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Channel and Frequency Offset Estimation for MIMO-OFDM Systems

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Abstract: In this study, we present a new reduced-complexity scheme for Maximum-Likelihood (ML) estimate of both Carrier-Frequency Offset (CFO) and channel coefficients in multi antenna OFDM transmission, assuming that a training sequence is available. Our scheme is also capable to accommodate any Space-Time Coded (STC) transmission. Moreover, to benchmark the performance of the proposed scheme, the Cramer-Rao Bounds (CRBs) are derived for both CFO and channel estimators. Simulation results show that the proposed scheme achieves almost ideal performance compared with the CRBs in all ranges of Signal-to-Noise Ratios (SNR) for both channel and frequency offset estimates.

Key words: Carrier frequency offset, channel impulse response, maximum-likelihood estimation, MIMO, orthogonal frequency division multiplexing

INTRODUCTION

High data-rate wireless access is demanded by many applications. Traditionally, more bandwidth is required for higher data-rate transmission. However, due to spectral limitations, it is often impractical or sometimes very expensive to increase bandwidth. In this case, using multiple transmit and receive antennas for spectrally efficient transmission is an alternative solution. Multiple transmit antennas can be used either to obtain transmit diversity, or to form Multiple-Input Multiple-Output (MIMO) channels.

The combination of MIMO signal processing with Orthogonal Frequency-Division Multiplexing (OFDM) has gained considerable interest in recent years (Stuber *et al*., 2004; Li *et al*., 2002). MIMO offers extraordinary throughput without additional power consumption or bandwidth expansion (Bolckei *et al*., 2002) and OFDM introduces overlapping but orthogonal narrowband subchannels to convert a frequency selective fading channel into a non-frequency selective one. Moreover, OFDM avoids Inter-Symbol Interference (ISI) by means of Cyclic Prefix (CP) (Engels, 2002). Hence, in the presence of frequency selectivity, it is beneficial to consider MIMO in the OFDM context (Lu *et al*., 2002).

Like single antenna OFDM, MIMO-OFDM is very sensitive to frequency synchronization and channel estimation errors (Ma *et al*., 2005). Carrier Frequency Offset (CFO) induced by the mismatches of local oscillators in transmitter and receiver causes Inter-Carrier Interferences (ICI), which may result in significant performance degradation. Several carrier frequency synchronization schemes for MIMO-OFDM systems are reported in the literature (Schenk and van Zelst, 2003; Priotti, 2004; Sun *et al*., 2005). Moreover, the coherent detection of MIMO-OFDM signals requires channel estimation to mitigate amplitude and phase distortions in a fading channel. Various channel estimation algorithms are also proposed for MIMO-OFDM systems (Ma *et al*., 2005; Wang *et al*., 2005; Minn and Al-Dhahir, 2006). In dealing with channel estimation, most investigators assume zero frequency offset between the carrier and the local reference at the receiver. In practice, this means that the offset is so small that the demodulated signal incurs only negligible phase rotations. Using stable oscillators is not a viable route to meet such conditions for; in general, the stability requirements would be too stringent. Furthermore, even ideal oscillators would be inadequate in a mobile communication environment experiencing significant Doppler shifts. The only solution is to measure the CFO accurately (Morelli and Mengali, 2000). The combination of CFO and channel estimation leads to particularly complex problems in MIMO-OFDM systems due to the number of unknowns (Ma *et al*., 2005).

More recently, joint channel and frequency offset estimation issue have received a lot of attentions (Ma *et al*., 2003; Cui and Tellambura, 2004) in OFDM context. The exact Maximum-Likelihood (ML) solutions of both frequency offset and Channel Impulse Response (CIR) is prohibitively complex. Therefore, in (Ma *et al*., 2003), the ML estimate for only frequency offset was obtained based on the Least Square (LS) CIR estimate. In

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(Cui and Tellambura, 2004), an adaptive approach (i.e., steepest descent algorithm) was employed to avoid the complexity of joint ML estimation, where firstly the channel estimation is performed assuming that the frequency offset is known and then the frequency offset is estimated assuming the channel state is known. However, all of these estimators (Ma *et al*., 2003; Cui and Tellambura, 2004) are designed for SISO-OFDM systems rather than MIMO systems.

In this study, we present a new reduced-complexity scheme for ML estimate of both CFO and CIR in multi antenna OFDM transmission, assuming that a training sequence is available. As we shall see, the solution consists of two separate steps: a CFO estimator and a channel estimator. It is known that the Expectation-Maximization (EM) algorithm (McLachlan and Krishnan, 2000; Moon, 1996) can provide the ML solutions in an iterative manner for ML estimation problems (Georghiades and Han, 1997; Feder and Weinstein, 1998). Therefore, to overcome the difficulty of ML estimation of CFO, we resort to the EM algorithm and propose a novel EM-based CFO estimator (first step). The CFO estimates are then exploited in the second step to estimate the MIMO channel coefficients. Moreover, to benchmark the performance of the proposed scheme, the Cramer-Rao Bounds (CRBs) are derived for both CFO and CIR estimators.

SYSTEM MODEL

We consider a MIMO-coded OFDM communication system with K subcarriers, N transmit and M receive antennas, signaling through frequency-selective fading channels in the presence of frequency offset. The system model is showed in Fig. 1. It is assumed that the fading channel processes are slowly time variant, such that the fading coefficients are assumed to remain constant during each OFDM word (one time slot) but it varies from one OFDM word to another and the fading processes associated with different transmit-receive antenna pairs are uncorrelated. (However, in a typical OFDM system, for a particular transmitter-receiver antenna pair, the fading processes are correlated in both frequency and time).

At the receiver, the signals are received from M receive antennas. Accordingly, the MIMO OFDM system model subject to CFO is given as (Sun *et al*., 2005).

$$y_i[p] = F(\varepsilon)W_K^I X[p]Wh_i[p] + z_i[p] \qquad i = 1,2,...,M, p = 1,..., P \tag{1}$$

where, the normalized frequency offset ε is presented in the matrix $F(\varepsilon)$ given by:

$$F(\varepsilon) = \mathrm{diag}(1, e^{j(2\pi\varepsilon/K)},..., e^{j(2\pi\varepsilon(K-1)/K)})_{K\times K} \tag{2}$$

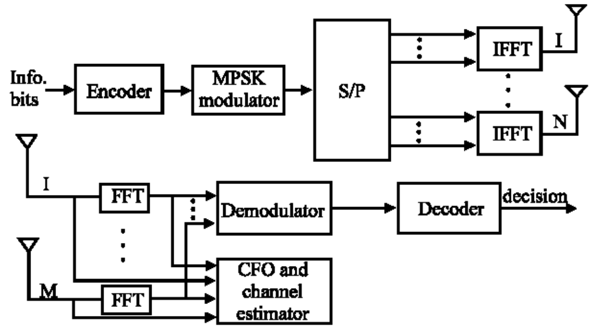

Fig. 1: MIMO-OFDM system model

and

$$y_i[p] = \left[y_i[p,0],..., y_i[p, K-1]\right]^T_{K\times 1} \tag{3}$$

$$\left[W_K^I\right]_{r,s} = \frac{1}{\sqrt{K}} e^{j(2\pi rs/K)} (r,s = 0,..., K-1) \tag{4}$$

$$X[p] = [X_1[p], X_2[p],..., X_N[p]]_{K\times N.K} \tag{5}$$

$$X_j[p] = \mathrm{diag}\left[x_j[p,0],..., x_j[p,K-1]\right]_{K\times K} \tag{6}$$

$$W = \mathrm{diag}\left[w,..., w\right]_{NK\times NL} \tag{7}$$

$$[w]_{r,s} = e^{-j(2\pi/K)rs} (r = 0,..., K-1 ; s = 0,..., L-1) \tag{8}$$

$$h_i[p] = \left[h_{i,1}^T[p],..., h_{i,N}^T[p]\right]^T_{NL\times 1} \tag{9}$$

$$h_{i,j}[p] = [h_{i,j}(p,0),..., h_{i,j}(p, L-1)]^T_{L\times 1} \tag{10}$$

Note that in Eq. 1 the additive white Gaussian noise (AWGN) vector $Z_i[p]$ contains independent zero-mean complex Gaussian random variables with power σ^2_{Zi}.

ML ESTIMATION OF CFO AND CIR

In practical OFDM applications, data transmission is organized in frames and training blocks (carrying known symbols) are located at the beginning of each frame. Our idea is to simultaneously make use of training symbols for both CFO and CIR estimation. Therefore, in the sequel to this study, we concentrate on a training block and omit the temporal index p for notational simplicity.

The transmission model in (1) contains two unknown parameters: the CFO ε and the channel parameters h_i (i = 1,..., M). The ML estimates of ε and, h_i are given by minimizing the following quadratic cost function:

$$\min_{\varepsilon, h_i}\left\{\sum_{i=1}^{M}\left|y_i - F(\varepsilon)Vh_i\right|^2\right\} \quad (11)$$

Where:

$$V = W_K^I X_{training} W \quad (12)$$

Problem of Eq. 11 lies in the fact that we have to estimate two parameters with only one cost function. We propose at first to estimate the parameter ε. The ML estimation of ε leads to the following mathematical development:

$$\begin{aligned}\tilde{\varepsilon} &= \arg\max_{\varepsilon}\sum_{i=1}^{M}\log p(y_i \mid \varepsilon) \\ &= \arg\max_{\varepsilon}\sum_{i=1}^{M}\log\int p(y_i \mid \varepsilon, h_i)\, p(h_i)dh_i\end{aligned} \quad (13)$$

It is seen in Eq. 13 that the direct computation of the optimal ML detection involves multiple-dimensional integral over the unknown random vector h_i and hence, is of prohibitive complexity. Instead of direct computation of Eq. 13, EM-type algorithms provide an iterative and more easily implementable solution. The basic idea of the EM algorithm is to solve problem Eq. 13 iteratively according to the following two steps

Step 1: Expectation (E)-step: Compute

$$Q(\varepsilon \mid \varepsilon^{(k)}) = E\left\{\left[\sum_{i=1}^{M}\log p\left(y_i \middle| \varepsilon, h_i\right)\right]\middle| y_i, \varepsilon^{(k)}\right\} \quad (14)$$

Step 2: Maximization (M)-step: Solve

$$\varepsilon^{(k+1)} = \arg\max_{\varepsilon} Q(\varepsilon \mid \varepsilon^{(k)}) \quad (15)$$

where, $\varepsilon^{(k)}$ denotes the estimated CFO value at the kth EM iteration. It is known that the likelihood function

$$\sum_{i=1}^{M}\log p\left(y_i \middle| \varepsilon^{(k)}\right)$$

is non-decreasing as a function of k and under regularity conditions the EM algorithm converges to a local stationary point (Poor, 1994).

In the E-step, the expectation is taken with respect to the hidden channel response h_i conditioned on y_i and $\varepsilon^{(k)}$. It is easily seen that, conditioned on y_i and $\varepsilon^{(k)}$, h_i is complex Gaussian distributed as (Stoica and Besson, 2003)

$$h_i \left| \left(y_i, \varepsilon^{(k)}\right) \sim N_c\left(\hat{h}_i, \hat{\Sigma}_{h_i}\right), \quad i = 1, \ldots, M \right. \quad (16)$$

with

$$\hat{\Sigma}_{h_i} = \left(V^H F^H(\varepsilon^{(k)}).\Sigma_{z_i}^{-1}.F(\varepsilon^{(k)})V + \Sigma_{h_i}^{\dagger}\right)^{-1} \quad (17)$$

$$\hat{h}_i = \hat{\Sigma}_{h_i}.V^H F^H(\varepsilon^{(k)}).\Sigma_{z_i}^{-1}.y_i \quad (18)$$

where, Σ_{z1} and Σ_{hi} denote respectively the covariance matrix of the ambient white Gaussian noise z_i and channel responses h_i. According to the above assumptions, both of them are diagonal matrices as

$$\Sigma_{z_i} = E\left(z_i.z_i^H\right) = \sigma_{z_i}^2.I \quad (19)$$

$$\begin{aligned}\Sigma_{h_i} &= E\left(h_i.h_i^H\right) \\ &= \mathrm{diag}\left[\sigma_{i,j,0}^2, \ldots, \sigma_{i,j,L-1}^2, \ldots, \sigma_{i,N,0}^2, \ldots, \sigma_{i,N,L-1}^2\right]\end{aligned} \quad (20)$$

where, $\sigma_{i,j,l}^2$ is the average power of the l-th tap between the j-th transmit and i-th receive antennas; $\sigma_{i,j,l}^2 = 0$ if the channel response at this tap is zero. Assuming Σ_{hi} is known, $\Sigma_{h_i}^{\dagger} = \mathrm{diag}[\gamma_{i,1,0}, \ldots, \gamma_{i,1,L-1}, \ldots, \gamma_{i,N,0}, \ldots, \gamma_{i,N,L-1}]$ is defined as the pseudo inverse of Σ_{hi} as

$$\gamma_{i,j,l} = \begin{cases} \dfrac{1}{\sigma_{i,j,l}^2} & \sigma_{i,j,l}^2 \neq 0 \\ 0 & \sigma_{i,j,l}^2 = 0 \end{cases} \quad (21)$$

As shown in Fig. 2, the diagonal elements of $V^H Y$ are equal to K/N. K/N is much larger than $\sigma_{z_i}^2/\sigma_{i,j,l}^2$, which is inversely proportional to the signal-to-noise ratio of the fading channel. Therefore, we can simplify Eq. 17 and 18 to

$$\hat{\Sigma}_{h_i} \approx \sigma_{z_i}^2\left(V^H V\right)^{-1} \quad (22)$$

$$\hat{h}_i \approx \left(V^H V\right)^{-1} V^H F^H(\varepsilon^{(k)})y_i \quad (23)$$

Hence, Eq. 14 reduces to

$$\begin{aligned}Q(\varepsilon \mid \varepsilon^{(k)}) &= -\sum_{i=1}^{M}\left\{E_{h_i|(y_i,\varepsilon^{(k)})}\left\{\left\|y_i - F(\varepsilon)V.h_i\right\|^2\right\}\right\} + \text{const.} \\ &= -\sum_{i=1}^{M}\left\{\left\|y_i - F(\varepsilon)V.\hat{h}_i\right\|^2 + \mathrm{trace}\left(V\hat{\Sigma}_{h_i}V^H\right)\right\} + \text{const.}\end{aligned} \quad (24)$$

The second term is independent of ε and hence has no contribution to the detector. Developing (24) and dropping terms irrelevant of ε, we obtain:

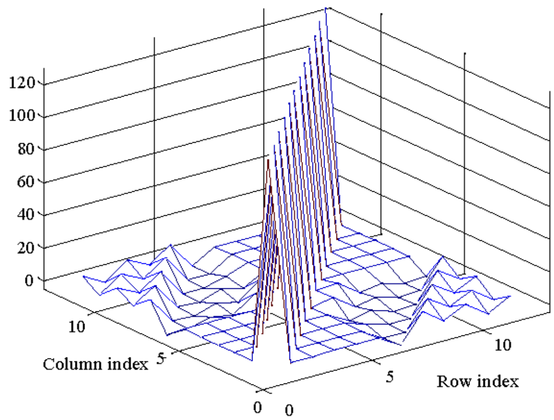

Fig. 2: Evaluation of matrix products V^HV (K = 128, N = 2)

$$Q(\varepsilon \mid \varepsilon^{(k)}) = 2\mathrm{Re}\left(\sum_{i=1}^{M} y_i^H F(\varepsilon)\underbrace{V\hat{h}_i}_{M_i}\right) + \text{const.}$$
$$= 2\mathrm{Re}\left(\sum_{i=1}^{M}\left\{\sum_{n=0}^{K-1} y_i^*(n).M_i(n).e^{j\frac{2\pi n\varepsilon}{K}}\right\}\right) + \text{const.} \qquad (25)$$

By denoting $ri(n) = y^*_i(n).\ M_i(n)$, the final expression for (k+1)-th estimate of ε is obtained as follows:

$$\varepsilon^{(k+1)} = \underset{\varepsilon}{\mathrm{argmax}}\left\{\mathrm{Re}\left(\sum_{i=1}^{M} R_i(\varepsilon)\right)\right\} \qquad (26)$$

Where:

$$R_i(\varepsilon) = \sum_{n=0}^{K-1} r_i(n).e^{j\frac{2\pi n\varepsilon}{K}}$$

can be easily computed using the FFT algorithm. Better frequency resolution can be achieved by zero-padding to effectively increase the length of the symbol sequence.

When $\tilde{\varepsilon}$ has been obtained, it is straightforward, using (23), to estimate h_i. The LS solution of (23) is also the ML channel estimate. The ML solution can be determined by:

$$\tilde{h}_i = \left(V^H.V\right)^{-1}.V^H F^H(\tilde{\varepsilon}).y_i \qquad (27)$$

CRB ANALYSIS

To benchmark the performance of our estimators, we evaluate the CRBs for the estimation of CFO and CIR. Remember that for each received antenna we have: $y_i = F(\varepsilon)\ V\ h_i + z_i$ (i = 1, ..., M). Stacking all the different vectors y_i in column to form the vector y and doing the same thing with vectors h_i to obtain h, we add together the contributions of all received antennas:

$$y = \left(I_M \otimes F(\varepsilon)V\right).h + z \qquad (28)$$

with $\otimes$ denoting the Kronecker product.

Let $\eta = [\varepsilon h_R\ h_I]^T$ denote the parameter vector of interest where h_R and h_I stand for the real and imaginary parts of h, respectively. Under all the made assumptions, the received signal y is a complex-valued circularly symmetric Gaussian vector with mean $\mu = (1_M \otimes F(\varepsilon)V)$ and covariance matrix $C_z = \sigma_z^2.I_{MK}$. For this type of problem, the Fisher Information Matrix (FIM) for estimation of $[\eta^T \sigma_z^2]$ is block diagonal i.e., the estimation of σ_z^2 is decoupled from that of η. Therefore, we only consider the FIM for η, which we denote by F. The latter is given by (Stoica and Besson, 2003):

$$F = \frac{2}{\sigma_z^2}\mathrm{Re}\left(\frac{\partial \mu^H}{\partial \eta}\frac{\partial \mu}{\partial \eta^T}\right) \qquad (29)$$

The CRB is obtained as the inverse of the FIM F. Using the same principle as those given in (Stoica and Besson, 2003), we obtain the final expression for CRB as:

$$\mathrm{CRB}(\varepsilon) = (K^2\sigma_z^2/8\pi^2)[h^H\left(I_M \otimes V^H D?\ DV\right)h]^{-1} \qquad (30)$$

with the following definitions:

$$D = \mathrm{diag}(0,1,\ldots,K-1) \qquad (31)$$

$$\Pi = I_K - V\left[V^H V\right]^{-1} V^H \qquad (32)$$

For the derivation of CRB for channel parameter estimation, we obtain (Stoica and Besson, 2003):

$$E[(\hat{h}-h)(\hat{h}-h)^H] \geq \frac{\sigma_z^2}{2}(2\lambda + \gamma^{-1}\beta\beta^H) = \mathrm{CRB}(h) \qquad (33)$$

Where:

$$\lambda = I_M \otimes \left(V^H V\right)^{-1} \qquad (34)$$

$$\gamma = h^H I_M \otimes \left[V^H D \Pi D V\right] h \qquad (35)$$

$$\beta = I_M \otimes \left[\left(V^H V\right)^{-1} V^H D V\right] h \qquad (36)$$

RESULTS AND DISCUSSION

The characteristics of the fading channels, specifically; the system performance is simulated in Typical Urban (TU) channel with six equal-power taps. In the following simulations, the available bandwidth is 1 MHz and is divided into 128 subcarriers. These correspond to a subcarrier symbol rate of 7.8 KHz and OFDM word duration of 128 μsec . In each OFDM word, a cyclic prefix interval of 32 μsec is added to combat the effect of ISI, hence, the duration of one OFDM word 160 μsec . For all simulations, two transmitter antennas and two receiver antennas are used. The information symbols are drawn from a Quaternary Phase-Shift Keying (QPSK) constellation. The simulated system transmits data in a burst manner. Each data burst includes 10 OFDM blocks. A preamble is applied at the beginning of each data burst for synchronization purposes. MIMO channel estimates are also drawn from the preamble. The performance of CFO estimator is evaluated with the mean-square error (MSE) of the estimated frequency offset. In the case of channel estimation parameters, $E\left(\left\|h-\hat{h}\right\|^2\right)$ is plotted.

Example 1 (performance of CFO estimator): First, we test the effect of the number of EM iterations on CFO estimation. The CFO is randomly selected in the range [-0.1, 0.1]. To start the iteration of proposed algorithm, we set the initial estimate $\varepsilon^{(0)} = 0$. In Fig. 3, the MSE performance of the CFO estimator versus the number of EM iterations is depicted. As it can be seen the MSE performance of the proposed estimator converges after three iterations at the SNR of 5 dB, while it is improved continually until the number of iterations increases to eleven for the SNR values greater than 20 dB.

In Fig. 4, we depict the MSE performance versus SNR. The curves denoted by EM Iter#4 and EM Iter#11 show the CFO estimator performance after the fourth and eleventh EM iteration, respectively. The CRB derived in (30) is also shown as a bench mark. As indicated in (30), CRB values are varying according to the channel state (Morelli and Mengali, 2000) and for a given SNR, the solid lines labeled Min/Max CRB indicate the minimum and maximum CRB obtained in 10^5 simulation runs. However, in high SNR above 20 dB, the proposed scheme needs eleventh iterations to maintain the improved performance between CRBs in all ranges of SNR. From Fig. 4, it is also deduced that proposed frequency estimator outperforms the one in (Sun *et al*., 2005) in all ranges of SNR.

Example 2 (performance of channel estimator): In this example, we test the performance of MIMO channel

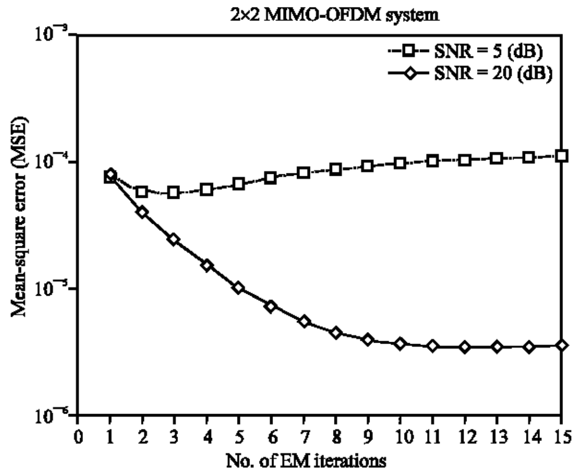

Fig. 3: MSE performance of the CFO estimator versus number of EM iterations for a 2×2 MIMO-OFDM communication system

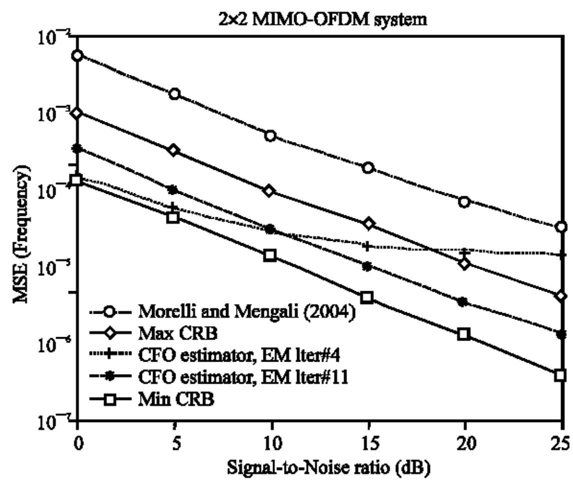

Fig. 4: MSE of CFO estimation versus SNR for a 2×2 MIMO-OFDM communication system

estimation with (N,M) = (2,2) and CFO being also randomly selected in the range [-0.1, 0.1]. In Fig. 5, the simulated MSE performances of channel estimator in (27) are presented and compared with the CRB in (33) and ideal case, where the CFO is perfectly known. It is seen that the channel estimator of the proposed algorithm shows almost ideal performance in all ranges of SNR. To study the disturbance effects of CFO on channel estimation performance, we also plot the MSE of channel estimator without estimating/compensating CFO, in curve denoted by Without CFO E/C. It is not surprising to see the significant performance loss due to CFO.

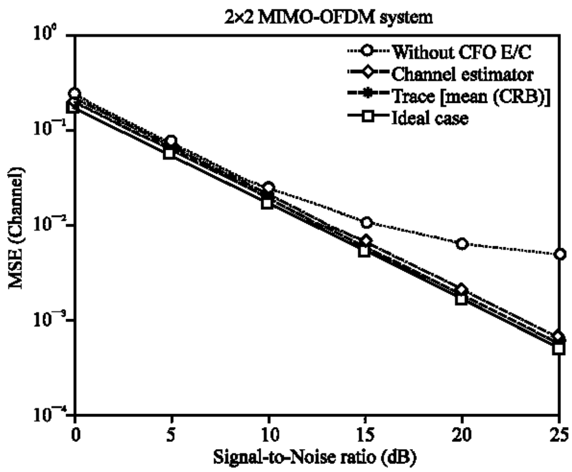

Fig. 5: Channel estimation performance of a 2×2 MIMO-OFDM communication system

CONCLUSIONS

The problem of estimating the frequency offset and channel coefficients in multi antenna OFDM transmission was investigated in this study. The frequency estimates were obtained using an ML approach and the EM algorithm was employed to reduce the computational complexity of ML solution. The CFO estimates were then exploited to estimate the MIMO channel responses. The performance of our estimators was benchmarked with CRBs and investigated by computer simulation. Simulation results show that the proposed algorithm achieves almost ideal performance compared with the CRBs for both channel and frequency offset estimations.

The possible directions for the future research include verifying the performance of our frequency offset and channel estimators for different space-time techniques such as the Bell labs layered space-time (BLAST) (Foschini, 1996), use of powerful coding schemes such as the low density parity check (LDPC) and the turbo codes (Berrou and Glavieux, 1996) and finally, extending the algorithm to the multi-user scenarios.

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