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Using the Complete Squares Method to Analysis the Global Optimal Policy for Vendor-Buyer Integrated Inventory System Within Just in Time Environment

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Abstract: The present study was carried out to investigate Yang's model by using an algebraic method (neither applying the first-order nor the second-order differentiations) to determine the optimal replenishment policy. The number of delivery and the integrated total cost is immediately provided by the proposed algebraic derivation as well. As a result, students who are unfamiliar with calculus may be able to understand the solution procedure with ease.

Key words: Integrated inventory system, just in time, algebraic method

INTRODUCTION

A vendor-buyer channel coordinates to achieve better joint profit by optimizing the integrated inventory policy. This vendor-buyer coordination policy has received significant attention among researchers over the past decades. Goyal (1977) first developed an integrated inventory model for a single supplier - single customer problem. Later, Banerjee (1986) developed a joint economic-lot-size model for a single-buyer single-vendor system, where the vendor has a finite production rate. Goyal (1988) extended Banerjee's model by relaxing the lot-for-lot policy and suggested that the vendor's economic production quantity should be an integer multiple of the buyer's purchase quantity, Goyal and Gupta (1989) reviewed related literature, afterward. Since then, Goyal and Srinivasan (1992), Lu (1995), Ha and Kim (1997), Hill (1997, 1999), Goyal and Nebebe (2000), Hoque and Goyal (2000), Wu and Ouyang (2003), Wu *et al.* (2007), Hsiao (2008) and Ben-Daya *et al.* (2008) have been devoted to developing integrated vendor-buyer inventory models under a variety of circumstances. Recently, Yang *et al.* (2007) proposed a global optimal policy for vendor-buyer integrated inventory system within just in time environment.

In a earlier study, Grubbström and Erdem (1999) showed that the formula for the Economic Order Quantity (EOQ) with backlogging could be derived algebraically without reference to derivatives. Cardenas-Barron (2001) extended the mentioned algebraic approach to the Economic Production Quantity (EPQ) model with shortage. Grubbström and Erdem (1999) stated that the algebraic approach had to be considered as a pedagogical advantage for explaining the EOQ concepts to the students who lacked the knowledge of derivatives, simultaneous equations and the procedure to construct and examine the Hessian matrix. Following, there is a vast inventory literature on algebraically method, the outline which can be found in review study by Chou *et al.* (2006), Lai *et al.* (2006), Shyu *et al.* (2006), Leung (2008) and others.

Recently, Yang *et al.* (2007) investigated a global optimal policy for vendor-buyer integrated inventory system within just in time environment using differential calculus to find the optimal solution. In this note, we refer to the algebraic approach method by Grubbstrom and Erdem's (1999) and solve the Yang *et al.* (2007) inventory problem without using derivatives (neither applying the first-order nor the second-order differentiations). Based on this approach, the exact expression of the number of deliveries and integrated total cost in optimum are obtained directly. As a result, students who are unfamiliar with differential calculus may be able to understand the solution procedure with ease.

ASSUMPTIONS AND NOTATIONS

The assumptions made in the research are defined as follows:

- Both the production and demand rate are constant
- The integrated system of single-vendor single-buyer is considered

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- The vendor and the buyer have complete knowledge of each other's information
- Shortage is not allowed

The following notations are used throughout the study:

Q : Buyer's lot size per delivery
n : Number of deliveries from the vendor to the buyer per vendor's replenishment interval
S : Vendor's setup cost per setup
A : Buyer's ordering cost per order
C_v : Vendor's unit production cost
C_b : Unit purchase cost paid by the buyer
r : Annual inventory carrying cost per dollar invested in stocks
P : Annual production rate
D : Annual demand rate
TC : Integrated total cost of the vendor and the buyer when both the vendor and the buyer collaborate instead of being independent

ALGEBRAIC DERIVATION OF THE INTEGRATED INVENTORY MODEL

From Eq. 1 of Yang *et al.* (2007), the vendor's average inventory level I_v is defined:

$$I_v = \frac{Q}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \tag{1}$$

and the integrated total cost of the vendor and the buyer per year is denoted as:

$$TC = \frac{DA}{Q}+\frac{rC_bQ}{2}+\frac{DS}{nQ}+\frac{rC_vQ}{2}\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] \tag{2}$$

Referring to Grubbström and Erdem's (1999) algebraic method, Eq. 2 can be rewritten as:

$$TC = \frac{X}{2Q}\left(Q-\sqrt{\frac{2DY}{X}}\right)^2+\sqrt{2DXY} \tag{3}$$

Where:

$$X = rC_b + rC_v\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right] > 0 \tag{4}$$

and

$$Y = A+\frac{S}{n} > 0 \tag{5}$$

For fixed integer n, Eq. 3 has a minimum value when the quadratic non-negative term is made equal to zero. Therefore, the optimal solution of Q (denoted by Q*) is:

$$Q^* = \sqrt{\frac{2DY}{X}} = \sqrt{\frac{2D\left(A+\frac{S}{n}\right)}{rC_b + rC_v\left[(n-1)\left(1-\frac{D}{P}\right)+\frac{D}{P}\right]}} \tag{6}$$

Substituting Eq. 6 into Eq. 3, we have the optimal integrated total cost for fixed n is:

$$TC^* = \sqrt{2DXY} = \sqrt{2rD\left(A+\frac{S}{n}\right)\left[C_b + C_v\left(n\left(1-\frac{D}{P}\right)+\frac{2D}{P}-1\right)\right]} \tag{7}$$

The results of Eq. 6 and Eq. 7 are the same as Yang *et al.*'s (2007) model.

Next, for convenience, we let:

$$X_1 = C_b + C_v\left(\frac{2D}{P}-1\right) \tag{8}$$

$$Y_1 = C_v\left(1-\frac{D}{P}\right) > 0 \tag{9}$$

Furthermore, in order to find the optimal value of n, the Eq. 7 can be rearranged as:

$$\frac{(TC^*)^2}{2rD} = \left(A+\frac{S}{n}\right)\left[C_b + C_v\left(n\left(1-\frac{D}{P}\right)+\frac{2D}{P}-1\right)\right] = U + Vn + W\frac{1}{n} \tag{10}$$

Where:

$$U = AX_1 + SY_1 \tag{11}$$

$$V = AY_1 = AC_v\left(1-\frac{D}{P}\right) > 0 \tag{12}$$

$$W = SX_1 = S\left[C_b + C_v\left(\frac{2D}{P}-1\right)\right] \tag{13}$$

Then, there are two cases to occur.

Case I: $X_1 \le 0$ (1.e., $Cb+C_v(2D/p-1)\le 0$).

The right hand side of Eq. 10 is a function of n; we denoted it by f(n), that is:

$$f(n) = \frac{(TC^*)^2}{2rD} = U + Vn + W\frac{1}{n}$$

For any two positive integers n_1 and n_2 (where, $n_1<n_2$), we have:

$$f(n_2)-f(n_1)=V(n_2-n_1)+W\left(\frac{1}{n_2}-\frac{1}{n_1}\right)>0$$

Since V>0 and W≤0 (from Eq. 12 and 13). It implies that f(n), or equivalently, $(TC^*)^2$ and thereby TC* is a strictly increasing function of n. Therefore, for optimal solution of n such that TC* has a minimum value, is n* = 1 and thus, from Eq. 6 and 7, we obtain the following results:

$$Q^*=\sqrt{\frac{2D(A+S)}{r\left(C_b+C_v\frac{D}{P}\right)}} \tag{14}$$

and

$$TC^*=\sqrt{2rD(A+S)\left(C_b+C_v\frac{D}{P}\right)} \tag{15}$$

The results of Eq. 14 and 15 are the same as the models proposed by Banerjee (1986) and Yang *et al.* (2007).

Case II: $X_1>0$ (1.e., $C_b+C_v(2D/p-1)>0$).

From Eq. (10), it can be rearranged as:

$$f(n)=\frac{(TC^*)^2}{2rD}=U+Vn+W\frac{1}{n}=U+\frac{V}{n}\left(n-\sqrt{\frac{W}{V}}\right)^2+2\sqrt{VW} \tag{16}$$

where, V>0 and W>0 (from Eq. 12 and 13).
As a result, the value of n that minimizes the Eq. 16 is:

$$n=\sqrt{\frac{W}{V}}=\sqrt{\frac{SX_1}{AY_1}}=\sqrt{\frac{S\left(C_b+C_v\left(\frac{2D}{P}-1\right)\right)}{AC_v\left(1-\frac{D}{P}\right)}} \tag{17}$$

Property 1: If, $X_1 = C_b+C_v(2D/p-1)>0$, TC* is decreasing on (0, n] and increasing on [0, ∞) where $n=\sqrt{W/V}=\sqrt{SX_1/AY_1}$.

Proof: From Eq. 16, for any two positive integers n_1 and n_2 (where, $n_1<n_2$), we have:

$$\begin{aligned} f(n_2)-f(n_1) &= V(n_2-n_1)+W\left(\frac{1}{n_2}-\frac{1}{n_1}\right). \\ &= (n_2-n_1)\left(V-\frac{W}{n_1n_2}\right)=\frac{n_2-n_1}{n_1n_2}(n_1n_2V-W) \end{aligned} \tag{18}$$

When, $n=\sqrt{W/V}=\sqrt{SX_1/AY_1}>n_2>n_1$, we have $\sqrt{W}>n_2\sqrt{V}>n_1\sqrt{V}>0$, i.e., $\sqrt{W}>n_2\sqrt{V}>0$ and $\sqrt{W}>n_1\sqrt{V}>0$. Thus, we obtain $W>n_1n_2V$. Hence, from Eq. 18, we have $f(n_2)-f(n_1)<0$, i.e., $f(n_2)<f(n_1)$. Therefore, f(n), or equivalently, $(TC^*)^2$ and thereby TC* is decreasing on (0, n).

Furthermore, for $n_2>n_1>n=\sqrt{W/V}=\sqrt{SX_1/AY_1}$, we have $n_2\sqrt{V}>n_1\sqrt{V}>\sqrt{W}>0$, i.e., $n_2\sqrt{V}>\sqrt{W}>0$ and $n_1\sqrt{V}>\sqrt{W}>0$, which implies $n_1n_2V>W>0$. Therefore, from Eq. 18, we have ,$f(n_2)-f(n_1)>0$, i.e., $f(n_2)>f(n_1)$. Therefore, f(n), or equivalently, $(TC^*)^2$ and thereby TC* is increasing on [n, ∞). The proof is completed.

Since the value of n is a positive integer, from Property 1, we can obtain the optimal value of n (denote by n*) as:

$$n^*=\begin{cases}[n] & \text{if } TC^*([n])\le TC^*([n]+1),\\ [n]+1 & \text{otherwise.}\end{cases}$$

Now, we can establish the following algorithm to obtain solution of (Q, n).

Algorithm

Step 1: Calculate $X_1 = C_b+C_v(2D/p-1)$. If $X_1>0$, go to Step 2; otherwise, go to Step 3

Step 2: Determine $Y_1 = C_v(1-D/P)$ and then n from Eq. 17 and compute the corresponding $TC^*(\lfloor n\rfloor)$ and $TC^*(\lfloor n\rfloor+1)$ from Eq. 7. If $TC^*(\lfloor n\rfloor)\le TC^*(\lfloor n\rfloor+1)$, then $n^*=\lfloor n\rfloor$; otherwise $n^*=\lfloor n\rfloor+1$. Go to Step 4

Step 3: The optimal solution n is obtained; i.e., n* = 1

Step 4: Substituting n* into Eq. 6 to evaluate Q*

Once n* is obtained the optimal integrated total cost TC*(n*) can be found from 7.

Example 1: In order to show the above solution procedure, let us consider an inventory system with the data as in Goyal (1988) and Yang *et al.* (2007). D = 1,000 units $year^{-1}$, P = 3,200 units $year^{-1}$, C_b = \$25 per unit, C_v = \$20 per setup, A = \$25 per order, S = \$400 per set-up, r = 0.2.

Applying the proposed algorithm, we check the condition:

$$X_1=C_b+C_v(2D/P-1)=25+20((2\times1{,}000)/3{,}200-1)=17.5>0$$

Then, determine $Y_1 = C_v(1-D/P) = 13.75$ and finding the value of n from Eq. 17, we get n = 4.5126. Since Tc (n = 5)-TC (n = 4) = 1,903.287-1,903.943<0, the optimal No. of deliveries is n* = 5. Substituting this value into Eq. 6, we obtain the optimal order quantity Q = 110 units. These results are the same as Yang *et al.* (2007).

Example 2: The data are the same as in Example 1, expect C_v = \$70. Under the parameter values given above, we check the condition:

$$X_1 = C_b + C_v(2D/P-1) = 25 + 70((2\times 1{,}000 - 3{,}200) - 1) = -1.25 < 0$$

Therefore, by using the proposed algorithm, the optimal No. of delivery is $n^* = 1$. Next, utilizing Eq. 14 and 15, we have the optimal order quantity $Q^* = 301$ units and the integrated total cost is \$2,822.9.

CONCLUSION

In this study, without reference to the use of derivatives, we algebraically derive the optimal order quantity and optimal number of delivery for the integrated single-buyer single-vendor inventory. Using this approach, the optimal solution of proposed model can be asserted without the need to apply the first-order and the second-order differentiations. Also, the exact expression of the number of deliveries and integrated total cost in optimum are obtained directly. As a result, students who are unfamiliar with differential calculus may be able to understand the solution procedure with ease.

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