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Optimum Production Control and Workforce Scheduling of Machining Project

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Abstract: Through the proposed model in this study, the production control with the consideration of workforce scheduling for advanced manufacturing systems becomes realistically and concretely solvable. This study not only meditates the concept of balancing machine productivity and human ability into the objective, but also implements Calculus of Variations to optimize the profit for a deterministic production quantity. In addition, the optimum solutions of dynamic productivity control and workforce scheduling are comprehensively provided. Moreover, the decision criteria for selecting the optimum solution and the sensitivity analysis of the critical variables are fully discussed. This study definitely contributes the applicable strategy to control the productivity and workforce in manufacturing and provides the valuable tool to conclusively optimize the profit of a machining project for operations research in today's manufacturing industry with profound insight.

Key words: Production control, workforce scheduling, parallel machines, calculus of variations, operations research

INTRODUCTION

For many years, the planning and scheduling of discrete manufacturing systems have mostly focused on the management of machines and the decision making process at the shop-floor level has been addressed by the complete idea in optimizing the usage of machines. Some of the present researchers claim to also manage workforce, but accomplish most of the time a local allocation of operators to machines (Grabot and Letouzey, 2000). For the corporations to efficiently adapt to new technologies, the interactions between tasks and human ability must be simultaneously matched. Therefore, the Just-In-Time (JIT) philosophy (Xiaobo and Ohno, 1997; Hasgül, 2005) is necessarily considered to promote a better balance between the machine and workforce management into this study. However, this is a combinatorial problem. Hence, the workforce is defined as the working hours so that it is introduced to be a continuous function during production into this study. Besides, the cost of labor is relatively small to the operational cost of machines; hence, an upper integer of the optimum workforce will reasonably represent the optimum manpower allocation.

A production line is generally configured by a sequence of workstations and each workstation consists of one or more parallel machines of the same type (Lan and Lan, 2000). As modern Computer Numerical Controlled (CNC) machines are widely used in the computer-based manufacturing systems, the workforce management is then condensed to the viewpoint of material handling rather than machine operating in shifting from workshops to Flexible Manufacturing Systems (FMS) (Wang and Luh, 1996; Kim and Moon, 2007). Therefore, to appropriately compose the manpower scheme for material handling among a group of parallel machines surely becomes consequential for operation. Practically, the loading-unloading workforce on the machine is normally one of the operator's jobs and it is considered fixed. Contrarily, the workforce for raw material and finished parts handling in between the machines and storage varies with the production rate. Thus, the workforce for material and part handling among a group of parallel machines is then mathematically contemplated as a function of production rate into this study.

Many simple models of workforce allocation have been solved with Kuhn's Hungarian Algorithm (Bazaraa *et al*., 1990). However, they all boiled down to assigning a number of workers to a number of jobs. Konno and Ishii (1995) have established a fuzzy analysis to obtain the optimal solution for a complex workforce scheduling problem. But, without the specificity of individual abilities from each worker, the mathematical model will never be adaptable. A modification of genetic annealing was developed through solving the real size manpower allocation problem (Abbound *et al*., 1998; Yong and Yong-quan, 2007); nevertheless, the corresponding performance of the proposed model was merely verified for small size versions and the optimal

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solution might not be obtainable from the real size of human resources. Thus, the average manpower performance is practically presented to extend the applicability in this study.

Production control is often modeled as optimization problems for constructing optimal profits. As the marginal operation cost is a linear increasing function of productivity (Kamien and Schwartz, 1991), the marginal operation cost of the machine is also considered to be a linear increasing function of production rate in this study. It is that the higher production rate results higher operational cost such as machine maintenance and depreciation. For most researches with this viewpoint, the production rate is fixed because of the difficulty in controlling the variable production rate. Nevertheless, through the modern computer-integrated interface to program the feed rate with fixed cutting speed and depth of cut on Computer Numerical Controlled (CNC) machines (Balazinski and Songmene, 1995; Wang *et al*., 2007), the production rate is suited of being dynamically controlled. In addition, while the machines are idle or breakdown, the operation cost is negligible (Lan and Lan, 2000; Chen and Lan, 2001). This is because that the consumption of input resources does not exist and electricity fees of idle machines are relatively small comparing with those of the whole system. Although several models engaged in a profit function were described by Kalir and Arzi (1998), none is related to the workforce. Actually, the overall profit and the productivity are both the most concerned problems confronting manufacturing industry.

The optimal production and workforce control to balance machine productivity and human ability in computer-based manufacturing systems is rebellious and crucial to industrial management. In addition, to complete an order earlier will freeze the capital, raise the inventory cost and indicate the sub-optimal resource utilization. On the other hand, an order accomplished later than the production deadline may lose customers. Therefore, meeting the production deadline is also the most desirable objective of management (Soroush, 1999; Qiu *et al*., 2007). With the reasons above, it is necessary to not only economically solve the workforce scheduling problem for material handling among all parallel machines, but also competently optimize the production rate for a deterministic production quantity and punctually match the production deadline to reach the maximum profit. With the model proposed in this study, this issue becomes realistically and concretely solvable.

ASSUMPTIONS AND NOTATIONS

Assumptions: Before formulating this study, several conditions are assumed. They are described as follows:

- The production project is a continuous machining operation with no breakdown. And, the order quantity is equivalently assigned to each parallel machine.
- No delay or scrapping of parts occurs during the machining process.
- The raw materials are shipped just in time from inventory to manufacturing for all parallel machines.
- All manufactured parts are collected and held in the shop until the whole production quantity is done.
- The marginal operation cost of machines is a linear increasing function of the production rate (Kamien and Schwartz, 1991).
- The operational cost for idle or breakdown machines is negligible (Lan and Lan, 2000; Chen and Lan, 2001). That is, the operation cost is considered only from the parallel machines assigned for production.
- The workforce for material handling is defined as the working hours and considered as a continuous function of production time.
- All products are shipped and sold at a given price immediately at the time the order quantity is done.

Parameters and Notations: Throughout the study, the parameters and notations are used. They are defined and listed as follows:

B	=	Maximum production rate of the machine, which is limited by the maximum machining conditions
$Bx'(t)$	=	Marginal operation cost for each machine at the production rate $x'(t)$, where b is a constant
$Bx'^2(t)$	=	Operational cost for each machine at time t
c	=	Product holding cost for unit part per unit time
c_1	=	Average labor cost of material handling per unit workforce
L	=	Average material handling ability per unit workforce, which denotes the average number of parts that is capable to be handled by unit workforce
M	=	Number of parallel machines assigned for production
p	=	Sales price per unit product
Q	=	Order quantity of the production project
T	=	Production deadline that is given by the customer or production schedule

Decision variables

k(t)	=	Workforce scheduling for material handling at time t, which is defined as working hours for material handling at time t
x(t)	=	Cumulative parts manufactured per unit machine during time interval [0, t]

$x'(t)$ = Production rate per unit machine at time t, which means number of parts manufactured per unit time

MODEL FORMULATION

The production quantity distributed to each parallel machine, Q/m, is not particular to be an integer. Therefore, to meet the production quantity of the machining project, a larger quantity Q + m is necessarily considered for production. In order to practically introduce the JIT philosophy in balancing the productivity and workforce, the constraint $L\int_0^t k(\tau)d\tau = mx(t)$ is also applied for $\forall\tau\in[0,T]$.

In this study, pQ describes the total revenue of the production quantity and $\int_0^T c_1 k(t)dt$ represents the workforce cost during the production period [0, T]. In addition, $m\int_0^T [bx'^2(t)+cx(t)]dt$ express the operation cost and part holding cost during the production period [0,T], respectively. Thus, the mathematical model in achieving the maximum profit and its constraints are then formulated as follows.

$$\max_{x,k}\left\{pQ-\int_0^T c_1 k(t)dt - m\int_0^T [bx'^2(t)+cx(t)]dt\right\}$$

$$\text{st}\quad L\int_0^t k(\tau)d\tau = mx(t)$$

$$0 \le x'(t) \le B$$

$$x(0)=0 \quad \text{and} \quad x(T)=\frac{Q+m}{m}$$

OPTIMAL SOLUTION

Set (x*, k*) to be the optimal solution of the mathematical model. And, assume that the time interval $[0,\tilde{t}]$ is the maximum subinterval of [0, T] to satisfy Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992). There are two feasible situations to be discussed in this study.

Situation 1: $x'(t)$ will never reach the maximum limit B before T.$(\tilde{t}=T)$

The optimum solution for situation 1 is shown as follows:

$$x^*(t)=\frac{c}{4b}t^2+\left(\frac{Q+m}{mT}-\frac{cT}{4b}\right)t \tag{1}$$

$$x^{*\prime}(t)=\frac{c}{2b}t+\left(\frac{Q+m}{mT}-\frac{cT}{4b}\right) \tag{2}$$

$$k^*(t)=\frac{mc}{2bL}t+\left(\frac{Q+m}{TL}-\frac{mcT}{4bL}\right) \tag{3}$$

The detail is described in Appendix A.

With Eq. 2 and 3, it is found that the optimal production rate $x^{*\prime}(t)$ and the optimal workforce scheme are both linear increasing functions of t before touching the maximum limit.

Before discussing the other situation, one PROPERTY is proposed and described as follow:

PROPERTY: If the line $y = x^{*\prime}(t)$ touches the line y = B, two lines should overlap to be y = B from the touch point $\tilde{t}$ to the end point T.

The proof of PROPERTY is discussed in Appendix B.

Situation 2: $x'(t)$ will reach the maximum limit B before T. $(\tilde{t}<T)$

$$\tilde{t}^* = T-\frac{1}{c}\left(pm-2bB-\frac{mc_1}{L}\right) \tag{4}$$

$$x^*(t)=\begin{cases}\frac{c}{4b}t^2+\left(\frac{Q+m}{mT}-\frac{cT}{4b}\right)t & ,\text{if}\ \ t\in[0,\tilde{t}]\\ x(\tilde{t})+B(t-\tilde{t}) & ,\text{if}\ \ t\in(\tilde{t},T]\end{cases} \tag{5}$$

$$x^{*\prime}(t)=\begin{cases}\frac{c}{2b}t+\left(\frac{Q+m}{mT}-\frac{cT}{4b}\right) & ,\text{if}\ \ t\in[0,\tilde{t}]\\ B & ,\text{if}\ \ t\in(\tilde{t},T]\end{cases} \tag{6}$$

$$k^*(t)=\begin{cases}\frac{mc}{2bL}t+\left(\frac{Q+m}{TL}-\frac{mcT}{4bL}\right) & ,\text{if}\ \ t\in[0,\tilde{t}]\\ \frac{mB}{L} & ,\text{if}\ \ t\in(\tilde{t},T]\end{cases} \tag{7}$$

The detail is described in Appendix C.

Decision criteria: With Eq. 4, two possible decision criteria are classified as follows:

- When $pm \le 2bB+\frac{mc_1}{L}$, it means $\tilde{t}\ge T$. This contradicts the assumption of Situation 2. It is that the optimal production rate $x^{*\prime}(t)$ will not touch the maximum limit B within the production deadline T. The optimal solution is situation 1.
- When $pm > 2bB+\frac{mc_1}{L}$, it means $\tilde{t}<T$. This is that the optimal production rate $x^{*\prime}(t)$ will reach the maximum limit B within the production deadline T. The optimal solution is situation 2.

SENSITIVITY ANALYSIS

The sensitivity analyses for situation 1: It is considered that $x^*(t)$, $x^{*\prime}(t)$ and $k^*(t)$ are the decision functions in this case. Q, m, T and L are the relevant parameters in the analysis.

From Eq. 1 and 2, it is claimed that the optimal cumulative products per unit machine $x^*(t)$ and the optimal production rate per unit machine $x^{*\prime}(t)$ are both in inverse proportion to the production deadline T and are both in direct proportion to the order quantity Q. In addition, it is derived from Eq. 3 that the optimal workforce scheduling for material handling $k^*(t)$ is an increasing function of both the order quantity Q and the number of parallel machines m. On the other hand, $k^*(t)$ is a decreasing function of the production deadline T and the average workforce ability L.

The sensitivity analysis on $x^*(t)$, $x^{*\prime}(t)$ and $k^*(t)$ with respect to q, m, T and L for situation 1 is shown in Table 1.

The sensitivity analyses for situation 2: It is considered that $\tilde{t}$, $x^*(\tilde{t})$, $x^{*\prime}(t)$ and $k^*(t)$ are the key variables in this case. p, b, B, Q, T and L are the relevant parameters in the analysis. From Eq. 4, it is claimed that $\tilde{t}$ is directly proportional to the marginal operation constant b, the maximum productivity B and the production deadline T and it is inversely proportional to the sales price per unit product p and the average material handling ability per unit workforce L.

In addition, it is asserted from Eq. 5 and 6 that the optimal cumulative products per unit machine $x^*(t)$ and the optimal production rate per unit machine $x^{*\prime}(t)$ are both increasing functions of the order quantity Q and the maximum production rate B. Moreover, through Eq. 7, the optimal workforce scheduling for material handling $k^*(t)$ is a decreasing function of the production deadline T and the average workforce ability L and it is an increasing function of maximum productivity B and the order quantity Q.

The sensitivity analysis on $\tilde{t}$, $x^*(\tilde{t})$, $x^{*\prime}(t)$ and $k^*(t)$ with respect to p, b, B, Q, T and L for Situation 2 are shown in Table 2.

Table 1: The sensitivity analyses for situation 1

Decision variables	Parameters Q	m	T	L	Reference
$x^*(t)$	+	#	−	#	Eq. 1
$x^{*\prime}(t)$	+	#	−	#	Eq. 2
$K^*(t)$	+	+	−	−	Eq. 3

+: Decision variable is an increasing function of the parameter, −: Decision variable is a decreasing function of the parameter, #: Decision variable depends on the changes of other relevant parameters

Table 2: The sensitivity analyses for situation 2

Decision variables	Parameters p	b	B	Q	T	L	Reference
$\tilde{t}^*$	−	+	+	#	+	−	Eq. 4
$x^*(t)$	#	#	+	+	#	#	Eq. 5
$x^{*\prime}(t)$	#	#	+	+	#	#	Eq. 6
$k^*(t)$	#	#	+	+	−	−	Eq. 7

+: Decision variable is an increasing function of the parameter, −: Decision variable is a decreasing function of the parameter, #: Decision variable depends on the changes of other relevant parameters

CONCLUSION

The interest of productivity and workforce management grows up in manufacturing systems with the necessity of being more and more flexible. The production quantity, number of parallel machines, maximum production rate, operation cost, product holding cost and the average ability of workforce are considered simultaneously in balancing humans and machines to optimize the profit for a manufacturing project. This is an extremely hard-solving and complicated issue and the existing researches are far from giving the satisfactory answer to this viewpoint. However, through our proposed model, the problem becomes practically and concretely solvable.

In addition, with the optimum solution and the decision criteria, the operational scheme on both production control and workforce scheduling is then precisely determined. Therefore, the production planning, production cost estimating and even the contract negotiation can be further approached through this study. With this viewpoint, the applicability of the proposed model is significantly extended.

The production rate and the workforce are both important control factors of a machining project. Besides, the control of machine productivity and the manpower scheduling are also critical for production planners. This study surely generates the idea of managing both machines and humans and also contributes the solution to optimize the overall profit matter.

Future researches for managing the machines and the humans under floating sales prices and variation of production quantities or orders are encouraged in this approach. In sum, this study surely generates a reliable and applicable idea of optimal control to the techniques and also provides a better and practical solution to this field.

Appendix A: The optimum solution for Situation 1.

With the boundary condition

$$x(T) = \frac{Q+m}{m}$$

and the constraint

$$L\int_0^t k(\tau)d\tau = mx(t),$$

the objective function is then rearranged as

$$\max\left\{p[mx(T)-m]-\frac{mc_1x(T)}{L}-m\int_0^T[bx'^2(t)+cx(t)]dt\right\}$$

From Euler Equation (Kamien and Schwartz, 1991; Chiang, 1992), it is found

$$c=\frac{d}{dt}[2bx'(t)]$$

There exists a constant k_1 satisfying

$$x'(t)=\frac{c}{2b}t+k_1 \tag{A1}$$

Integrating Eq. A1 with t, then

$$x(t)=\frac{c}{4b}t^2+k_1t+k_2 \tag{A2}$$

Using the boundary conditions, x(0) = 0 and

$$x(T)=\frac{Q+m}{m}$$

into Eq. A2 separately; we have

$$k_2=0 \tag{A3}$$

$$k_1=\frac{Q+m}{mT}-\frac{cT}{4b} \tag{A4}$$

Applying Eq. A3 and A4 into Eq. A1 and A2, $x^*(t)$ and $x^{*\prime}(t)$ are thus obtained.

Substituting $x^*(t)$ into the constraint $L\int_0^t k(\tau)d\tau = mx(t)$ then $k^*(t)$ is found.

Appendix B: The proof of PROPERTY.

From Eq. 2, $x^{*\prime}(t)$ is a strictly increasing linear function of t. And it holds for any subinterval during [0, T] satisfying $0 \le x^{*\prime}(t) \le B$. Therefore, $x^* \le (t)$ in the time interval $[\tilde{t}, T]$ (shown in Fig. 1) cannot exist because it contradicts to be a decreasing linear function of t, the PROPERTY of is thus verified.

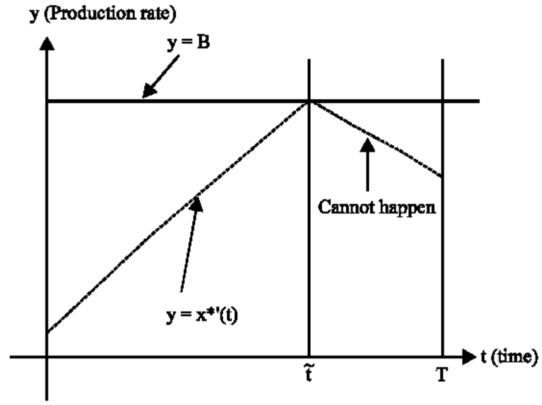

Fig. 1: Possible condition of line y = B and y = x*′(t)

Appendix C: The optimum solution for Situation 2.

Before reaching the maximum limit B, Eq. 1, 2 and 3 are satisfied for this situation either. By applying the PROPERTY into the objective function, it is then modified as:

$$\begin{aligned}-\min\{ &\int_0^{\tilde{t}}\left[bx'^2(t)+cx(t)\right]dt+bB^2(T-\tilde{t})+cx(\tilde{t})(T-\tilde{t})+\frac{cB}{2}(T^2-\tilde{t}^2)\\ &-cB\tilde{t}(T-\tilde{t})-p[mx(\tilde{t})+mB(T-\tilde{t})-m]\\ &+\frac{mc_1}{L}[x(\tilde{t})+B(T-\tilde{t})]\ \}\end{aligned}$$

From the transverality condition of salvage value for free end value $x(\tilde{t})$ (Kamien and Schwartz, 1991; Chiang, 1992), it is obtained that

$$2bx'(\tilde{t})+c(T-\tilde{t})-pm+\frac{mc_1}{L}=0 \tag{C1}$$

Using $x'(\tilde{t})=B$ into Eq. C1, the optimal time $\tilde{t}^*$ to reach the maximum limit is obtained.

Introducing $\tilde{t}^*$ and the PROPERTY into the Eq. 1 and 2, the optimal solution $x^*(t)$ and $x^{*\prime}(t)$ are then found.

Substituting $x^*(t)$ into the constraint $L\int_0^t k(\tau)d\tau = mx(t)$ then $k^*(t)$ is found.

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