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Developing and Solving a New Model for the Location Problems: Fuzzy-Goal Programming Approach

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Abstract: This study surveys facility location problems and their objectives. For this aim, this problem is solved in the three phases. First, finding the least number of distribution centers (DCs), second, locating them in the best possible location, that this expresses the quality of the DCs locations which is evaluated by studying the value of appropriate attributes affecting the quality of location, so the value of each location is determined by using multi attribute decision making models and finally, finding the minimum costs of locating the facilities. Then regarding the obtained value the functions are formed and with using fuzzy-goal programming, the locations of DCs are determined. In the last phase, locating some agency in a real-world for a cooler factory is determined via., lingo software.

Key words: Location, multi criteria decision making, entropy, fuzzy-goal programming

INTRODUCTION

One of the critical factors for successful production or services organization is marketing. Perhaps, we can say the most important area in marketing is analyzing product's market and establish agents for organization's products in the best locations. This identify organization's products to customers and increase customer's satisfy with increase in quality of product.

For helping organization's management in order to recognize various locations of organization's agents, we can use multi criteria decision methods. Operation research science is one of the largest sciences to solve production, services agriculture, etc. problems.

In a problem relatively similar to that of this study, Hultz *et al*. (1981) studied on multiactivity-multifacility problems and proposed an interactive solution method to compute non-dominated solutions to compare and choose each others. Fortenberry and Mitra (1986), an application of integer goal programming for facility location with multiple competing objectives are addressed. Brandeau and Chiu (1989) present a survey of representative problems that have addressed in location problems (Ghosh and Rushton, 1987). Schilling *et al*. (1993) applies an approximation scheme to generate a set of non-dominated solutions to a bi-objective location problem.

There are several problems that are accepted as classical ones: the point objective problem (Wendell and Hurter, 1973), the continuous multi criteria min-sum facility location problem (Hamacher and Nickel, 1997). Hamacher *et al*. (1999) worked on the network multicriteria median location problem. Ogryczak (1999) applied symmetrically efficient location patterns in a multi criteria discrete location problem.

Nowadays, multiobjective combinatorial optimization (MOCO), provides an appropriate framework to tackle various types of discrete multicriteria problems (Klamroth and Wiecek, 2000). Within this emergent research area several methods are known to handle different problems such as dynamic programming enumeration (Villarreal and Karwan, 1981).

It is necessary to note that most of multi objective combinatorial optimization (MOCO) problems are NP-hard and intractable (Gandibleux *et al*., 2000). In the case of incapacitated plant location problem, the single-objective version is already NP-hard (Krarup and Pruzan, 1983).

In this study, using TOPSIS technique which is one of the most important methods of multi attribute decision making set and entropy for locations indexes weight calculation and also with using heuristic method that is based on fuzzy-goal programming method, we select best location for facility location.

USING MULTI CRITERIA DECISION MAKING MODEL

In recent century, researchers focused on multi criteria decision making for complex decisions. In these decisions, we may use one optimum criterion in replace multi criteria. There are two kinds of these models:

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- Multi objective models
- Multi attribute models

Multi objective models are used for designing and multi attribute models are used for selecting better alternative (Yoon and Hwang, 1981).

But more exact investigations shows that some techniques like TOPSIS and ELECRE are the best for this case, because we have both desirable directions (min and max) (Yoon and Hwang, 1981). However, due to better results of implementation, we have used TOPSIS.

TOPSIS METHOD

TOPSIS assumes that each attribute in decision matrix takes either monotonically increasing or monotonically decreasing utility in other phase, the best alternative should have the shortest distance from ideal solution and farthest from the negative ideal solution (Yoon and Hwang, 1981). The details of each step in TOPSIS method are presented in the following section:

Step 1: Constructing the decision matrix and changing to normalized matrix can be calculated as:

$$r_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^{m} X^2_{ij}}} \qquad j = 1, 2, ..., n \tag{1}$$

Step 2: Constructing the weighted normalized decision matrix (V). The matrix can be calculated by multiplying each column of the matrix R with associated weight W.

$$w = (w_1, w_2, ..., w_n) \tag{2}$$

Step 3: Determining ideal and negative ideal solution. Let the two artificial alternatives which be defined as:

$$A^+ = \left\{(\max_i Vij \middle| j \in J) \text{ and } (\min_i Vij \middle| j \in J') \middle| i = 1, 2, ..., m\right\} \tag{3}$$

$$A^- = \left\{(\min_i Vij \middle| j \in J) \text{ and } (\max_i Vij \middle| j \in J') \middle| i = 1, 2, ..., m\right\} \tag{4}$$

where, $j = 1,2,...,n$ is associated with benefit criteria and $j' = 1,2,...,n$ is associated with cost criteria. Then it is certain that the two created alternatives, A^+ and A^-, indicate the most preferable alternative (ideal solution) and the least preferable alternative (negative ideal solution), respectively.

Step 4: Calculation the separation measure, the separation between each alternative and the ideal alternative can be measured by Euclidean distance:

$$di^+ = \left\{\sum_{i=1}^{m}(vij - vj(max)^2\right\}^{0.5} \tag{5}$$

$$di^- = \left\{\sum_{i=1}^{m}(vij - vj(min)^2\right\}^{0.5} \tag{6}$$

$i = 1,2,...,m$

where, di^+ is the separation of each alternative from the ideal one and d_i^- is the separation from the negative ideal one.

Step 5: Calculating relative closeness to ideal solution:

$$cl_i^- = \frac{d_i^-}{d_i^+ + d_i^-} \qquad 0 \leq cli^- \leq 1 \tag{7}$$

$$cl_i^+ = \frac{d_i^+}{d_i^+ + d_i^-} \qquad i = 1, ..., m \tag{8}$$

It is clear, if cli^+ is close to 1, Vj is close to Vj(max) and $cli^+ = 1$ if Vj = Vj (max).

Step 6: Ranking the preference order based upon cli^- (Yoon and Hwang, 1981). However, here do not use ranked alternatives in our approach and the value of relative closeness is enough to continue the procedure.

CALCULATING WEIGHTS FOR DECISION MATRIX INDEXES WITH ENTROPY TECHNIQUE

Entropy is criteria that have been expressed for unreliability with discrete probability distribution (P_i), this unreliability is as follows (E):

$$E \approx S\{P_1, P_2, ..., P_n\} = -k\sum_{i=1}^{m}[P_i.Lnpi] \tag{9}$$

Which k is a positive constant number: $0 \leq E \leq 1$.

In MADM model, a decision matrix has information that entropy method has been used as criteria for evaluation (Yoon and Hwang, 1981). The decision matrix is as follows (Table 1) (Yoon and Hwang, 1981):

Firstly, we normalized decision matrix as follow:

$$P_{ij} = \frac{rij}{\sum_{i=1}^{m} rij}; \forall i, j \tag{10}$$

Table 1: The decision matrix

A	r_{11}	r_{12}	r_{1n}
	....		
A_n	r_{m1}	r_{m2} .	r_{mn}.

And for each Ej from set P_{ij}, we have:

$$Ej = -k\sum_{i=1}^{m}[Pij.LnPij]; \forall j \tag{11}$$

$$\text{Which} : k = \frac{1}{Lnm}$$

Now, unreliability for each criterion is as follow:

$$dj = 1\text{-}Ej; \forall j \tag{12}$$

Finally, weights of criterias will be:

$$wj = \frac{dj}{\sum_{j=1}^{n} dj}; \forall j \tag{13}$$

If decision maker has a mind judgment (λ_j) as a relative importance for each criterion (j), we can obtain the weights with Entropy method are as follows:

$$wj' = \frac{\lambda jwj}{\sum_{j=1}^{n}\lambda j.wj}; \forall j \tag{14}$$

MULTI OBJECTIVE DECISION MODEL IN RELATED TO LOCATION PROBLEMS AND SOLUTION METHODOLOGY

In general, covering problems and proposed techniques for solving them may be important to model the service facility location problems (Karasakal and Karasakal, 2004). Set covering model is as follows (Francis *et al*., 1992; Mirchandani and Francis, 1990):

$$\begin{aligned} &\min z_1 = \sum_{i=1}^{m} Xi \\ &\sum_{i=1}^{m} aji \geq 1 st: \\ &i = 1, 2, \ldots, m \\ &j = 1, 2, \ldots, n \\ &X_i \in \{0,1\} \end{aligned} \tag{15}$$

where, X_i is a binary variable that is equal to one if the feasible alternative, i is suitable for locating distribution centers, otherwise it is equal to 0.

In this expression A = [a_{ij}] is called covering matrix; is equal to 1 if a potential supportive center located in location i, is able to cover the supported center located in location j.

However, the objective functions of the problem are as follows:

- Minimizing the number of distribution centers or warehouses
- Maximizing the utility of the selected locations
- Minimizing facility location cost

In order to constructing objective function in relation to maximizing the utility of the selected locations, we use cli^{-} which are TOPSIS algorithm results:

$$\min z_1 = \sum_{i=1}^{m} Xi \tag{16}$$

$$\max z_2 = \sum_{i=1}^{m} CiXi \tag{17}$$

$$\min z_3 = \sum_{i=1}^{m} SiXi \tag{18}$$

$$\begin{aligned} st: &\sum_{i=1}^{m} aji \geq 1 \\ &i = 1,2,\ldots,m \\ &j = 1,2,\ldots,n \\ &X_i \in \{0,1\} \end{aligned} \tag{19}$$

where, C_i has been already defined as TOPSIS algorithm result. S_i is facility locating cost for location i.

Constraint Eq. 19 ensures that all of the supported centers are covered. Some optimum heuristics and meta-heuristics like GA have been proposed for solving covering problems. However, LINGO 8.00 as powerful software for solving problems based on the selected method and the size of problem is used. It is in the nature of the MODM problems to have conflicting objectives. Therefore, in this phase we face a MODM problem. Note that the first and second objective functions must be maximized and third objective function must be minimized. For solving this model, firstly we solve model only with first objective function and obtain optimum amount of Z_1 (say Z_1*).

Then we have fixed the value of Z_1 in a constraint to find the value of Z_2 and Z_3, then we repeat this procedure for second and third objective functions. The results are presented in Table 2.

Table 2: The procedure of proposed

	Z1	Z2 Z3	X		
$\max Z1 = \sum_{i=1}^{m} {}_iX_i$	Max	$Z_1^* = a^*$	b	c	...
$\max Z2 = \sum_{i=1}^{m} C_iX_i$	Max	$Z_2^* = a'$	b*	c'	...
$\min Z3 = \sum_{i=1}^{m} S_iX_i$	Min	$Z_3^* = a''$	b''	c*	...

After obtain the optimum of the each objective function, in order to use fuzzy approach, firstly we have to define membership degree for each objectives as follow:

$$\mu(Z_i) = \begin{cases} 0 & Z_i < L_i \\ \dfrac{Z_i - L_i}{U_i - L_i} & L_i \le Z_i < U_i \\ 1 & Z_i \ge U_i \end{cases} \tag{20}$$

Also, for each of objective functions, we defined α_i as follow:

α_i = The percent of utility that each function arrived to optimum amount

Now, we can formulate fuzzy-goal model as follows:

$$\begin{aligned} & \text{MAX}: \sum_{i=1}^{m} w_i \alpha_i \\ & \text{st}: \alpha_i \le \frac{Z_i - l_i}{U_i - L_i} \\ & g_i(x_1, x_2, \ldots, x_m) \ge \text{or} \le b_i \\ & x_i = \{0,1\} \\ & \sum_{i=1}^{m} w_i = 1 \quad 0 \le \alpha_i \le 1; \quad i = 1,2,\ldots,m \end{aligned} \tag{21}$$

With solving this model, optimum amounts of each objective (α_i) and distribution centers locations (x_i) can be obtained. With changing w_i, we can propose various amounts of X_i and α_i to decision maker to select the best X_i as the best locations.

NUMERICAL EXAMPLE

A producer which is producing gassy cooler in suburb of Tehran wants to establish agents in some city centers in the country, in order to products marketing and increasing in sale's amount.

This company selects only 15 cities after survey that be done to establish agents and wants to establish minimum 8 agents in the best cities. For selection the best cities, company selects indexes areas follows:

A : People population in city
B : Level of people life in city
C : Accumulation level and existing similar agent
D : Distance between city and company
E : Road availability for transportation
F : Transportation cost
G : City's weather

Also, the company allocates weight to each index that shows importance of indexes. The allocated indexes are as follows:

$$\lambda_j = w = (0.456, 0.57, 0.474, 0.392, 0.369, 0.364, 0.63) \tag{22}$$

The cities that company candidate for establish agents are:

(1) Tehran, (2) Mashhad, (3) Esfehan, (4) Shiraz, (5) Yazd, (6) Rasht, (7) Qazvin, (8) Ardabil, (9) Kermanshah, (10) Lorestan, (11) Khoozestan, (12) Hormozgan, (13) Golestan, (14) Booshehr and (15) Kohkilooye va booyerahmad

Also, this company is defined costs of establish agents in different cities as Table 3.

The Table 4 is a decision matrix. It is necessary to say, that we can obtain information of this matrix, with getting from similar companies after recognize cities.

For solving this example, firstly weights with entropy technique in respect to defined algorithm should be obtained:

$$wj = \left(\overset{A}{0.053937},\ \overset{B}{0.268462},\ \overset{C}{0.191972},\ \overset{D}{0.091712},\ \overset{E}{0.110571},\ \overset{F}{0.091712},\ \overset{G}{0.191634} \right) \tag{23}$$

Also, the mind judgments of decision maker are as follows:

$$\lambda_j = \left(\overset{A}{0.13}, \overset{B}{0.2}, \overset{C}{0.17}, \overset{D}{0.1}, \overset{E}{0.05}, \overset{F}{0.05}, \overset{G}{0.3} \right) \tag{24}$$

Finally criteria's weights are as follows:

$$w\acute{j} = (\overset{A}{0.041218}, \overset{B}{0.315624}, \overset{C}{0.191843}, \overset{D}{0.053912}, \overset{E}{0.032499}, \overset{F}{0.026956}, \overset{G}{0.337949}) \tag{25}$$

Then, with using w'j and TOPSIS method, we obtain cli^- that show quality of each city as Table 5.

Table 3: Costs of establish agents in different cities

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
350	430	450	530	500	450	400	620	530	510	480	450	540	440	430

Table 4: Decision matrix

Indexes alternatives		A	B	C	D	E	F	G
Tehran	x_1	8	9	8	1	3	1	6
Mashhad	x_2	9	5	4	4	3	4	8
Esfehan	x_3	8	7	6	3	3	3	8
Shiraz	x_4	7	5	4	3	3	3	8
Yazd	x_5	6	7	6	4	3	4	8
Rasht	x_6	6	7	6	3	2	3	4
Qazvin	x_7	5	5	4	2	3	2	6
Ardabil	x_8	7	5	4	4	2	4	2
Kermanshah	x_9	6	3	2	3	3	3	2
Lorestan	x_{10}	7	3	2	3	2	3	2
Khoozestan	x_{11}	6	3	2	4	3	4	8
Hormozgan	x_{12}	5	1	2	5	1	5	8
Golestan	x_{13}	7	5	4	4	2	4	2
Booshehr	x_{14}	4	1	2	5	1	5	8
Kohkilooye va booyerahmad	x_{15}	3	1	2	5	1	5	6

Table 5: The quantities of cli^- for each city

Cl_1^-	Cl_2^-	Cl_3^-	Cl_4^-	Cl_5^-	Cl_6^-	Cl_7^-	Cl_8^-	Cl_9^-	Cl_{10}^-	Cl_{11}^-	Cl_{12}^-	Cl_{13}^-	Cl_{14}^-	Cl_{15}^-
0.655	0.625	0.696	0.626	0.692	0.565	0.569	0.423	0.376	0.375	0.538	0.454	0.423	0.45	0.399

Table 6: Constrain table

	Z1	Z2	Z3	Xi
max Z1:	**15**	**7.872232**	**7160**	**all x_i**
max Z2:	**15**	**7.872232**	**7160**	**all x_i**
min Z3:	**8**	**4.390796**	**3430**	**$x_1, x_2, x_3, x_7, x_{11}, x_{12}, x_{14}, x_{15}$**

Table 7: The quantities of variables

$\alpha_1 = 0.43$	$Z^*_1 = 11.00$
$\alpha_2 = 0.54$	$Z^*_2 = 6.274578$
$\alpha_3 = 0.60$	$Z^*_3 = 4950.00$
$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = x_{11} = x_{12} = x_{14} = x_{15} = 1$	

In this problem, three objectives are as follows:

- Maximizing the number of agents
- Maximizing the utility of the selected locations
- Minimizing facility location cost

CONSTRAINTS FOR THIS PROBLEM

All of the supported cities must be covered with agents. According to company's survey, there are 26 cities that must be covered. Also, minimum 8 agents must be establishing in the best cities. At it was mentioned earlier, firstly, the constrain table is defined as Table 6.

Now, with weights which are proposed from decision maker, we can formulate fuzzy-goal programming for solving this problem:

$$\begin{aligned}
&\text{MAX}: 0.2\times\alpha_1 + 0.3\times a_2 + 0.5\times a_3 \\
&\quad \text{st}: \sum_{i=1}^{15} X_i \geq 15 - 7(1-\alpha_1) \\
&\quad \sum_{i=1}^{15} C_i X_i \geq 7.872232 - 3.481436(1-\alpha_2) \\
&\quad \sum_{i=1}^{15} S_i X_i \leq 3430 + 3730(1-\alpha_3) \\
&\quad 0 \leq \alpha_i \leq 1; \quad i = 1, 2, 3
\end{aligned} \tag{26}$$

After solving this model via LINGO software, the quantities of variables are obtained as Table 7.

Therefore, the cities: Tehran, Mashhad, Esfehan, Shiraz, Yazd, Rasht, Qazvin, Khoozestan, Hormozgan, Booshehr and Kohkilooye va booyerahmad were selected for establish agents.

CONCLUSION

In this study, an algorithm for modeling and solving location problems using fuzzy-goal programming is proposed. Also a case study about finding the locations of the agents with the maximum number of agents, maximum quality of the locations and minimum cost of the locations for increasing in company's sale was investigated and finally the best cities for establishing agents are selected with fuzzy-goal programming. However, in such cases due to the size, complexity and large number of attributes in the problem, it is solved sequentially and in each stage regarding the situation we used different tools and models.

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