{-½}$$

Consider the following orthonormal transformations

$$|Z| = |H|^t|I| = (|M|^t|M|)^{-½}\ |M|^t|I|= (|M|^t|M|)^{-½}\ |\beta'|$$

Since, |H| is unitary,

$$|I|=|H||Z|=|M|\ |\beta|=\sum_{i=0}^{2}\sum_{j=0}^{2}\beta_{ij}\left|O_{ij}\right|$$

Where

$$(|\ M\ |^t|\ M\ |)^{-1}|\beta'| \tag{11}$$

As per Eq. 11 the signal |I| can be expressed as a linear combination of the 9 basis operators of which $|O_{00}|$ is the local averaging operator and the remaining 8 are finite difference operators (Eq. 9). From Eq. 11, we obtain the completeness relation or Bessel's equlity as follows.

$$\left(|I|,|I|\right)=\left(|Z|,|Z|\right)\text{i.e.,}\sum_{i=0}^{2}\sum_{j=0}^{2}I_{ij}^2=\sum_{i=0}^{2}\sum_{j=0}^{2}Z_{ij}^2 \,\square$$

The code word difference matrix ΔM is defined as the difference between the transmitted code word and the word received by the receiver. Where ΔM has the same structure as the code matrix M shown above and explicit evaluation of the determinant shows that the value will be zero only when the code matrix and received matrix are same. As determinant is non-zero the code difference matrix is full rank and hence satisfies the rank criteria.

Maximum-likelihood decoding: Here we follow the ML decoding as described by (Damen *et al*., 2003) and the same is detailed below. In several communication problems, the received signal is given by a linear combination of the data symbols corrupted by additive noise, where linearity is defined over the field of real numbers. The input-output relation describing such channels can be put in the form of the real multiple input-multiple output (MIMO) linear model

$$Y = Bx + z \tag{12}$$

Where, $x \in R^m$, y, z, $\in R^n$ denote the channel input, output and noise signal and $B\in R^{n\times m}$ is a matrix representing the channel linear mapping. Typically the noise components z_j, j = 1….n are independent and identically distributed zero-mean Gaussian random variables with a common variance and the information signal x is uniformly distributed over a discrete and finite set $C \subset R^m$, representing the transmitter code book. Under such conditions and assuming B perfectly known at the receiver, the optimal detector $g: y \rightarrow \hat{x} \in C$ that minimizes the average error probability.

$$P(e) \overset{\Delta}{=} P(\hat{x} \ne x)$$

is the maximum-likelihood (ML) detector given by

$$\hat{x} = \underset{x\in C}{\arg\min}|y-Bx|^2 \tag{13}$$

For the sake of simplicity, we assume that $C = X^m$, where X is a pulse shift key modulation (PSK) signal set (Proakis, 2002) of size Q,
i.e.,

$$\overset{\Delta}{X}=\{u=2\pi(q-1/Q)\} \tag{14}$$

with $Z_Q = \{0,1,\ldots Q\}$
Under the assumption (14), by applying a suitable translation and scaling of the received signal vector (13) takes on the normalized form

$$\hat{x} = \underset{x \in Z_Q^m}{\arg\min}\,|y-Bx|^2$$

where the components of the noise z have a common variance equal to 1.

SIMULATION RESULTS

Here, we present the simulation results for STBC constructed using the proposed orthogonal polynomial based coding scheme. In our simulation experiment we use the number of symbols to be 300, with SNR ranging from 10 to 22 dB. With a fixed 3 transmitting antennas and one receiving antenna, we simulate the proposed orthogonal polynomial based STBC. For the modulation type PSK, the Bit Error Rate (BER) obtained by the proposed STBC, is plotted against the SNR range and is shown in the Fig. 1. For the modulation type QASK, the bit error rate by the proposed STBC, against the same range of SNR is shown in Fig. 2. In order to measure the performance of our proposed STBC, we also conduct simulation experiments with the same inputs on the GSTBC. The output of the GSTBC is shown in dotted lines in the Fig. 1 and 2. From the outputs, it is evident that the proposed orthogonal polynomial based STBC is superior than GSTBC.

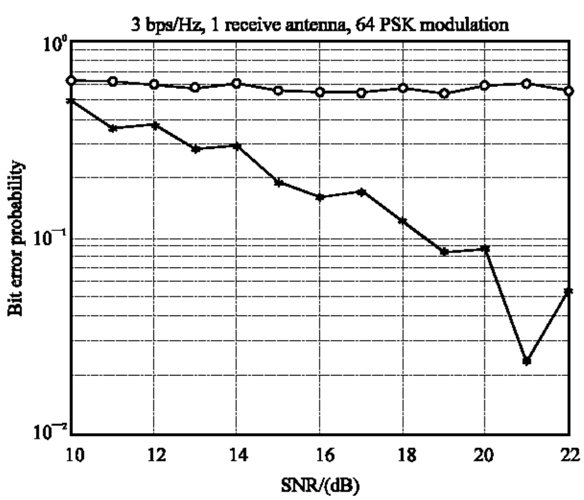

Fig. 1: BER performance comparison of PSK (3Tx and 1 Rx) OPSTB C = --*--, GSTBC = --°--

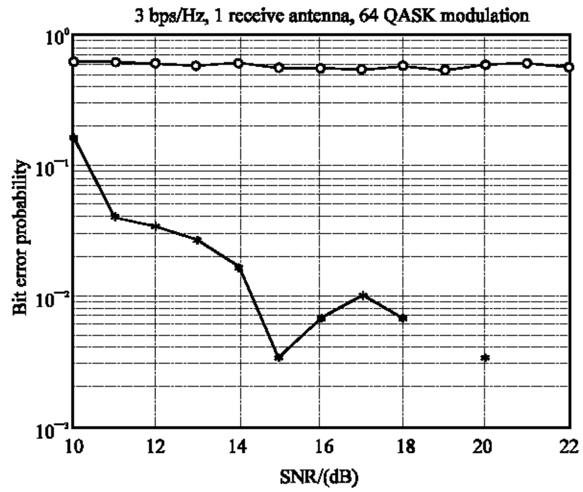

Fig. 2: BER performance comparison of QASK (3Tx and 1 Rx) OPSTB C = --*--, GSTBC = --°--

CONCLUSIONS

In this research, a new space time block coding based on a set of orthogonal polynomials that could effectively handle multipath-fading has been proposed. The proposed coding is of full rank, full rate and low complexity. The proposed coding technique exploits the maximum diversity order for a given number of transmit and receive antennas. The proposed STBC is also compared with the GSTBC. Simulation experiments are conducted with PSK and QASK modulation schemes. At the receiver end the ML decoding is used. It is interesting to note that the proposed STBC is superior to GSTBC in the sense of configuration of code operator of any size (odd or even). The proposed STBC also fully satisfies the code design criteria.

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