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Inelastic Collision of Optical Solitons

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Abstract: In this research, we study a nonsimultaneous three-soliton collision in the presence of third-order dispersion in WDM systems. The interaction between solitons may be viewed as an inelastic collision in which energy is lost to continuous radiation owing to nonzero third-order dispersion. We develop a perturbation theory with two small parameters; the third order dispersion coefficient d_3 and the reciprocal of the interchannel frequency difference, $1/\beta$. In the leading order the amplitude of the emitted radiation after each collision is proportional to d_3/β^2. In addition, the only other effects up to the combined third order of the perturbation theory are phase changes and position shifts of the solitons. It has been shown that after each collision the rate of emitted energy is the same.

Key words: Nonlinear Schrödinger equation, third order dispersion, interchannel interaction, perturbation theory, WDM systems

INTRODUCTION

In recent studies, it has been shown that Wavelength-Division Multiplexing (WDM) can be a successful method to increase the transmission capacity of optical fiber systems (Evangelides and Gardon, 1996; Etrich *et al*., 2001). However, there are some limitations on the performance of these systems. One of the major ones is due to the nonlinear interchannel interaction of data signals from different channels (Agrawal, 2002). The increasing demand for the development of optical communication systems for high data rate transmission and high quality information leads to the increase in the number of channels used and a decrease in the width of the pulses launched. As a result, the importance of the interchannel collisions between optical pulses is expected to increase and an accurate description of the effects of these collisions is required.

In WDM method, solitons at different wavelengths are injected at the input of the fiber. Since, the spectral separation between these solitons (channels) is $\lambda \leq 0.5$ nm and they have different velocities in different channels, collisions among them inevitably take place (Agrawal, 2002). In here, full attention is given to the effect of third-order dispersion on the interchannel collisions between solitons, which is expected to be dominant (in comparison with other inelastic effects) near the zero-dispersion wavelength.

In an ideal fiber, interchannel collisions between solitons can be modeled by using the nonlinear Schrödinger equation (NLSE) (Hasegawa and Matsumoto, 2002). In that case since the collisions are elastic no radiation is emitted. Moreover, by using NLSE the amplitude, frequency and shape of the solitons do not change. Using this model for an ideal collision only leaves us a phase shift proportional to $1/|\beta|$ and a position shift proportional to $1/(|\beta|\beta)$, where, β is the frequency difference between the solitons. In real optical fibers, however, this ideal elastic nature of soliton collisions breaks down owing to the presence of high-order corrections (perturbations), such as third-order dispersion, to the ideal NLSE. In this case, collisions between solitons from different frequency channels might lead to the emission of radiation, change in the soliton's amplitude and frequency, corruption of the soliton's shape, stronger shift in the soliton's position and other undesirable effects.

Accurate analysis of the effects of perturbations on interchannel collisions is a very complicated and long-standing problem. The main technical issue in this case is how to develop a perturbation theory around the multisoliton solutions of the ideal NLSE. In spite of the existence of exact expressions for the multisoliton solutions of the ideal NLSE, direct perturbative analysis around the complex multisoliton solutions has not yet been successfully implemented.

Our aim in this work is to study the effects of the second collision the perturbative soliton from the origin channel (It means a soliton that has shifted its phase and position because of a collision with another soliton from the other channel and partially its energy is lost at continuous radiation) with stationary soliton from the

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other channel. Moreover, we calculate the dynamics and the total intensity of the continuous radiation emitted as a result of two collisions and also the change, induced by the collision, in the soliton parameters. That before is shown (Plege *et al*., 2003) interaction between stationary two-soliton in presence of third order dispersion lead to $O(1/\beta)$ phase shift, $O(1/\beta^2)$ position shift. In addition, the amplitude of the emitted radiation is proportional to $O(d_3/\beta^2)$. It should also be mentioned that the propagation of a single pulse in the presence of third-order dispersion was studied in detail by Kodama, (1985) and Horikis and Elgin (2001). It was found that even if the pulse launched into the fiber is not exactly of the stationary form it evolves into the stationary form after a transient (Elgin *et al*., 1995).

Notice that the major technical tool used in the analytical calculations is singular perturbation theory that is an appropriate extension of the technique developed (Kaup, 1990).

MATERIALS AND METHODS

Propagation of an electrical field wave packet ψ (t, z) through an optical fiber under the influence of third order dispersion is described by the following modification of the nonlinear Schrödinger (Agrawal, 2001)):

$$i\partial_z\psi + \partial_t^2\psi + 2|\psi|^2\psi = id_3\partial_t^3\psi \tag{1}$$

Where, z is the dimensionless position along the fiber, $z = x\,(\kappa p_0/2)$, x is the actual position along the fiber, P_0 is the peak soliton power and κ is the kerr nonlinearity coefficient. The dimensionless retarded time is $t = \tau/\tau_0$, where, τ is the retarded time associated with the reference channel and τ_0 is the soliton width. Figure 1 describe these parameters. The term $id_3\partial^3{}_t\psi$ (where, d_3 is a small constant) on the right-hand side of Eq. 1 accounts for the effect of third order dispersion.

For $d_3 \neq 0$, Eq. 1 is not integrable. However, in many particle cases $d_3 << 1$, allowing a perturbative calculation about the integrable $d_3 = 0$ limit. Fiber losses in Eq. 1 are neglected. In practice, this can be achieved by compensating for losses in a fiber span by means of distributed optical amplification, e.g., Raman amplification.

Let us assume that $d_3 << 1$ and derive perturbatively a z-independent (stationary) single-soliton solution of Eq. 1. When $d_3 = 0$, the single-soliton solution of Eq. 1 in a β frequency channel is described by

$$\psi_\beta(t,z) = \eta_\beta \frac{\exp\,(ia_\beta + i\beta(t - y_\beta) - i(\beta^2 - \eta_\beta^2)z)}{\cosh\,(\eta_\beta(t - y_\beta - 2\beta z))} \tag{2}$$

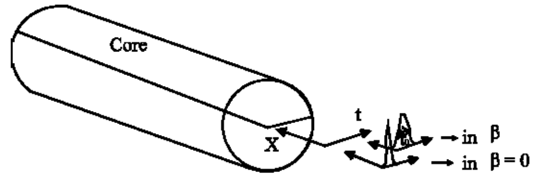

Fig. 1: Propagation two solitons in channels 0 and β at a fiber

Where, α_β, η_β and y_β are the soliton phase, amplitude and position, respectively. Assuming that $d_3 << 1$, we will be looking for a stationary perturbative single-soliton solution of Eq. 1 in the form

$$\psi_\beta(t,z) = e^{i\chi_\beta}\left(\tilde{\psi}_{\beta 0}(h_\beta) + \tilde{\psi}_{\beta 1}(h_\beta) + ...\right) \tag{3}$$

Where:

$$\tilde{\psi}_{\beta 0}(h_\beta) = \frac{\eta_\beta}{\cosh(h_\beta)},$$

$$h_\beta = \frac{\eta_\beta(t - y_\beta - 2\beta(1 + 3d_3\beta/2)z)}{(1 + 3d_3\beta)^{-1/2}}$$

and

$$\chi_\beta = a_\beta + \beta(t - y_\beta) + (\eta_\beta^2 - \beta^2(1 + d_3\beta))z$$

The term $\tilde{\Psi}_{\beta 0}(h_\beta)$ is ideal single-soliton soluton and $\tilde{\psi}_{\beta 1}$ is the first order (in d_3) correction. To calculate this term, we adopt the perturbation method (Kaup, 1990). In Kaup's theory, the differential operator $\hat{L}_\eta$ is used to describe a linear perturbation around the ideal soliton solution. We expand $\tilde{\psi}_{\beta 1}$ in terms of the eigenfunctions of $\hat{L}_\eta$ and calculate the coefficients of this expansion. It is shown in (Plege *et al*., 2003) that $\tilde{\psi}_{\beta 1}$ is stable and localized. Stationary of the solution Eq. 3 means that each of the solitons propagates without any change in their parameters and without shedding any radiation; thus effects of radiation emission and parameter change are due only to soliton collisions. We develop a double perturbation theory with the two small parameters d_3 and $1/\beta$. The perturbation theory presented here is valid for any value of β, provided $d_3 << 1$ and $d_3 << 1 + d_3\beta$. For simplicity andwithout any loss of generality, one of the two channels is chosen as a reference one with $\beta = 0$. We also assume that for the second channel $|\beta| >> 1$.The stationary two-soliton solution Eq. 1 with $d_3 \neq 0$ is calculated (Plege *et al*., 2003). In this study we investigate the effects of the perturbative soliton from the origin channel in order to its collision again with stationary

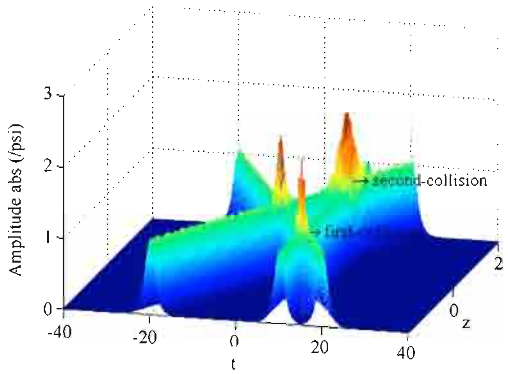

Fig. 2: Non simultaneous, Three-soliton collision: $\eta_2 = \eta_1 = \eta_0 = 1$, $\beta_2 = 10$, $\beta_1 = -10$, $\beta_0 = 0$, $y_2 = 0$, $y_1 = 0$, $y_0 = 10$

soliton from a typical β channel up to the third order of the theory, in which appear in terms of Φ' (Fig. 2). Symbolically we assume the perturbative soliton as an ideal soliton Ψ_0 in which there are some changes in the former one due to the first collision that can be used as initial conditions for the second collision. We are looking for a two-soliton solution of Eq. 1 in the form, $\Psi_{two} = \Psi_0+\Psi_\beta+\Phi'$ where, ψ_0 and ψ_β are described by Eq. 3 with $\beta = 0$ and β, respectively and Φ' is a small correction due to collision. It is straightforward to check that the exact two-soliton solution of Eq. 1 at $d_3 = 0$ acquires the form

$$\Psi_{two} = \Psi_0 + \Psi_\beta + \varphi_0 + \varphi_\beta + \varphi_{-\beta} + \varphi_{2\beta} + O\left(\frac{1}{\beta^3}\right) \qquad (4)$$

Where, Φ_0 and Φ_β are corrections of the leading order $1/\beta$ in the channels 0 and β, respectively. The terms $\Phi_{-\beta}$ and $\Phi_{2\beta}$ correspond to O $(1/\beta^2)$ corrections in channels-β and 2β, respectively; the two latter corrections are exponentially small outside the collision region. By analogy with the ideal $d_3 = 0$ case, one substitutes a solution of the form Eq. 4 into Eq. 1 and calculates Φ'_0. Since Φ'_0 oscillates together with ψ_0 and $d_3 \ll 1$, one neglects the exponentially small contributions from the terms rapidly oscillating with t and z. Then the equation describing Φ'_0 is

$$\begin{aligned}&\partial_z \tilde{\Phi}'_0 - i\left[\left(\partial_t^2-\eta_0^2\right)\tilde{\Phi}'_0 + 4\left|\psi_0\right|^2\tilde{\Phi}'_0 + 2\tilde{\psi}_0{}^2\,\tilde{\Phi}'^*_0\right]\\ &= 4i[\left|\psi_\beta\right|^2\tilde{\psi}_0 + \left|\psi_\beta\right|^2\tilde{\Phi}'_0 + \tilde{\psi}_0\left(\psi_\beta\Phi'^*_\beta + \psi^*_\beta\Phi'_\beta\right)\\ &+\tilde{\psi}_0{}^*\psi_\beta\Phi'_{-\beta} + \frac{1}{2}\psi_\beta^2\Phi'^*_{2\beta} + \tilde{\psi}_0\left|\Phi'_0\right|^2 + \frac{1}{2}\tilde{\psi}_0{}^*\tilde{\Phi}'^2_0 +\\ &\tilde{\psi}_0\left|\Phi'_\beta\right|^2 + \tilde{\Phi}'_0\left(\psi_\beta\Phi'^*_\beta + \psi^*_\beta\Phi'_\beta\right)] + d_3\partial_t^3\tilde{\Phi}'_0\end{aligned} \qquad (5)$$

Where:

$$\tilde{\Phi}'_0 = \Phi'_0\, e^{-i\chi_0}$$

and

$$\tilde{\psi}_0 = \psi_0\, e^{-i\chi_0}$$

Vicinity (in z) of the collision event is given by

$$\left[z_1 - \frac{\tilde{z}}{|\beta|}, z_1 + \frac{\tilde{z}}{|\beta|}\right]$$

Where, $|\beta| \gg \tilde{z} \gg 1$ and is naturally separated from the region before and after the collision. In the collision region, $\tilde{\Phi}'_0$ acquires a fast (with respect to z) change. Since for this region $\Delta z \sim 1/\beta$, the $\partial_z\tilde{\Phi}'_0$ and $\left|\psi_\beta\right|^2\tilde{\psi}_0$ terms give leading contributions to Eq. 5, whereas the $\partial_z\tilde{\Phi}'_0$ term can be neglected together with all the order terms. In the successive orders of the perturbation theory, one should carefully consider contributions coming from terms such as $\partial^2_t\tilde{\Phi}'_0$ and $\partial^3_t\tilde{\Phi}'_0$. In the region before and after the collision, the interaction between the solitons is exponentially small, so that all the interaction terms there can be neglected. Formally, separation into three well-defined regions means that one can replace all the in Eq. 5, except for $\partial^2_t\tilde{\Phi}'_0$, with C $\delta[(\beta(z-z_1)]$, where, δ (z) is the Dirac delta function and the constant C is simply the integral of all these terms over z. This separation results in the three well-formulated Cauchy problems for Φ'_0 in the three regions. The pre-collision region is

$$z_0 + \frac{\tilde{z}}{|\beta|} \le z \le z_1 - \frac{\tilde{z}}{|\beta|}$$

we use this region as

$$z \le z_1 - \frac{\tilde{z}}{|\beta|}$$

Even though the rigorous calculation of the effects of the collision in successive orders of the perturbation theory is quite complicated, the main result can be derived in a straightforward manner by use of just a few equations. The initial condition for $\Phi'^{(0)}_0$ before the second collision is equal to solution of after the first collision in the first order perturbation theory

$$\tilde{\Phi}'_{01}\left(\tilde{h}_0, z \le z_1 - \frac{\tilde{z}}{|\beta|}\right) = i\delta\alpha'^{(0)in}_{01}\,\tilde{\psi}_{00} = -i\delta\alpha^{(0)in}_{01}\,\tilde{\psi}_{00} \qquad (6)$$

Where:

$$\delta\alpha'^{(0)in}_{01} = -\delta\alpha^{(0)in}_{01} = 2\frac{\eta_\beta\left(1+3d_3\beta\right)^{\frac{1}{2}}}{\left(1+3d_3\frac{\beta}{2}\right)|\beta|} \qquad (7)$$

In Eq. 7 and the following equations the superscript in stands for initial values of phase, position, etc., while the superscript out represents final values of the same parameters.

In the collision region Eq. 5 reduces to

$$\partial_z \tilde{\Phi}_{01}'^{(0)} = 4i\left|\psi_{\beta 0}\right|^2 \tilde{\psi}_{00} = \frac{4i\eta_0\eta_\beta^2}{\cosh(h_0)\cosh(h_\beta)^2} \tag{8}$$

Integrating Eq. 8 over the collision region and using the initial condition 6 at

$$z = z_1 - \frac{\tilde{z}}{|\beta|}$$

one arrives at

$$\tilde{\Phi}_{01}'\left(\tilde{h}_0, z \geq z_1 + \frac{\tilde{z}}{|\beta|}\right) = i\delta\alpha_{01}'^{(0)\,out}\,\tilde{\psi}_{00} \tag{9}$$

And

$$\delta\alpha_{01}'^{(0)\,out} = 3\delta\alpha_{01}'^{(0)\,in} = -3\delta\alpha_{01}^{(0)\,in} \tag{10}$$

Comparing Eq. 9 and 6, we see that the only effect of the collision in the first order of the perturbation theory is a change of the soliton phase

$$\Delta\alpha_{01}'^{(0)} = \delta\alpha_{01}'^{(0)\,out} - \delta\alpha_{01}'^{(0)\,in} = \frac{4\eta_\beta\left(1+3d_3\beta\right)^{1/2}}{\left(1+3d_3\beta/2\right)|\beta|} \tag{11}$$

Notice that Eq. 11 is also equal to the result obtained from the first collision.

Calculation of higher order terms requires knowledge of the complete z dependence of $\tilde{\Phi}_{01}'$. To achieve this aim, integrating Eq. 8 from -8 to some general z, one obtains

$$\tilde{F}_{01}'^{(0)}(t,z) = \frac{2i\eta_0\eta_\beta\left(1+3d_3\beta\right)^{-1/2}}{\beta\left(1+3d_3\beta/2\right)\cosh\left(h_0\right)}\left(2-\tanh\left(h_\beta\right)\right) \tag{12}$$

For second order perturbation theory similar to the first order perturbation theory, the initial condition for the $0(1/\beta^2)$ term $\Phi_{02}'^{(0)}$ in the pre-collision region is:

$$\tilde{\Phi}_{02}'^{(0)}\left(\tilde{h}_0, z \leq z_1 - \frac{\tilde{z}}{\beta}\right) = -\frac{1}{2}\left(\delta\alpha_{01}'^{(0)\,in}\right)^2\tilde{\psi}_{00} - \eta_0\delta y_{02}'^{(0)\,in}\tilde{\psi}_{00}' \tag{13}$$

Where:

$$\delta y_{02}'^{(0)in} = -\delta y_{02}^{(0)in} = -2\frac{\eta_\beta\left(1+3d_3\beta\right)^{\frac{1}{2}}}{\left(1+3d_3\frac{\beta}{2}\right)\beta|\beta|} \tag{14}$$

In the collision region the $0(1/\beta^2)$ part of Eq. 5, one can show that the only change in the solitons's parameters comes from the term $i\partial_t^2\tilde{\Phi}_{01}'$:

$$\partial_z\tilde{\Phi}_{02}'^{(0)} = i\partial_t^2\tilde{\Phi}_{01}' = \frac{-4\eta_0^2\eta_\beta^2}{\left(1+3d_3\beta/2\right)\beta}\frac{\tanh(\tilde{h}_0)}{\cosh\left(\tilde{h}_0\right)\cosh^2\left(\tilde{h}_\beta\right)} + i H\eta_0\,\mathrm{sech}(\tilde{h}_0)\tanh^2(\tilde{h}_0) - iH\eta_0\,\mathrm{sech}^3(\tilde{h}_0) \tag{15}$$

Where:

$$H = \frac{4i\eta_0\eta_\beta\left(1+3d_3\beta\right)^{1/2}}{\left(1+3d_3\beta/2\right)\beta\cosh\left(\tilde{h}_0\right)} \tag{16}$$

Integrating over the collision region and using the initial condition (13) at $z = z_1 - \frac{\tilde{z}}{|\beta|}$, one derives

$$\tilde{\Phi}_{02}'^{(0)}\left(\tilde{h}_0, z \geq z_1 + \frac{\tilde{z}}{\beta}\right) = -\frac{1}{2}\left(\delta\alpha_{01}'^{(0)\,in}\right)^2\tilde{\psi}_{00} - \eta_0\delta y_{02}'^{(0)\,out}\tilde{\psi}_{00}' \tag{17}$$

Where:

$$\delta y_{02}'^{(0)\,out} = 3\delta y_{02}'^{(0)\,in} = -3\delta y_{02}^{(0)\,in} \tag{18}$$

and

$$\tilde{\psi}_{00}' = \frac{d\tilde{\psi}_{00}}{d\tilde{h}_0}$$

Because of $\frac{\tilde{z}}{|\beta|} \ll 1$, we neglected result obtained from last two terms on the rhs of Eq. 15.

Comparing Eq. 17 and 13, we see that the only effect of the collision in $1/\beta^2$ order is a position shift (time retardation) give by

$$\Delta y_{02}'^{(0)} = \delta y_{02}'^{(0)\,out} - \delta y_{02}'^{(0)\,in} = 2\delta y_{02}'^{(0)\,in} = \frac{-4\eta_\beta\left(1+3d_3\beta\right)^{1/2}}{\left(1+3d_3\beta/2\right)\beta|\beta|} \tag{19}$$

Also, we shown that after each collision the rate of position shift is the same.

Moreover, we see that the only effect of the collision in $O(d_3/\beta)$ is a $O(1/\beta)$ change of phase, on top of the $O(d_3)$ stationary solution $\tilde{\Psi}_{01}$. This change also is equal to those from the first collision.

Emission of radiation comes from the $O(d_3/\beta^2)$ term. To analyze the $O(d_3/\beta^2)$ correction, we first write it in the form

$$\tilde{\Phi}_{03}'^{(1)} = \tilde{\Phi}_{03}'^{(1)\,NR} + \tilde{\upsilon}_{03}' \tag{20}$$

Where, $\tilde{\upsilon}'_{03}$ is the leading, $O(d_3/\beta^2)$, contribution to the radiation emitted due to the collision and $\tilde{\Phi}'^{(0)\,NR}_{02}$ corresponds to the nonradiative part. The initial collision for $\tilde{\Phi}'^{(1)}_{03}$ is the changes that is caused by the first collision

$$\tilde{\Phi}'^{(1)}_{03} = \tilde{\Phi}'^{(1)\,NR1}_{03} + \tilde{\Phi}'^{(1)\,R}_{03} \qquad (21)$$

The initial condition for $\tilde{\Phi}'^{(1)}_{03}$ is taken to be

$$\begin{aligned}\tilde{\Phi}'^{(1)}_{03}\left(\tilde{h}_0, z \le z_1 - \frac{\tilde{z}}{|\beta|}\right) &= \tilde{\Phi}'^{(1)NR}_{03}\left(\tilde{h}_0, z \le z_1 - \frac{\tilde{z}}{|\beta|}\right)\\ +\tilde{v}'^{in}_{03}\left(\tilde{h}_0, z \le z_1 - \frac{\tilde{z}}{|\beta|}\right) &= i\delta\alpha'^{(1)in}_{03}\tilde{\psi}_{00}\left(\tilde{h}_0\right)\\ -\frac{1}{2}\left(\delta\alpha'^{(0)in}_{01}\right)^2\tilde{\psi}_{01}\left(\tilde{h}_0\right) &- \eta_0\delta y'^{(0)in}_{02}\tilde{\psi}'_{01}\left(\tilde{h}_0\right)\\ +\tilde{v}'^{in}_{03}\left(\tilde{h}_0, z \le z_1 - \frac{\tilde{z}}{|\beta|}\right)\end{aligned} \qquad (22)$$

Where:

$$\begin{pmatrix}\tilde{v}'^{in}_{03}(t,z)\\ \tilde{v}'^{*in}_{03}(t,z)\end{pmatrix} = B\int_{-\infty}^{\infty} ds\left[a_s(z)\psi_s\left(\tilde{h}_0\right) + a_s^*(z)\overline{\psi}_s\left(\tilde{h}_0\right)\right] \qquad (23)$$

i.e., the initial condition contains radiation.

The equation of source term for the emitted radiation $\Phi^{\sim(1)R}_{03}$ is

$$\partial_z\tilde{\Phi}'^{(1)R}_{03} = d_3\partial_t^3\tilde{\Phi}'^{(0)}_{01} \qquad (24)$$

Substituting Eq. 12 into Eq. 24 and integrating over the collision region, one obtains

$$\begin{pmatrix}\tilde{\Phi}'^{(1)R}_{03}\left(t, z_1 + \frac{\tilde{z}}{|\beta|}\right)\\ \tilde{\Phi}'^{(1)R*}_{03}\left(t, z_1 + \frac{\tilde{z}}{|\beta|}\right)\end{pmatrix} = 2iB\frac{1}{\cosh(\tilde{h}_0)} + \begin{pmatrix}\tilde{v}'^{out}_{03}\left(t, z_1 + \frac{\tilde{z}}{|\beta|}\right)\\ \tilde{v}'^{*out}_{03}\left(t, z_1 + \frac{\tilde{z}}{|\beta|}\right)\end{pmatrix} \qquad (25)$$

Where, the coefficient B is defined by

$$B = -\frac{6\eta_0^3\eta_\beta\left(1+3d_3\beta\right)^{1/2}d_3}{\left(1+3d_3\beta/2\right)^2\beta|\beta|} \qquad (26)$$

and

$$\begin{pmatrix}\tilde{\upsilon}'^{out}_{03}\left(t, z_1 + \tilde{z}/|\beta|\right)\\ \tilde{\upsilon}'^{*out}_{03}\left(t, z_1 + \tilde{z}/|\beta|\right)\end{pmatrix} = 2B\int_{-\infty}^{\infty} ds\begin{bmatrix}a_s\left(z_1 + \tilde{z}/|\beta|\right)f_s\left(\tilde{h}_0\right) +\\ a_s^*\left(z_1 + \tilde{z}/|\beta|\right)\overline{f}_s\left(\tilde{h}_0\right)\end{bmatrix} \qquad (27)$$

The functions $f_s(\tilde{h}_0)$ and $\overline{f}_s(\tilde{h}_0)$ are eigenfunctions of operator $\hat{L}_\eta$; The expansion coefficients $a_s(z_1 + \tilde{z}/|\beta|)$ appearing in Eq. 27 are given by

$$a_s\left(z_1 + \tilde{z}/|\beta|\right) = \frac{-i(s+i)^2}{4\cosh(\pi s/2)} \qquad (28)$$

Where, $s = k/\eta_0$. After second collision, we find

$$\begin{pmatrix}\tilde{\upsilon}'^{out}_{03}(t,z)\\ \tilde{\upsilon}'^{*out}_{03}(t,z)\end{pmatrix} = 2B\int_{-\infty}^{\infty} ds\left[a_s(z)f_s\left(\tilde{h}_0\right) + a_s^*(z)\overline{f}_s\left(\tilde{h}_0\right)\right] \qquad (29)$$

Dynamics of the coefficients $a_k(x)$ are given by

$$a_s\left(z \ge z_1 + \tilde{z}/|\beta|\right) = a_s\left(z_1 + \tilde{z}/|\beta|\right)\exp\left(i\,\eta_0^2\left(s^2+1\right)\left(z - z_1\right)\right) \qquad (30)$$

Equations 29 and 30 describe the dynamics of the term $\tilde{\upsilon}'_{03}$, which is the leading contribution responsible for radiation.

The emitted radiation after each collision is of order d_3/β^2. The absolute value of this emission for four different values of $z = z_1$, $z = z_1+1$, $z = z_1+3$ and $z = z_1+7$ are shown in Fig. 3.

Since $\tilde{\upsilon}'_{03} \approx O(d_3/\beta^2)$, the leading contribution to the radiation intensity emitted due to the collision is of order d^2_3/β^4. This contribution give by

$$\begin{aligned}E_0^R &= \int_{-\infty}^{\infty} dt\,\left|\Delta\tilde{v}'_{03}(t,z)\right| = \frac{2\pi B^2}{\eta_0}\\ &\int_{-\infty}^{\infty}\left\{ds|a_s|^2 + [\text{crossterms, oscillating with z}]\right\}\end{aligned} \qquad (31)$$

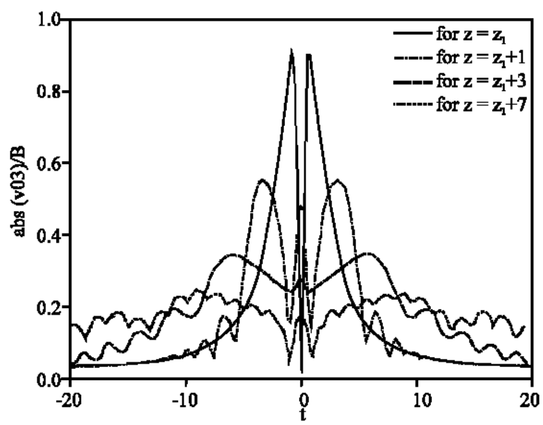

Fig. 3: Absolute value of the radiation profile function normalized to B after the second collision, i.e., $|\upsilon'_{03}|/B = |\tilde{\upsilon}'_{03}|/B$, is shown as a function of t, for four values of z: $z = z_1$(—), $z = z_1+1$ (-·-·-), $z = z_1+3$(- - -), $z = z_1+7$(......)

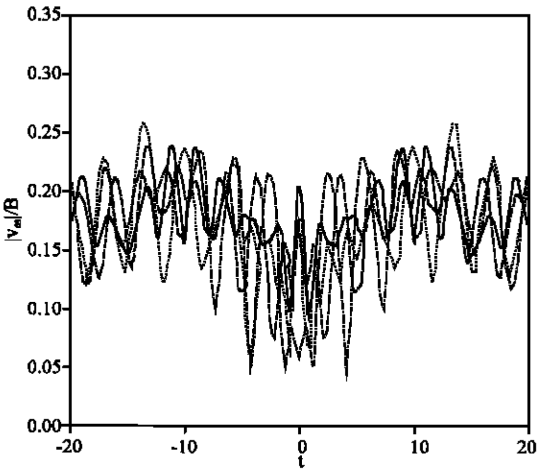

Fig. 4: Absolute value of the radiation profile function normalized to B, i.e., $|\upsilon'_{03}|/B = |\tilde{\upsilon}'_{03}|/B$, is shown as a function of t, for four values of z: $z = z_1+10$ (—), $z = z_1+13$ (- - -), $z = z_1+15$ (▬▬), $z = z_1+17$ (......)

So, according to the Fig. 4, for $z \rangle\rangle z_1+1$, all z dependent contributions to E^R_O decay algebraically with $z-z_1$. Thus, far away from the collision region the only nonvanishing contribution to E^R_O is

$$E_0^R\left(z\rangle\rangle z_1+1\right) = \frac{2\pi B^2}{\eta_0}\int_{-\infty}^{\infty} ds|a_s|^2 = \frac{16B^2}{15\eta_0} \quad (32)$$

Using Eq. 26, we can calculate the radiation energy emitted E^R_O by the reference channel soliton after each collision in far away from the collision region:

$$E_0^R \approx \frac{192\eta_0^5\eta_\beta^2(1+3d_3\beta)d_3^2}{5(1+3d_3\beta/2)^4\beta^4} \quad (33)$$

RESULTS AND DISCUSSION

We finds that the only changes in the pulse parameters up to the third order of the theory are the $O(1/\beta)$ phase shift, $\Delta\alpha_0 \sim 4\eta_\beta(1+3d_3\beta)^{1/2}[(1+3d_3\beta/2)|\beta|]^{-1}$ and the $O(1/\beta^2)$ position shift, $\Delta y_0 = -4\eta_\beta(1+3d_3\beta)^{1/2}[(1+3d_3\beta/2)^2\beta|\beta|]^{-1}$.

Thus, the radiations propagate away from the soliton (in t) with velocity, is of the first order. The soliton retains its shape (such that, at each instant, the soliton is close to a stationary solution, width phase anaphase velocity) while evolving slowly. Note that the rate of emitted energy after each collision is always the same. Also, the rate of changes soliton parameters after each collision is always the same. Moreover, neglecting the decrease in the soliton amplitude, the total energy emitted by the reference channel soliton as a result of many collisions with solitons from the β channel grows linearly with the number of collisions. Taking $\eta_0 = 1$ and requiring that the widths of the colliding solitons are equal, we obtain $\eta_\beta = (1+3d_3\beta)^{1/2}$ with the assumption that (typical setup for a short pulse optical fiber experiment) $\tau_0 = 0.5$ ps, $\beta_2 = -1$ ps^2 km^{-1}, $\beta_3 = 0.1$ ps^3 km^{-1}, $d_3 = 0.07$, $\beta = 10$, $p_0 = 0.4$ W, $\kappa = 10$ W^{-1} km^{-1}, $\Delta\nu = 2.03\times10^{12}$ Hz. Thus for the parameters introduced, we calculated that the mean distance passed by the soliton until it experiences 20000 collisions and loses about 10% of its energy is approximately 2500 km (Plege *et al*., 2003). The soliton amplitude and phase velocity do not acquire any change up to third order of dispersion in WDM systems. The result for the soliton amplitude is consistent with the conservation law for the total energy, which requires $\eta = 1+O(d^2_3/\beta^4)$ for both solitons. Results obtained from the second collision the perturbative soliton from origin channel with stationary soliton from other channel at third order perturbation collision is similar with ones after the first collision the ideal soliton from origin channel with stationary soliton from other channel. Moreover, after the second collision at this order of theory, we obtained also, the $O(1/\beta)$ phase shift on top of the $O(d_3)$ stationary solution $\tilde{\Psi}_{01}$. An interesting feature of the collision is that the leading contributions to the observed effects come from terms in the equations that involve $\tilde{\Phi}'_{01}$. Thus, the leading, $O(1/\beta)$, contribution to the phase shift, which is due to the term $4i|\Psi_{\beta 0}|^2\Psi_{00}$ is simply given by $\tilde{\Psi}_{01}$. Then, the leading, $O(1/\beta^2)$, contribution to the position shift is due to the $\partial_t^2\tilde{\Phi}'_{01}$ term. Finally, the leading, $O(d_3/\beta^2)$, contribution to the radiation emission is due to the $d_3\partial_t^3\tilde{\Phi}'_{01}$ term that does not exist in the ideal two-soliton collision problem.

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